MIXING EFFICIENCY VIA ALTERNATING INJECTION IN A HETEROGENEOUS POROUS MEDIUM

J. S. Li

Department of Mechanical Engineering
National Chiao Tung University
Hsinchu, Taiwan

Q. Li **W. H. Cai** **F. C. Li**

School of Energy Science and Engineering
Harbin Institute of Technology
Harbin, China

C. Y. Chen*

Department of Mechanical Engineering
National Chiao Tung University
Hsinchu, Taiwan

ABSTRACT

We numerically verify better fluid mixing efficiency can be achieved by alternating injection scheme in a heterogeneous porous medium, whose permeability heterogeneity is characterized by two statistical parameters, i.e., the variance s and the correlation length l. Nevertheless, the fingering pattern is strongly affected by permeability distribution to result in similar fingering interface on each of injected layer of less viscous fluids. Instead of randomly chaotic fingering interaction in a homogeneous condition, more orderly channeling interaction occurs in a heterogeneous medium. As a result, higher Peclet number *Pe* (relative measure of advection and diffusion effects) generally leads to worse mixing efficiency in a heterogeneous medium, which might contradict the result found in a homogeneous case. By the same token, in the cases which strong chaotic fingering interaction already exists in homogeneous conditions, e.g., sufficiently short alternating injection interval Δt, large viscosity contrast A and high Pe, the presence of permeability heterogeneity would constrain the randomly chaotic fingering interaction and favors the more orderly channeling interaction, so that mixing efficiency is deteriorated compared with the corresponding homogeneous case.

Keywords: Heterogeneous porous media, Viscous fingering, Alternating injection, Mixing efficiency.

1. INTRODUCTION

The viscous fingering problem, or the so-called Saffman-Taylor instability [1], considers the evolution of interfacial instability when a less viscous fluid displaces another fluid of higher viscosity in a porous medium or between the narrowly-spaced plates of a Hele-Shaw cell. It should be noticed that, the Hele-Shaw cell can be mathematically treated in lieu of a two-dimensional homogeneous porous media. Comprehensive reviews of the viscous fingering instability are referred to Refs. [2-3]. One of important practical applications of this fingering phenomenon relates to enhance oil recovery efficiency. In general, fingering instability is not favorable during the process of oil recovery, and leads to recent efforts focusing on suppressing the instability and control of the shape of emergent patterns by time-dependent injection schemes in a Hele-Shaw cell [4-8] or a heterogeneous porous medium [9]. On the other hand, mixing of another fluid with the fluid originally occupied in a confined domain of porous medium or Hele-Shaw cell are important for certain applications, e.g., micro-fluidic devices [10], ocean biochemistry [11] and population genetics [12]. It has been shown that

* Corresponding author (chingyao@mail.nctu.edu.tw)

DOI : 10.1017/jmech.2017.32

mixing efficiency is improved with the presence of vigorous viscous fingering, which increases fluid-fluid interfacial area in rectangular Hele-Shaw cells [13]. Furthermore, in a follow-up paper [14], the same group of researchers has demonstrated that alternating injection can effectively enhance mixing in a rectangular Hele-Shaw setup, in which the local displacing velocity remains constant through entire process. In the condition of radial injection flow in a Hele-Shaw cell, where the local displacing velocity decays proportionally to the radial distance away from the injection origin, similar mixing enhancement via alternating injection is also confirmed [15]. Concluded in the same study, there are two major factors to enhance fluid mixing via alternating injection scheme of constant injected amount of fluids. On the one hand, alternating injection generates multiple layers of fluid annuluses to effectively increase the interfacial contact area between the two fluids, leading to more diffusive mixing. On the other hand, fingering instability on the thin layers of less viscous fluid annuluses produced during different injection cycle results in strong randomly chaotic fingering interaction, which greatly enhances dispersive fluid mixing. It is noticed that these previous studies of viscous fingering in alternating injection merely analyzed the displacement in a homogeneous condition. Due to the practical and academic relevance, it is also of interest to understand the emergence of similar phenomena for radial displacements in heterogeneous media. The major goal of this research is to investigate how the presence of permeability heterogeneity affects mixing via alternating injection in radial porous medium flows by intensive numerical simulations.

2. PHYSICAL PROBLEM AND GOVERNING EQUATIONS

We numerically study a binary system containing two incompressible viscous fluids, which are miscible to each other, in a two-dimensional heterogeneous porous medium with a permeability distribution $k(x,y)$. The viscosities of the two fluids are denoted as η_1 (fluid 1), and η_2 (fluid 2), respectively, which $\eta_2 \geq \eta_1$ is assumed. The porous medium is initially fully occupied by fluid 2. In the process of alternating injection scheme, equal amount of the two fluids are injected alternatively in sequence. The process may continue until a maximum time $t = t_f$, when the area of the total injected fluid expands to $\pi D_f^2/4$ in a stable injection condition without fingering instability, e.g. $\eta_1 = \eta_2$. A diffusive fluid-fluid interface in a smaller circular core region of diameter D_0 is initially assumed, and a Cartesian coordinate system (x,y) is defined in such a way that its origin is located at the center of this circular core. Consequently, the constant injection strength can be obtained as $Q = \pi(D_f^2 - D_0^2)/4t_f$, in which the total injection duration for both fluid 1 and fluid 2 is $t_f/2$ each. In the condition of $\eta_2 > \eta_1$, the fluid-fluid interface is viscously unstable during the injection of the less viscous fluid 1.

The system in such a two-dimensional heterogeneous porous medium is governed by the following set of Darcy's equations [16-20]

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\nabla p = -\frac{\eta}{k}\mathbf{u} \tag{2}$$

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = D\nabla^2 c \tag{3}$$

Here, $\mathbf{u}$, p and η denote the velocity vector, the pressure and the viscosity of the binary system, respectively. c represents fluid concentration, such that c = 1 and c = 0 for the less viscous fluid 1 and the more viscous fluid 2, respectively. The constant D is the coefficient of diffusion. The correlation of viscosity (η) and concentration (c) is assumed to be related as

$$\eta(c) = \eta_1 e^{[R(1-c)]}, \quad R = \ln\left(\frac{\eta_2}{\eta_1}\right) \tag{4}$$

where R is a control parameter of the viscosity contrast. The heterogeneous permeability field $k(x,y)$ associated with desired statistical features is expressed in terms of a characteristic value K and a random function $g(x,y)$, whose Gaussian distribution is characterized by the variance s and the spatial correlation length l with a vanishing mean $(\overline{g} = 0)$. The permeability distribution [16-21] is generated by employing an algorithm originally provided by Shinozuka and Jan [22], which is described by

$$k(x,y) = Ke^{g(x,y)} \tag{5}$$

$$< g,g >= s^2 \exp\left(-\pi\left[\left(\frac{x}{l}\right)^2 + \left(\frac{y}{l}\right)^2\right]\right) \tag{6}$$

with a logarithmic mean permeability of $\overline{\log[k(x,y)]} = log[K]$. Within this description, changes in the magnitude of the permeability are determined by the variance s, while the typical size of more permeable regions is prescribed by the correlation length l. Note that, by considering the limit of vanishing variance, i.e., s = 0, one reproduces a homogeneous medium situation.

In order to render the governing equations and relevant variables dimensionless, D_f, t_f and K are taken as the characteristic scales. Furthermore, the pressure is scaled by $(\eta_1 D_f^2)/(Kt_f)$. Thus, the dimensionless versions of the governing equations can be expressed as

$$\nabla \cdot \mathbf{u} = 0 \tag{7}$$

$$\nabla p = -\frac{\eta}{k}\mathbf{u} \tag{8}$$

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = \frac{1}{Pe}\nabla^2 c \tag{9}$$

In the context of our problem, dimensionless control-

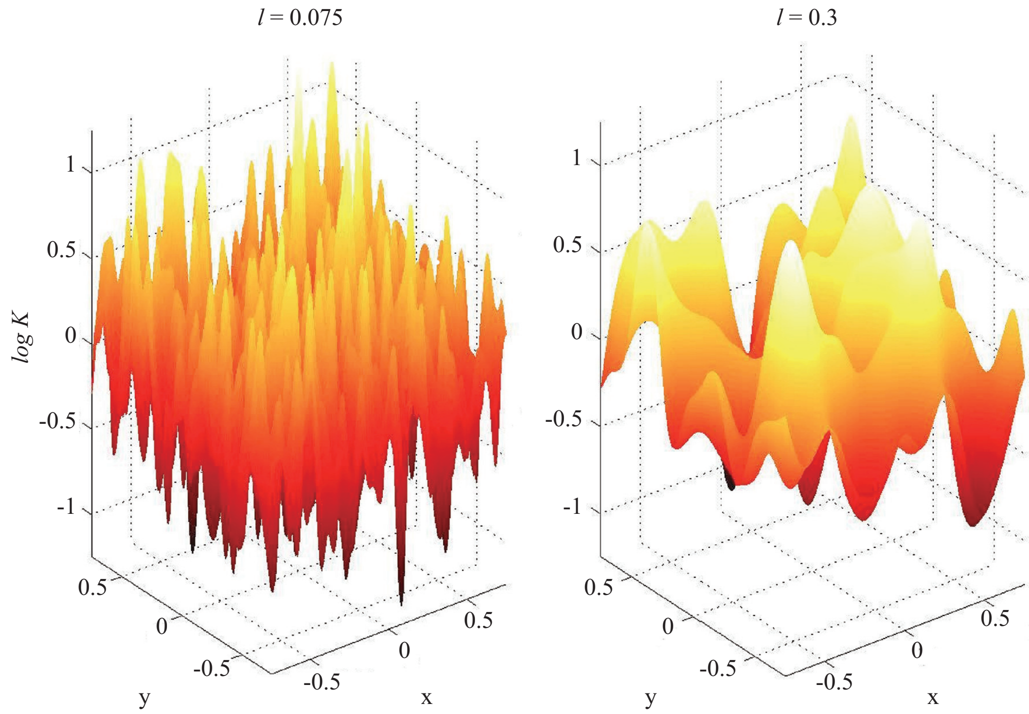


Fig. 1 Distributions of heterogeneous permeability fields for s = 0.6 of l = 0.075 and 0.3. The fields follow the log-Gaussian distributions with vanishing logarithm means, i.e., $\overline{\log k(x,y)} = 0$.

ling parameters such as the Peclet number Pe (relative measure of advection and diffusion effects), and the Atwood number A (dimensionless viscosity difference) are defined as

$$Pe = \frac{D_f^2}{Dt_f}, \quad A = \frac{e^R - 1}{e^R + 1}$$

The permeability is scaled by K, and two representative dimensionless heterogeneous permeability distributions $(\overline{\log[k(x,y)]} = 0)$, e.g., l = 0.075 and 0.3 associated with s = 0.6, are demonstrated in Fig. 1.

The entire alternating injection procedure is carried out by sequentially injecting even amounts of the less viscous fluid and the more viscous fluid, for n full cycles till the completion of the process at dimensionless time t = 1. Each cycle contains two alternating injection stages, i.e., first/second stage of injecting less/more viscous fluid, whose injection interval lasts Δt to yield the same total amount of injection, e.g., $n\Delta t$ = 0.5, for each fluid. As a result, five dimensionless control parameters (s, l, Pe, A and Δt) will be used in the rest of this work to investigate how the permeability heterogeneity, alternating injection interval and viscous fingering affect fluid mixing in porous media flows.

The numerical methods we employed in this work are similar to the ones develop in Refs. [8, 17, 19, 20], in which the governing equations are reformulated into the well-known streamfunction (ϕ) - vorticity (ω) system and yielding

$$u = \frac{\partial \phi}{\partial y}, \quad v = -\frac{\partial \phi}{\partial x} \tag{10}$$

$$\nabla^2 \phi = -\omega \tag{11}$$

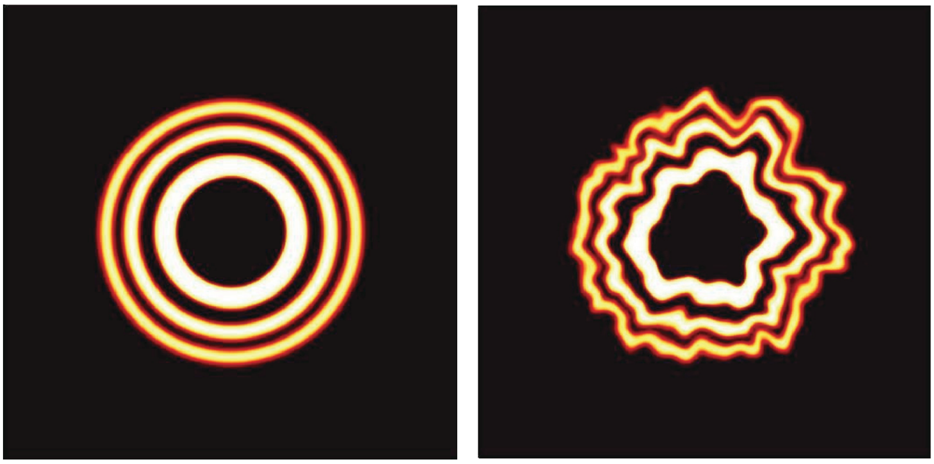
where

$$\omega = -R\left(u\frac{\partial c}{\partial y} - v\frac{\partial c}{\partial x}\right) - \frac{1}{k}\left(u\frac{\partial k}{\partial y} - v\frac{\partial k}{\partial x}\right)$$

In the present framework of the Darcy's formulation, the rotational component of the velocity induced by vorticity can be obtained numerically with high accuracy. On the contrary, the potential part of the radial velocity arisen by source injection is related to a flow singularity at the injecting origin, in which accurate computations are impossible. To avoid numerical instability near the origin, we smooth out the source by distributing its strength in a Gaussian way over the initially circular core region, i.e., $r \le D_c$, where r and D_c denote the dimensionless radial distance away from the origin and the dimensionless core diameter ($D_c = D_0/D_f$), respectively. Scaled by a characteristic injection rate D_f^2/t_f, the dimensionless potential radial velocity, denoted as $\mathbf{u}_{pot}$, satisfying these requirements can be expressed as [8, 17, 19, 21]

$$\mathbf{u}_{pot} = -\frac{Q}{2\pi r}\left[1 - \exp\left(-\frac{4r^2}{D_c^2}\right)\right]\hat{\mathbf{r}}, \tag{12}$$

where $\hat{\mathbf{r}}$ represents the unit vector along the radial direction. The dimensionless injection strength Q in the above expression takes the form of $Q = \pi(1 - D_c^2)/4$.



(a) $s = 0$ (b) $s = 0.6$

Fig. 2 Concentration images in (a) homogeneous (s = 0) and (b) heterogeneous (s = 0.6) condition at t = 0.75 for Δt = 0.125, l = 0.075, A = 0 and Pe = 6000. For the homogeneous condition, circular annuluses of the less viscous fluids (lighter) are well preserved. On the contrary, the interfaces of the fluid annuluses appear irregularly in a heterogeneous medium because of perturbation of permeability heterogeneity.

The simulations are performed up to the maximum time at t = 1 in a square computational domain with dimensionless length of 3/2. In cases which the most outward finger reaches closely to boundary, simulations are terminated before t = 1 to minimize possible effects from boundary. A small circular core with a diameter of D_c = 0.15 initially filled with the less viscous fluid is placed at the center of the domain to start the injection procedure. To proceed with the alternating injection, fluids inside the circular core will be swapped when the injected fluid is changed. Since the flux cross the boundary is prescribed by the potential part, the rotational components of velocity are confined in the computational domain. Vanishing streamfunction can be applied on the boundary, so that the boundary conditions are prescribed as follows.

$$x = \pm\frac{3}{4} : \phi = 0, \quad \frac{\partial c}{\partial x} = 0, \tag{13}$$

$$y = \pm\frac{3}{4} : \phi = 0, \quad \frac{\partial c}{\partial y} = 0, \tag{14}$$

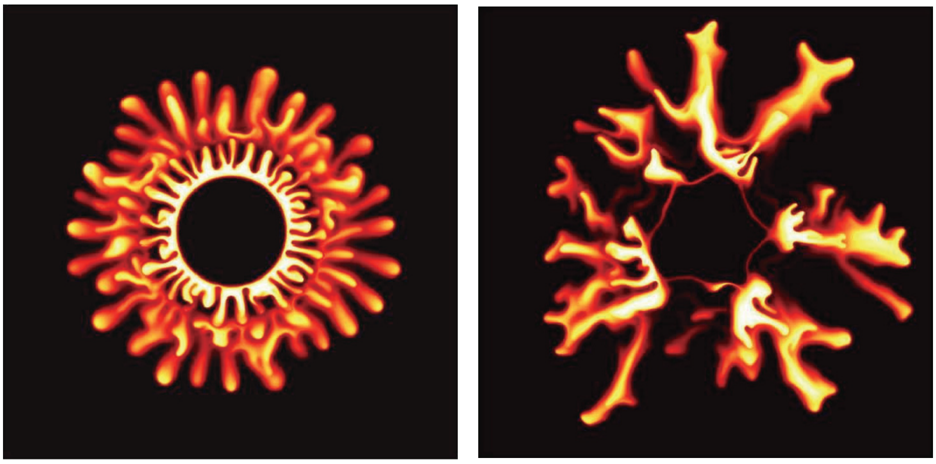
To reproduce extremely fine structures of induced fingers, spatial derivatives in the concentration equation are discretized by compact finite differences with fourth and sixth order of accuracy for convective and diffusive terms, respectively, associated with third order Runge-Kutta procedure for time integration. The Poisson equation is solved by pseudospectral method associated with discretization of sixth order compact finite difference in space. The simulations by using a similar approach [17, 19] were validated by comparing the growth rates in a homogeneous condition with the respective values obtained from linear stability theory. For a more detailed account about these numerical schemes and their validations, the reader is referred to Refs. [8, 17, 19, 21].

3. RESULTS AND DISCUSSION

3.1 Fluid and Flow Patterns

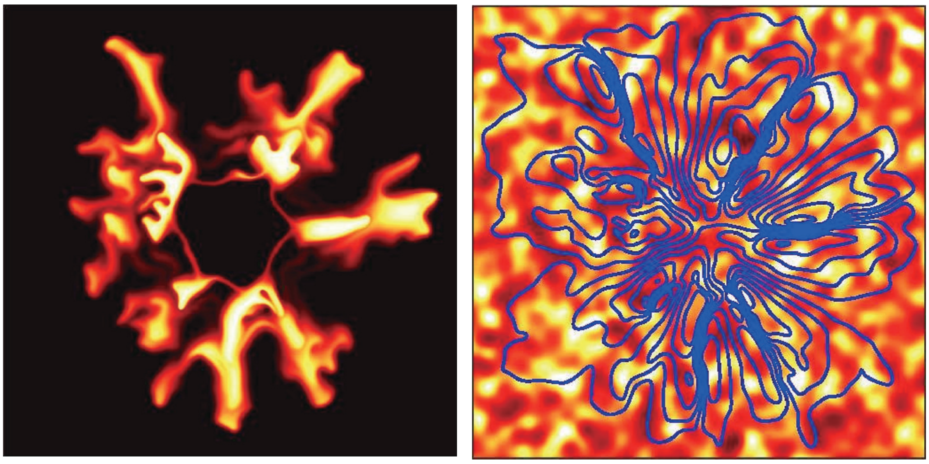
Shown in Fig. 2 are concentration images at t = 0.75 of two viscously stable cases for s = 0 (homogeneous condition) and 0.6 of l = 0.075 (permeability distribution shown in Fig. 1), Pe = 6000, A = 0 and Δt = 0.125 (n = 4). It is noticed that 3 cycles of alternating injection have completed at t = 0.75, so that 3 annuluses of the less viscous fluid are generated. In the present situation of matched viscosity, i.e., A = 0, the interfaces remain stable and circular shapes are well preserved in the homogeneous condition (s = 0). All the fluid mixing is purely caused by molecular diffusion. On the other hand, the presence of permeability heterogeneity (s = 0.6) results in irregular fluid-fluid interfaces between fluid annuluses. As reported in Ref. [15], one of the major factor for the alternating injection to enhance mixing is by increasing the contact circumference of fluid-fluid interface. It is apparent that the permeability heterogeneity leads to longer contact circumference, so that better diffusive mixing is expected which will be quantitatively confirmed in the next section.

Nevertheless, very different scenario occurs for the cases with significant viscosity contrast, e.g., A = 0.848 as shown in Fig. 3. Because of unfavorable viscosity contrast, vigorous fingering instability is triggered even for the homogeneous condition, as shown in Fig. 3(a). Without influences of permeability heterogeneity, the induced fingers on the interface of each layer of the less viscous fluid annulus are randomly generated. The randomly generated fingers of inner fluid annulus grow continuously to catch up the outer fluid annulus, and lead to chaotic fingering interaction between the fluid annuluses. This randomly chaotic fingering interaction between different layers of fluid annuluses is the second and major factor for mixing enhancement by alternating injection scheme [15]. On the contrary, instead of the



(a) $s = 0$ (b) $s = 0.6$

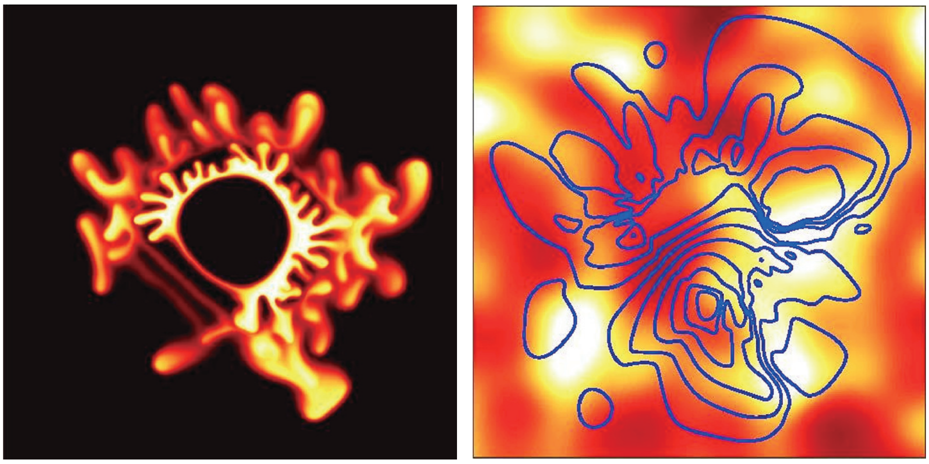
Fig. 3 Concentration images in (a) homogeneous ($s = 0$) and (b) heterogeneous ($s = 0.6$) condition at $t = 0.75$ for $\Delta t = 0.125$, $l = 0.075$, $A = 0.848$ and $Pe = 6000$. Because of viscosity contrast, vigorous fingering instability is triggered even in the homogeneous condition. Fingers on the interfaces of the less viscous fluid annuluses are generated randomly to cause chaotic interaction. In a heterogeneous medium, fingering phenomenon is strongly affected by the permeability distribution. Fingering patterns are similar for all the less viscous fluid annuluses to transform the randomly chaotic fingering interaction toward the channeling interaction among the fewer dominant fingers.



(a) (b)

Fig. 4 Concentration image (a), and streamlines superimposed on permeability distribution (b) at $t = 0.75$ for $\Delta t = 0.125$, $s = 0.6$, $l = 0.075$, $A = 0.848$ and $Pe = 3000$. The lighter (darker) region shown in (b) represents higher (lower) permeability. More prominent fluid dispersion can be visibly observed, compared with the correspondent case of higher $Pe = 6000$ shown in Fig. 3(b). A few main flow channels toward regions of higher permeability are clearly demonstrated in (b), which correspond the developments of dominant fingers formed in (a).

randomly natural perturbation, fingering patterns are strongly affected by the permeability distribution in a heterogeneous medium [9, 20, 21]. Shapes of the dominant fingers on all layers of the less viscous fluid annuluses are similar, which can also be easier observed in Fig. 2(b). As a result, only these dominant fingers interact to each other, i.e., penetrations of the dominant fingers on the inner annulus into the ones on the outer annulus, and form a few apparently longer and slimmer channels as shown in Fig. 3(b). This more orderly fingering interaction is referred to as *channeling* interaction hereafter. The channeling interaction of the few dominant fingers restrains comprehensive chaotic fingering interaction, so that the mixing efficiency is lower for a heterogeneous medium, which will also be verified quantitatively in the latter section. It is noticed that similar channeling interaction leads to lower mixing efficiency was also reported in the homogeneous condition associated with sufficiently high viscosity contrast and longer alternating injection interval, e.g., case of $Pe = 3000$, $A = 0.922$ and $\Delta t = 0.25$ shown in Fig. 7 of Ref. [15].



(a) (b)

Fig. 5 Concentration image (a), and streamlines superimposed on permeability distribution (b), at t = 0.75 for Δt = 0.125, s = 0.6, l = 0.3, A = 0.848 and Pe = 3000. The lighter (darker) region shown in (b) represents higher (lower) permeability. Influence of correlation length can be realized by direct comparison with Fig. 4.

One of the interesting findings [13-15] regarding the mixing enhancement via alternating injection in homogeneous condition is the inconsistent role of the Peclet number Pe, which represents the relative measure of advection and diffusion effects. For cases of prominent chaotic fingering interactions, e.g. large A and high Pe associated with shorter Δt, higher Pe is preferred for better mixing performance. Nevertheless, lower Pe performs better mixing in diffusion dominated condition. This inconsistent influence is re-examined coupled with the presence of heterogeneity. Shown in Fig. 4 is a case with lower Pe = 3000, which can be directly compared with the correspondent case of Pe = 6000 shown in Fig. 3(b). The overall fingering patterns of these two cases remain similar, only dispersion is apparently stronger in the condition of Pe = 3000 to achieve higher mixing efficiency, see next section for quantitative verification. This observation is true for numerous simulated cases with various combinations of control parameters, so that lower Pe is generally favorable for better mixing in heterogeneous media. The consistent influence of Pe to mixing efficiency can be realized by the dominant fingering mechanism in heterogeneous media. As mentioned in the previous paragraphs, fingering pattern is dominated by the permeability distribution, so that the randomly fingering interaction emerged in homogeneous conditions is restrained. Instead, channeling interaction is more prominent in heterogeneous media. This more orderly channeling interaction can be clearly observed in Fig. 4(b), in which streamlines are superimposed on permeability field. A few main channels with highly concentrated streamlines within regions of higher permeability flow through the originally separated fluid annuluses, which can be compared with the apparently separated layers of vortex pairs appear in the homogeneous condition, e.g., Fig. 3 in Ref. [15]. These highly concentrated streamlines result in the prominent development of channeling interaction of the dominant fingers formed in Fig. 4(a). Because of the more prominent channeling effect in a heterogeneous medium, more vigorous fingering instability does not necessary improve mixing efficiency. On the other hand, lower Pe always ensures stronger diffusive mixing. This explains why lower Pe can result in better mixing in heterogeneous media, which might contradict the findings reported in homogeneous conditions [13-15].

Another parameter in a heterogeneous medium is the correlation length l. It has been well concluded that overall fingering pattern, e.g., sizes of dominant fingers and their evolving orientations, is strongly determined by l [9, 20, 21]. Shown in Fig. 5 is the concentration image and streamlines superimposed on permeability distribution for Δt = 0.125, s = 0.6, A = 0.848, Pe = 3000 and l = 0.3, in which the permeability distribution is also demonstrated in Fig. 1 and can be directly compared with Fig. 4 to realize influence of the correlation length. Change of the correlation length significantly alters the fingering pattern. Because of larger area within every high permeability region, more fingers are allowed to develop on each layer of fluid annulus as shown in Fig. 5(a). The fact of higher number of evolved fingers can also be confirmed by the more evenly distributed streamlines shown in Fig. 5(b). Generation of higher number of evolved fingers leads to shorter length of each finger by the same injection volume. Shorter fingers associated with the intrinsic feature of channeling effect in heterogeneous media leads to less prominent fingering interaction as observed in Fig. 5(a). Consequently, the overall fluid mixing would be less effective in the present case of l = 0.3.

3.2 Quantitative Measures

To better present influences of the control parameters, i.e., s, Pe, A and Δt, to mixing efficiency for a fixed l = 0.075, a quantitative measure of the concentration variance (σ^2) is shown in Fig. 6. The variance of concentration distribution is calculated by [15]

$$\sigma^2 = \sum \frac{A_{i*}(c_i - c_m)^2}{A_0}. \tag{15}$$

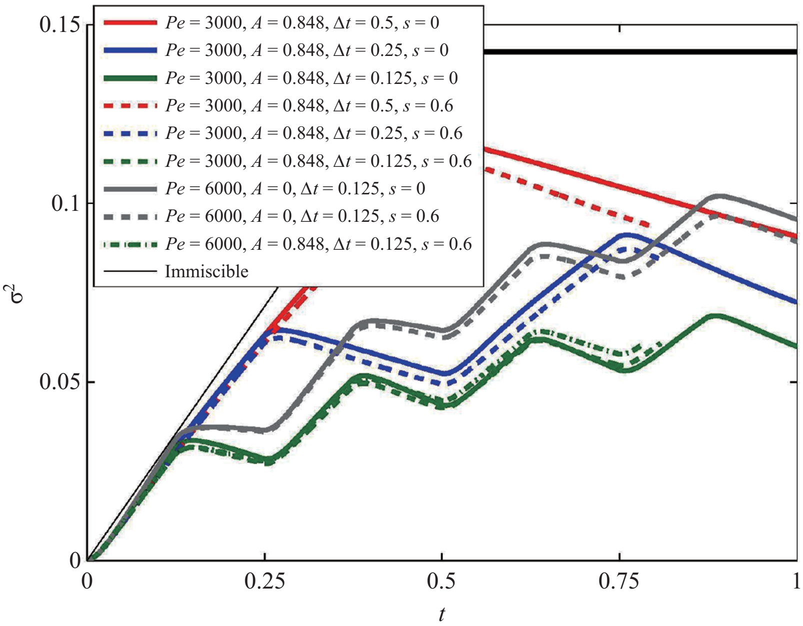


Fig. 6 Temporal evolution of concentration variance (σ^2) for various control parameters $Pe = 3000$, 6000, $A = 0$, 0.848, $\Delta t = 0.5$, 0.25, 0.125 and $s = 0$, 0.6 of $l = 0.075$. The black solid-line is a reference case of immiscible condition associated with a non-diffusive sharp interface. Lower variance (σ^2) represents better mixing efficiency.

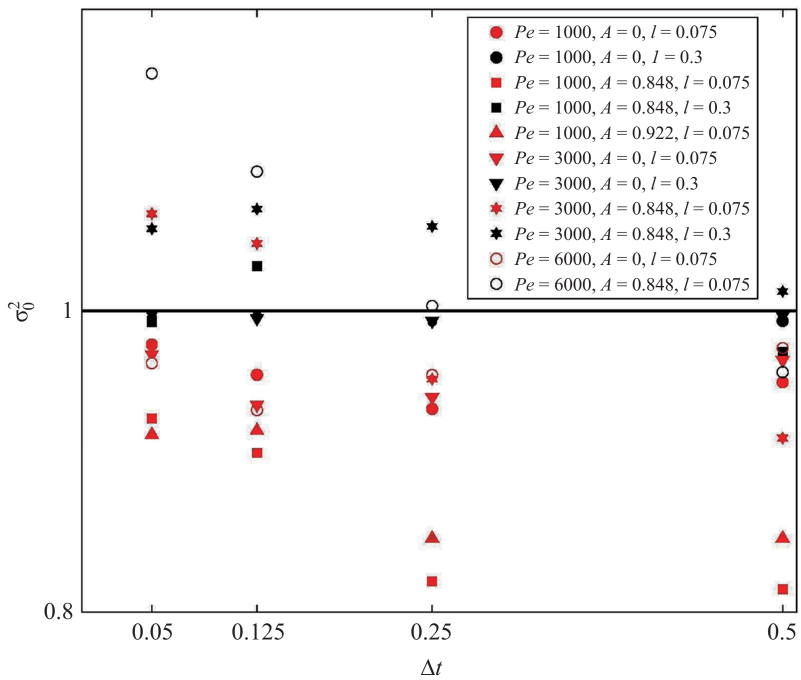


Fig. 7 Normalized concentration variance (σ_0^2) in heterogeneous conditions for various control parameters Δt, Pe, A and l of $s = 0.6$. $\sigma_0^2 < 1$ represents mixing enhancement is achieved by the presence of permeability heterogeneity.

Here, A_i, c_i and A_0 are the area of every discretized mesh, the local concentration in the mesh, and the entire computational domain excluding the core area, respectively. c_m is the instantaneous mean concentration in A_0, which is obtained theoretically as $c_m(t) = Q_1(t)/A_0$ with Q_1 being the instantaneous injected area of the less viscous fluid 1. A smaller σ^2 always indicates better fluid mixing. In general, injection of the fluid 1 increases the variance, while the variance remains constant or less if injection is switched to the fluid 2. For the reader's better information, a stable injection for $\Delta t = 0.5$ ($n = 1$) of sharp interface without diffusion, e.g. $A = 0$ and $Pe \to \infty$, is also plotted in Fig. 6. Under such special condition, the variance yields $\sigma^2(t) = Q_1(A_0\text{-}Q_1)/A_0^2$. The variance shows a rapid increase during the first injection stage of fluid 1 for $t \le 0.5$, and reaches a maximum of $\sigma_m^2 = 0.1424$. Afterward, the variance keeps constant when the injection is alternated to fluid 2 for $0.5 < t \le 1$.

We first verify the mixing enhancement by alternating injection coupled with permeability heterogeneity, e.g., the curves of s = 0.6 and l = 0.075 of Δt = 0.5 (n = 1), 0.25 (n = 2) and 0.125 (n = 4) for Pe = 3000 and A = 0.848 shown in Fig. 6. It is noticed that, because of prominent channeling effect, the most outward finger of these cases reaches close to the boundary, so that the simulations are terminated earlier. The results are consistent with the findings reported in literatures [13-15], such that shorter Δt always gives better mixing efficiency, i.e., smaller σ^2. However, the presence of permeability heterogeneity in the present viscously unstable condition leads to inconsistent effect to the mixing efficiency, compared with the correspondent case in a homogeneous condition (s = 0) also shown in Fig. 6. While fluid mixing is apparently improved for cases of longer Δt = 0.5 and 0.25, the influence is insignificant or even slightly worse for a shorter Δt = 0.125. This inconsistency is also mainly caused by the channeling interaction. The reason of better mixing for shorter Δt is attributed to more fingering interaction between different layers of fluid annuluses. As described in the previous section, the presence of heterogeneity would alter the behavior of fingering interaction toward channeling interaction, instead of randomly chaotic interaction in a homogeneous condition. As a result, for the cases associated with extremely fingering interactions, e.g., in a very shorter injecting interval of Δt = 0.125 (Fig. 3) or less, the presence of permeability heterogeneity might deteriorate mixing efficiency. Nevertheless, for the cases with no or milder fingering interactions, e.g., Pe = 6000 and A = 0 whose images shown in Fig. 2, fluid mixing is improved by effectively increasing the diffusive contact area in heterogeneous media as described in the previous section. Also quantitatively confirmed in Fig. 6 is a previously mentioned observation in a heterogeneous medium, such that lower Pe leads to better mixing even for situations associated with prominent fingering interaction, e.g., cases of Pe = 3000 and 6000 for Δt = 0.125, s = 0.6 and A = 0.848, which is contradictory with the earlier finding in homogeneous conditions. More detailed presentations regarding the influence of permeability heterogeneity coupled with various control parameters will be discussed in the next paragraphs.

To comprehensively evaluate how the presence of permeability heterogeneity affects to the mixing efficiency, numerous combinations of all the control parameters are simulated and their results in terms of the normalized variance σ_0^2 are presented in Fig. 7. The normalized variance is defined as $\sigma_0^2 = \sigma_h^2 / \sigma_r^2$, in which σ_h^2 and σ_r^2 denote the variance in a heterogeneous medium and the reference variance in a homogeneous condition with the same set of Δt, Pe and A, respectively. By the present expression, normalized variance less than unity ($\sigma_0^2 < 1$) indicates better mixing for the presence of permeability heterogeneity.

Two general trends can be easily identified by inspecting Fig. 7, which the presence of permeability heterogeneity generally improves mixing in the conditions of (1) longer interval of alternating injection Δt and (2) shorter correlation length l. The better mixing in a heterogeneous medium for longer Δt is realized by the enhancement of overall fingering interaction as well as the diffusive contact area. As thoroughly reported in Ref. [15], fingering interaction is less prominent in homogeneous case of longer Δt, to result in worse mixing efficiency. On the contrary, more irregular fingering pattern triggered by the presence of heterogeneity can effectively enhance the fingering interaction, so that mixing efficiency is improved as well. On the other hand, for those cases in homogeneous media already with vigorous chaotic fingering interactions, e.g., $Pe \ge 3000$ and A $\ge$ 0.848, the fingering interactions in heterogeneous media appear in forms of channeling interactions to reduce the overall mixing efficiency. As to influence by the correlation length of heterogeneity, worse mixing in longer l is mainly because of shorter finger lengths associated with strong channeling effect to weaken the overall fingering interaction, which is described in the previous section.

To summarize the influence of permeability heterogeneity to mixing efficiency, prominence of the channeling interaction, which is an intrinsic feature always associated with permeability heterogeneity, is the main factor to be considered. In general, permeability heterogeneity will enhance the fingering instability and increase the diffusive fluid-fluid interface. These two effects are both favor for fluid mixing in the situations with no or milder fingering instability. Nevertheless, in the cases which chaotic fingering interaction already exists in homogeneous conditions, e.g., sufficiently short Δt, high Pe and large A, the presence of permeability heterogeneity would change the randomly chaotic fingering interaction into channeling interaction, so that mixing efficiency is deteriorated.

4. CONCLUSIONS

Alternating injection scheme is numerically employed to study the mixing performance of two fluids in a heterogeneous porous medium, whose permeability heterogeneity is characterized by two statistical measures of the variance s and the correlation length l. It has been concluded that this cyclic injection scheme can significantly improve the mixing performance in a homogeneous condition because of two main factors, i.e., increase of contact area for stronger molecular diffusion and triggering randomly chaotic fingering interaction for better convective dispersion [13-15]. Also reported in these literatures, for cases with vigorous fingering instability, the chaotic fingering interaction is the dominant factor of mixing enhancement, so that higher Pe is favor for better mixing, even the molecular diffusion is weaker. In this study, we emphasize the coupling effect of the permeability heterogeneity with other control parameters, such as alternating injection interval Δt, the dimensionless diffusive strength Pe and the dimensionless viscosity contrast A, to the overall mixing efficiency.

We first verify that the alternating injection scheme can also effectively improve mixing efficiency in a heterogeneous porous medium. In line with the finding in a homogeneous condition, the shorter alternating injection interval Δt is applied, the better mixing efficiency is achieved. Nevertheless, the influence of Pe in a heterogeneous porous medium shows different trend from the homogeneous condition, in which higher Pe might be favor better fluid mixing. The presence of permeability heterogeneity triggers more irregular development of fingering pattern to significantly increase the diffusive contact area. In the meantime, the fingering pattern is dominated by the permeability distribution to form similar interface on each layer of separated fluid annuluses. Instead of the chaotic fingering interaction occurring in a homogeneous condition, these similar patterns result in the channeling interaction in a heterogeneous condition, which is less favor for fluid mixing. As a result, lower Pe generally gives better mixing efficiency in a heterogeneous medium, which might contradict the finding reported in a homogeneous case.

It is also revealed, whether the mixing efficiency can be improved by the presence of heterogeneity is mainly determined by prominence of the intrinsic channeling interaction. In the cases which strong chaotic fingering interactions already exist in homogeneous conditions, e.g., sufficiently short Δt, high Pe and large A, the presence of permeability heterogeneity would change the randomly chaotic fingering interaction into the channeling interaction, so that mixing efficiency is deteriorated. On the contrary, the heterogeneity can effectively improve mixing efficiency in the conditions with milder fingering interactions and dominated by diffusive mixing. This suggests that, compared with the corresponding homogeneous cases, the presence of heterogeneity generally gives better mixing efficiency in the conditions of longer interval of alternating injection Δt or shorter correlation length l.

ACKNOWLEDGEMENTS

Supports by the Ministry of Science and Technology, Taiwan (MOST 105-2221-E-009-074-MY3) and the National Natural Science Foundation of China (Grant Nos. 51421063 and 51576051) are acknowledged.

REFERENCES

1. Saffman, P. G. and Taylor, G. I., "The Penetration of a Fluid into a Porous Medium or Hele-Shaw Cell Containing a More Viscous Liquid," *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, **245**, pp. 312-329 (1958).
2. Homsy, G. M., "Viscous Fingering in Porous Media," *Annual Review of Fluid Mechanics*, **19**, pp. 271-311 (1987).
3. McCloud, K. V. and Maher, J. V., "Experimental Perturbations to Saffman-Taylor flow," *Physics Reports*, **260**, pp. 139-185 (1995).
4. Li, S., Lowengrub, J. S., Fontana, J. and Palffy-Muhoray, P., "Control of Viscous Fingering Patterns in a Radial Hele-Shaw Cell, Phys," *Physical Review Letters*, **102**, 174501 (2009).
5. Chen, C.-Y., Huang, C.-W., Wang, L.-C. and Miranda, J., "Controlling Radial Fingering Patterns in Miscible Confined Flows," *Physical Review E*, **82**, 56308 (2010).
6. Dias, E. O. and Miranda, J., "Control of Radial Fingering Patterns: A Weakly Nonlinear Approach," *Physical Review E*, **8**, 016312 (2010).
7. Dias, E. O., Alvarez-Lacalle, E., Carvalho, M. S. and Miranda, J. A., "Minimization of Viscous Fluid Fingering: A Variational Scheme for Optimal Flow Rates," *Physical Review Letters*, **109**, 144502 (2012).
8. Huang, Y.-S. and Chen, C.-Y., "A Numerical Study on Radial Hele-Shaw Flow: Influence of Fluid Miscibility and Injection Scheme," *Computational Mechanics*, **55**, pp. 407-420 (2015).
9. Chen, C.-Y. and Yan, P.-Y., "Radial Flows in Heterogeneous Porous Media with a Linear Injection Scheme," *Computers & Fluids*, **142**, pp. 30-36, (2017).
10. Stone, H. A., Stroock, A. D. and Ajdari, A., "Engineering Flows in Small Devices: Microfluidics Toward a Lab-on-a-Chip," *Annual Review of Fluid Mechanics*, **36**, pp .381-411 (2004).
11. Katija, K. and Dabiri, J. O., "A Viscosity-Enhanced Mechanism for Biogenic Ocean Mixing," *Nature*, **460**, pp. 624-626 (2009).
12. Korolev, K. S., Avlund, M., Hallatschek, O. and Nelson, D. R., "Genetic Demixing and Evolution in Linear Stepping Stone Models," *Reviews of Modern Physics*, **82**, pp. 1691-1718 (2010).
13. Jha, B., Cueto-Felgueroso, L. and Juanes, R., "Fluid Mixing from Viscous Fingering," *Physical Review Letters*, **106**, 194502 (2011).
14. Jha, B., Cueto-Felgueroso, L. and Juanes, R., "Synergetic Fluid Mixing from Viscous Fingering and Alternating Injection," *Physical Review Letters*, **111**, 144501 (2013).
15. Chen, C.-Y., Huang, Y.-C., Huang, Y.-S. and Miranda, J. A., "Enhanced Mixing via Alternating Injection in Radial Hele-Shaw Flows," *Physical Review E*, **92**, 043008 (2015).
16. Tan, C. T. and Homsy, G. M., "Viscous Fingering with Permeability Heterogeneity," *Physics of Fluids A*, **4**, pp. 1099-1101 (1992).
17. Chen, C.-Y. and Meiburg, E., "Miscible Porous Media Flows in the Quarter Five-Spot Configuration. Part 1: The Homogeneous Case," *Journal of Fluid Mechanics*, **371**, pp. 233-268 (1998).
18. Chen, C.-Y. and Meiburg, E., "Miscible Porous Media Flows in the Quarter Five-Spot Configuration.

Part 2: Effect of Heterogeneities," *Journal of Fluid Mechanics*, **371**, pp. 269-299 (1998).

19. Meiburg, E. and Chen, C.-Y., "High-Accuracy Implicit Finite-Difference Simulations of Homogeneous and Heterogeneous Miscible Porous Medium Flows," *SPE Journal*, **5**, pp. 129-137 (2000).
20. Chen, C.-Y. and Yan, P.-Y., "A Diffuse Interface Approach to Injection-Driven Flows of Different Miscibility in Heterogeneous Porous Media," *Physics of Fluids*, **27**, 083101 (2015).
21. Chen, C.-Y., Lin, T.-S., and Miranda, J. A., "Rotationally Induced Fingering Patterns in a Two-Dimensional Heterogeneous Porous Medium," *Physical Review E*, **94**, 053105 (2016).
22. Shinozuka, M. and Jan, C.-M., "Digital Simulation of Random Processes and Its Applications," *Journal of Sound and Vibration*, **25**, pp.111-128 (1972).

(Manuscript received December 30, 2016,
accepted for publication March 21, 2017.)