The effects of chemical reaction, in which surfactants are produced on the interface of two immiscible fluids, on viscous fingerings in a radial Hele-Shaw flow are numerically investigated. In the present study, influences of reaction rate and dispersion of produced surfactants are evaluated systematically. For the case of high Damköhler number side-branching is preferred. Nevertheless, side-branching is suppressed in the cases associated with low Péclet number of surfactant. The patterns obtained by the simulations qualitatively agree with the observations in the experiments [1].

Viscous Fingering on an Immiscible Reactive Interface with Variation of Interfacial Tension

Governing Equations

\[ \psi \cdot u = 0, \]
\[ \psi p = -\rho u - \frac{2 \psi}{\alpha} \left( [\psi (\psi - \alpha)] \right)^2, \]
\[ \psi \frac{\partial u}{\partial t} + u \cdot \nabla u = \frac{1}{\rho} \nabla \mu, \]
\[ \psi = \frac{\partial H}{\partial \psi} - C \psi^2 \psi, \]
\[ \Delta \psi + u \cdot \nabla \psi = \frac{1}{\rho} \nabla \psi \psi + \Delta (1 - \psi), \]
\[ \psi (\psi) = 1 - \Gamma \ln (1 + \kappa \psi) \quad \text{if} \quad \psi \geq \psi_s, \]
\[ \psi = \psi_s \quad \text{if} \quad 1 - \Gamma \ln (1 + \kappa \psi_s) \leq \psi, \]
\[ u : \text{velocity vector}, \]
\[ p : \text{pressure}, \]
\[ \alpha : \text{viscosity}, \]
\[ \psi : \text{coefficient of capillarity}, \]
\[ \psi : \text{phase-field variable}, \]
\[ \mu : \text{chemical potential}, \]
\[ H : \text{Heimholtz free energy}, \]
\[ \kappa : \text{adsorption constant}, \]
\[ C : \text{Cahn number}, \]
\[ f_s : \text{injection strength}, \]
\[ \psi_s : \text{Péclet number of surfactant}, \]
\[ \psi_s : \text{Péclet number of surfactant}, \]
\[ Da : \text{Damköhler number}, \]

\[ f_s = \psi_s / \psi_s. \]

(a) \( I = 1, Da = 200, Pe_s = 1200 \)
(b) \( I = 2, Da = 100, Pe_s = 2400 \)
(c) \( I = 4, Da = 50, Pe_s = 4800 \)
(d) \( I = 8, Da = 25, Pe_s = 9600 \)

**Figure** The green curve represents the interface of the immiscible fluids. Brightness of background indicates the concentration of surfactant. Da*I = 200, Pe_s*I = 1200.