



Figure 1 Viscous fingering is a morphological pattern in an unstable interface between two fluids in a porous medium or in a Hele-Shaw cell. It occurs when a less viscous fluid displaces a more viscous one. The images demonstrate the temporal evolutions of injection (blue)/suction (orange) of an immiscible fluid of lower/higher viscosity by simulations of phase field methods.

Fingering: Source and Sink^{*}

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Among natural non-equilibrium growth processes, the viscous fingering problem in Hele-Shaw cells has attracted much attention, since the pioneer work by Saffman and Taylor [1]. This classic hydrodynamic instability emerges when a less viscous fluid displaces a more viscous one in the narrow gap of a Hele-Shaw cell, leading to the formation of striking fingerlike structures. In the radial flow version of the problems, two opposite conditions could be applied, such as the less viscous fluid injected outwardly in a point source or a less viscous fluid sucked inwardly in a point sink, e.g. cases in a miscible condition [2]. On the other hand, phase-field (or, diffuse-interface) modeling offers a convenient alternative in an immiscible condition to usual sharp-interface approaches, which can not handle interface topological changes such as finger pinch-off and merging [3]. In the framework of the diffuse-interface theory, the fluid-fluid boundary between two immiscible fluids is represented by a thin layer of finite thickness, and not as a sharp interface.

In the present simulations, we consider a diffuse-interface approach that is based on a Boussinesq Hele-Shaw-Cahn-Hilliard (BHSCH) model [3]. The dimensionless parameters presented include the mobility

ratio (Peclet number) Pe , the viscosity contrast (Atwood number) A , the Cahn number C , and an effective capillary number Ca . Shown in figure are the evolutions of fingering patterns in an injection of less viscous fluid ($Pe=3,000$, $A=0.922$, $C=10^{-5}$, $Ca=6,700$), and suction of a more viscous fluid ($Pe=12,000$, $A=0.935$, $C=10^{-5}$, $Ca=6,700$). Depending on the balance among injection/suction and the surface tension a gallery of pattern-forming structures is obtained, i.e. fanlike patterns for outward source flow, and a characteristic hierarchy of screened-off fingers for inward sink flow.

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