

# SAS/STAT® 14.1 User's Guide The DISTANCE Procedure



This document is an individual chapter from SAS/STAT® 14.1 User's Guide.

The correct bibliographic citation for this manual is as follows: SAS Institute Inc. 2015. SAS/STAT® 14.1 User's Guide. Cary, NC: SAS Institute Inc.

### SAS/STAT® 14.1 User's Guide

Copyright © 2015, SAS Institute Inc., Cary, NC, USA

All Rights Reserved. Produced in the United States of America.

**For a hard-copy book**: No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, or otherwise, without the prior written permission of the publisher, SAS Institute Inc.

For a web download or e-book: Your use of this publication shall be governed by the terms established by the vendor at the time you acquire this publication.

The scanning, uploading, and distribution of this book via the Internet or any other means without the permission of the publisher is illegal and punishable by law. Please purchase only authorized electronic editions and do not participate in or encourage electronic piracy of copyrighted materials. Your support of others' rights is appreciated.

**U.S. Government License Rights; Restricted Rights:** The Software and its documentation is commercial computer software developed at private expense and is provided with RESTRICTED RIGHTS to the United States Government. Use, duplication, or disclosure of the Software by the United States Government is subject to the license terms of this Agreement pursuant to, as applicable, FAR 12.212, DFAR 227.7202-1(a), DFAR 227.7202-3(a), and DFAR 227.7202-4, and, to the extent required under U.S. federal law, the minimum restricted rights as set out in FAR 52.227-19 (DEC 2007). If FAR 52.227-19 is applicable, this provision serves as notice under clause (c) thereof and no other notice is required to be affixed to the Software or documentation. The Government's rights in Software and documentation shall be only those set forth in this Agreement.

SAS Institute Inc., SAS Campus Drive, Cary, NC 27513-2414

July 2015

 $SAS^{@}$  and all other SAS Institute Inc. product or service names are registered trademarks or trademarks of SAS Institute Inc. in the USA and other countries. @ indicates USA registration.

Other brand and product names are trademarks of their respective companies.

# Chapter 36

# The DISTANCE Procedure

### **Contents**

Overview: DISTANCE Procedure	2287
Levels of Measurement	2288
Symmetric versus Asymmetric Nominal Variables	2289
Standardization	2290
Getting Started: DISTANCE Procedure	2290
Creating a Distance Matrix as Input for a Subsequent Cluster Analysis	2290
Syntax: DISTANCE Procedure	2295
PROC DISTANCE Statement	2296
VAR Statement	2302
ID Statement	2306
COPY Statement	2307
BY Statement	2307
FREQ Statement	2307
WEIGHT Statement	2307
Details: DISTANCE Procedure	2309
Proximity Measures	2309
Missing Values	2315
Formatted versus Unformatted Values	2316
Output Data Sets	2316
Examples: DISTANCE Procedure	2317
Example 36.1: Divorce Grounds – the Jaccard Coefficient	2317
Example 36.2: Financial Data – Stock Dividends	2323
References	2329

# **Overview: DISTANCE Procedure**

The DISTANCE procedure computes various measures of distance, dissimilarity, or similarity between the observations (rows) of an input SAS data set, which can contain numeric or character variables, or both, depending on which proximity measure is used.

The proximity measures are stored as a lower triangular matrix or a square matrix (depending on the SHAPE= option) in an output data set that can then be used as input to the CLUSTER, MDS, and MODECLUS procedures.

The number of rows and columns in the output data set equals the number of observations in the input data set. If the input data set contains BY groups, an output matrix is computed for each BY group with the size determined by the maximum number of observations in any BY group.

The output data set is of type TYPE=DISTANCE or TYPE=SIMILAR, depending on the value of the METHOD= option. See the METHOD= option for more information about the association between the method and the output data set type.

Data set types do not persist when you copy or modify a data set. You must specify the TYPE= data set option for the new data set, as in the following example:

```
data dist2(type=distance);
    set dist;
run;
```

See the OUT= option for more information about data set type persistence.

PROC DISTANCE also provides various nonparametric and parametric methods for standardizing variables. Different variables can be standardized with different methods.

Distance matrices are used frequently in data mining, genomics, marketing, financial analysis, management science, education, chemistry, psychology, biology, and various other fields.

### **Levels of Measurement**

Measurement of some attribute of a set of objects is the process of assigning numbers or other symbols to the objects in such a way that properties of the numbers or symbols reflect properties of the attribute being measured. There are different levels of measurement that involve different properties (relations and operations) of the numbers or symbols. Associated with each level of measurement is a set of transformations of the measurements that preserve the relevant properties; these transformations are called permissible transformations. A particular way of assigning numbers or symbols to measure something is called a scale of measurement.

The most commonly discussed levels of measurement are as follows:

Nominal	Two objects are	e assigned the sa	me symbol if they	have the same va	lue of the attribute.

Permissible transformations are any one-to-one or many-to-one transformation, although

a many-to-one transformation loses information.

Ordinal Objects are assigned numbers such that the order of the numbers reflects an order rela-

tion defined on the attribute. Two objects x and y with attribute values a(x) and a(y) are assigned numbers m(x) and m(y) such that if m(x) > m(y), then a(x) > a(y). Permissible transformations are any monotone increasing transformation, although a

transformation that is not strictly increasing loses information.

Interval Objects are assigned numbers such that differences between the numbers reflect differ-

ences of the attribute. If m(x) - m(y) > m(u) - m(v), then a(x) - a(y) > a(u) - a(v). Permissible transformations are any affine transformation t(m) = c \* m + d, where c and d are constants; another way of saying this is that the origin and unit of measurement

are arbitrary.

the attribute. If m(x)/m(y) > m(u)/m(v), then a(x)/a(y) > a(u)/a(v). Permissible transformations are any power transformation  $t(m) = c * m^d$ , where c and d are constants.

Ratio Objects are assigned numbers such that differences and ratios between the numbers

reflect differences and ratios of the attribute. Permissible transformations are any linear (similarity) transformation t(m) = c \* m, where c is a constant; another way of saying

this is that the unit of measurement is arbitrary.

Absolute Objects are assigned numbers such that all properties of the numbers reflect analogous

properties of the attribute. The only permissible transformation is the identity transforma-

tion.

Proximity measures provided in the DISTANCE procedure accept four levels of measurement: nominal, ordinal, interval, and ratio. Ordinal variables are transformed to interval variables before processing. This is done by replacing the data with their rank scores, and by assuming that the classes of an ordinal variable are spaced equally along the interval scale. See the RANKSCORE= option in the section "PROC DISTANCE Statement" on page 2296 for choices on assigning scores to ordinal variables. There are also different approaches for how to transform an ordinal variable to an interval variable. See Anderberg (1973) for alternatives.

## **Symmetric versus Asymmetric Nominal Variables**

A binary variable contains two possible outcomes: 1 (positive/present) or 0 (negative/absent). If there is no preference for which outcome should be coded as 0 and which as 1, the binary variable is called *symmetric*. For example, the binary variable "is evergreen?" for a plant has the possible states "loses leaves in winter" and "does not lose leaves in winter." Both are equally valuable and carry the same weight when a proximity measure is computed. Commonly used measures that accept symmetric binary variables include the Simple Matching, Hamann, Roger and Tanimoto, Sokal and Sneath 1, and Sokal and Sneath 3 coefficients.

If the outcomes of a binary variable are not equally important, the binary variable is called *asymmetric*. An example of such a variable is the presence or absence of a relatively rare attribute, such as "is color-blind" for a human being. While you say that two people who are color-blind have something in common, you cannot say that people who are not color-blind have something in common. The most important outcome is usually coded as 1 (present) and the other is coded as 0 (absent). The agreement of two 1's (a present-present match or a positive match) is more significant than the agreement of two 0's (an absent-absent match or a negative match). Usually, the negative match is treated as irrelevant. Commonly used measures that accept asymmetric binary variables include Jaccard, Dice, Russell and Rao, Binary Lance and Williams nonmetric, and Kulcynski coefficients.

When nominal variables are employed, the comparison of one data unit with another can only be in terms of whether the data units score the same or different on the variables. If a variable is defined as an asymmetric nominal variable and two data units score the same but fall into the absent category, the absent-absent match is excluded from the computation of the proximity measure.

### Standardization

Since variables with large variances tend to have more effect on the proximity measure than those with small variances, it is recommended that you standardize the variables before the computation of the proximity measure. The DISTANCE procedure provides a convenient way to standardize each variable with its own method before the proximity measures are computed. You can also perform the standardization by using the STDIZE procedure, with the limitation that all variables must be standardized with the same method.

### **Mandatory Standardization**

Variable standardization is not required if any of the following conditions is true:

- if there is only one level of measurement
- if only asymmetric nominal and nominal levels are specified
- if the NOSTD option is specified in the PROC DISTANCE statement

Otherwise, standardization is mandatory.

When standardization is mandatory and no standardization method is specified, a default method of standardization will be used. This default method is determined by the measurement level. In general, the default method is STD for interval variables and is MAXABS for ratio variables except when METHOD=GOWER or METHOD=DGOWER is specified. See the STD= option in the section "VAR Statement" on page 2302 for the default methods for GOWER and DGOWER as well as methods available for standardizing variables.

When standardization is mandatory, PROC DISTANCE ignores the REPONLY option, if it is specified.

# **Getting Started: DISTANCE Procedure**

# Creating a Distance Matrix as Input for a Subsequent Cluster Analysis

The following example demonstrates how you can use the DISTANCE procedure to obtain a distance matrix that will be used as input to a subsequent clustering procedure.

The following data, originated by A. Weber and cited in Hand et al. (1994, p. 297), measure the amount of protein consumed for nine food groups in 25 European countries. The nine food groups are red meat (RedMeat), white meat (WhiteMeat), eggs (Eggs), milk (Milk), fish (Fish), cereal (Cereal), starch (Starch), nuts (Nuts), and fruits and vegetables (FruitVeg). Suppose you want to determine whether national figures in protein consumption can be used to determine certain types or categories of countries; specifically, you want to perform a cluster analysis to determine whether these 25 countries can be formed into groups suggested by the data.

The following DATA step creates the SAS data set Protein:

```
data Protein;
  length Country $ 14;
  input Country &$ RedMeat WhiteMeat Eggs Milk
                    Fish Cereal Starch Nuts FruitVeg;
  datalines;
Albania
                 10.1 1.4
                           0.5
                                 8.9 0.2
                                         42.3 0.6
                                                    5.5
                                                        1.7
Austria
                  8.9 14.0
                           4.3 19.9 2.1
                                          28.0
                                               3.6
                                                    1.3
                                                         4.3
                 13.5 9.3
                           4.1 17.5 4.5
                                          26.6
                                               5.7
                                                    2.1
                                                         4.0
Belgium
Bulgaria
                  7.8 6.0
                           1.6
                                8.3
                                     1.2
                                          56.7
                                               1.1
                                                    3.7
                                                         4.2
                           2.8 12.5 2.0
Czechoslovakia
                 9.7 11.4
                                          34.3
                                               5.0
                                                    1.1
                                                         4.0
                               25.0 9.9
Denmark
                 10.6 10.8
                           3.7
                                          21.9
                                               4.8
                                                    0.7
                                                         2.4
E Germany
                  8.4 11.6
                           3.7 11.1 5.4
                                         24.6
                                               6.5
                                                    0.8
                                                        3.6
Finland
                 9.5 4.9
                           2.7
                                33.7
                                     5.8
                                         26.3
                                              5.1
                                                    1.0
                                                        1.4
France
                 18.0 9.9 3.3 19.5 5.7
                                         28.1
                                              4.8
                                                    2.4
                                                         6.5
Greece
                 10.2 3.0
                           2.8 17.6 5.9
                                          41.7
                                               2.2
                                                    7.8
                                                         6.5
                 5.3 12.4
                           2.9
                                9.7 0.3
                                          40.1
                                               4.0
Hungary
                                                    5.4
                                                         4.2
Ireland
                 13.9 10.0
                           4.7
                                25.8 2.2 24.0
                                               6.2 1.6
                                                        2.9
Italy
                 9.0 5.1
                           2.9
                                13.7 3.4
                                         36.8
                                               2.1
                                                    4.3
                                                        6.7
Netherlands
                  9.5 13.6
                           3.6 23.4 2.5 22.4
                                                        3.7
                                               4.2
                                                    1.8
Norway
                  9.4 4.7
                           2.7 23.3 9.7
                                          23.0
                                               4.6
                                                    1.6
                                                         2.7
                  6.9 10.2
                           2.7
                                19.3 3.0
                                         36.1
                                               5.9
                                                    2.0
                                                        6.6
Poland
Portugal
                  6.2 3.7
                           1.1
                                 4.9 14.2
                                         27.0
                                               5.9
                                                    4.7
                                                         7.9
                  6.2 6.3
                           1.5 11.1 1.0
                                          49.6
                                               3.1
                                                    5.3
                                                        2.8
Romania
                  7.1
                      3.4
                           3.1
                                 8.6
                                     7.0
                                          29.2
                                               5.7
                                                    5.9
                                                         7.2
Spain
                  9.9 7.8
                           3.5
                                 4.7
                                     7.5
                                          19.5
                                                    1.4
                                                         2.0
Sweden
                                               3.7
Switzerland
                 13.1 10.1
                           3.1
                                23.8 2.3
                                          25.6
                                               2.8
                                                    2.4
                                                         4.9
                                20.6 4.3
UK
                 17.4 5.7
                           4.7
                                          24.3
                                               4.7
                                                    3.4
                                                         3.3
USSR
                  9.3 4.6
                           2.1 16.6 3.0
                                          43.6
                                               6.4
                                                    3.4
                                                        2.9
                 11.4 12.5 4.1 18.8 3.4 18.6 5.2 1.5 3.8
W Germany
Yugoslavia
                  4.4 5.0 1.2
                                 9.5 0.6 55.9 3.0 5.7 3.2
```

The data set Protein contains the character variable Country and the nine numeric variables representing the food groups. The \$14. in the INPUT statement specifies that the variable Country has a length of 14.

The following statements create the distance matrix and display part of it:

```
title 'Protein Consumption in Europe';
proc distance data=Protein out=Dist method=Euclid;
   var interval(RedMeat--FruitVeg / std=Std);
   id Country;
run;

proc print data=Dist(obs=10);
   title2 'First 10 Observations in Output Data Set from PROC DISTANCE';
run;
title2;
```

An output SAS data set called Dist, which contains the distance matrix, is created through the OUT= option. The METHOD=EUCLID option requests that Euclidean distances (which is the default) should be computed and produces an output data set of TYPE=DISTANCE.<sup>1</sup>

The VAR statement lists the variables (RedMeat—FruitVeg) along with their measurement level to be used in the analysis. An interval level of measurement is assigned to those variables. Since variables with large variances tend to have more effect on the proximity measure than those with small variances, each variable is standardized by the STD method to have a mean of 0 and a standard deviation of 1. This is done by adding "/STD=STD" at the end of the variables list.

The ID statement specifies that the variable Country should be copied to the OUT= data set and used to generate names for the distance variables. The distance variables in the output data set are named by the values in the ID variable, and the maximum length for the names of these variables is 14.

There are 25 observations in the input data set; therefore, the output data set Dist contains a 25-by-25 lower triangular matrix.

The PROC PRINT statement displays the first 10 observations in the output data set Dist as shown in Figure 36.1.

<sup>&</sup>lt;sup>1</sup>Data set types do not persist when you copy or modify a data set. You must specify the TYPE= data set option for the new data set. See the METHOD= and OUT= options for more information about data set types.

Figure 36.1 First 10 Observations in the Output Data Set from PROC DISTANCE

**Protein Consumption in Europe** First 10 Observations in Output Data Set from PROC DISTANCE

Obs	Country	Albania	Austria	Belgium	Bulgaria	Czechoslovakia	Denmark	E_Germany	Finland
1	Albania	0.00000							
2	Austria	6.12388	0.00000						
3	Belgium	5.94109	2.44987	0.00000					
4	Bulgaria	2.76446	4.88331	5.22711	0.00000				
5	Czechoslovakia	5.13959	2.11498	2.21330	3.94761	0.00000			
6	Denmark	6.61002	3.01392	2.52541	6.00803	3.34049	0.00000	•	
7	E Germany	6.39178	2.56341	2.10211	5.40824	1.87962	2.72112	0.00000	
8	Finland	5.81458	4.04271	3.45779	5.74882	3.91378	2.61570	3.99426	0.00000
9	France	6.29601	3.58891	2.19329	5.54675	3.36011	3.65772	3.78184	4.56796
10	Greece	4.24495	5.16330	4.69515	3.74849	4.86684	5.59084	5.61496	5.47453
Obs	France Greece	e Hunga	ry Irelan	d Italy N	etherland	s Norway Polan	d Portuga	l Romania S	pain
1	•								
2									
3						•	•		•
•	•								
4									•
_			·	· · · · · · · · · · · · · · · · · · ·					
4				· · · · · · · · · · · · · · · · · · ·			•		
4 5				· · · · · · · · · · · · · · · · · · ·		· · · · · · · · · · · · · · · · · · ·			
4 5 6	· · · · · · · ·		· · · · · ·	· · · · · · · · · · · · · · · · · · ·			•		

Obs	Sweden	Switzerland	UK	USSR	W_Germany	Yugoslavia
1						
2						•
3						
4						•
5						
6						•
7						
8						
9						
10						

**10** 4.54456

The following statements produce the dendrogram in Figure 36.2:

```
ods graphics on;
proc cluster data=Dist method=Ward plots=dendrogram(height=rsq);
   id Country;
run;
```

The CLUSTER procedure performs a Ward's minimum-variance cluster analysis based on the distance matrix created by PROC DISTANCE. PROC CLUSTER, along with ODS Graphics, produces the dendrogram shown in Figure 36.2. The option PLOTS=DENDROGRAM(HEIGHT=RSQ) specifies the squared multiple correlation as the height variable in the dendrogram.

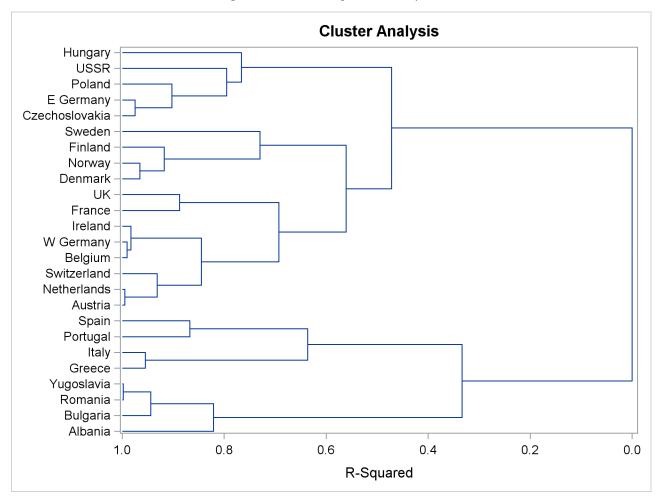


Figure 36.2 Dendrogram of R Square

After inspecting the dendrogram in Figure 36.2, you will see that when the countries are grouped into six clusters, the proportion of variance accounted for by these clusters is slightly less than 70% (69.3%). The 25 countries are clustered as follows:

- Balkan countries: Albania, Bulgaria, Romania, and Yugoslavia
- Mediterranean countries: Greece and Italy
- Iberian countries: Portugal and Spain
- Western European countries: Austria, Netherlands, Switzerland, Belgium, former West Germany, Ireland, France, and U.K.
- Scandinavian countries: Denmark, Norway, Finland, and Sweden
- Eastern European countries: former Czechoslovakia, former East Germany, Poland, former U.S.S.R., and Hungary

# **Syntax: DISTANCE Procedure**

The following statements are available in the DISTANCE procedure:

```
PROC DISTANCE < options > ;
BY variables;
COPY variables;
FREQ variable;
ID variable;
VAR level (variables < / options >);
WEIGHT variable;
```

Both the PROC DISTANCE statement and the VAR statement are required.

### **PROC DISTANCE Statement**

### **PROC DISTANCE** < options > ;

The PROC DISTANCE statement invokes the DISTANCE procedure. Table 36.1 summarizes the options available in the PROC DISTANCE statement. These options are discussed in the following section.

Table 36.1 Summary of PROC DISTANCE Statement Options

Option	Description
Standardize v	variables
ADD=	Specifies the constant to add to each value after standardizing and multiplying by
	the value specified in the MULT= option
FUZZ=	Specifies the relative fuzz factor for writing the output
INITIAL=	Specifies the method for computing initial estimates for the A-estimates
MULT=	Specifies the constant to multiply each value by after standardizing
NORM	Normalizes the scale estimator to be consistent for the standard deviation of a normal distribution
NOSTD	Suppresses standardization
SNORM	Normalizes the scale estimator to have an expectation of approximately 1 for a standard normal distribution
STDONLY	Standardizes variables only (suppresses computation of the distance matrix)
VARDEF=	Specifies the variances divisor

### **Generate distance matrix**

ABSENT=	Specifies the value to be used as an absence value for all the asymmetric nominal
	variables
METHOD=	Specifies the method for computing proximity measures
PREFIX=	Specifies a prefix for naming the distance variables in the OUT= data set
RANKSCORE=	Specifies the method of assigning scores to ordinal variables
SHAPE=	Specifies the shape of the proximity matrix to be stored in the OUT= data set
UNDEF=	Specifies the numeric constant used to replace undefined distances

### Replace missing values

NOMISS	Omits observations with missing values from computation of the location and scale
	measures, if standardization applies; outputs missing values to the distance matrix
	for observations with missing values
REPLACE	Replaces missing data with zero in the standardized data
REPONLY	Replaces missing data with the location measure (does not standardize the data)

### Specify data set details

DATA=	Specifies the input data set
OUT=	Specifies the output data set
OUTSDZ=	Specifies the output data set for standardized scores

These *options* and their abbreviations are described (in alphabetical order) in the remainder of this section.

### ABSENT=number | qs

specifies the value to be used as an absence value in an irrelevant absent-absent match for *all* of the asymmetric nominal variables. If you want to specify a different absence value for a particular variable, use the ABSENT= option in the VAR statement. See the ABSENT= option in the section "VAR Statement" on page 2302 for details.

An absence value for a variable can be either a numeric value or a quoted string consisting of combinations of characters. For instance, ., -999, and "NA" are legal values for the ABSENT= option.

The default absence value for a character variable is "NONE" (notice that a blank value is considered a missing value), and the default absence value for a numeric variable is 0.

### ADD=c

specifies a constant, *c*, to add to each value after standardizing and multiplying by the value you specify in the MULT= option. The default value is 0.

### DATA=SAS-data-set

specifies the input data set containing observations from which the proximity is computed. If you omit the DATA= option, the most recently created SAS data set is used.

### FUZZ=c

specifies the relative fuzz factor for computing the standardized scores. The default value is 1E–14. For the OUTSDZ= data set, the score is computed as follows:

if |standardized scores|  $< m \times c$ , then standardized scores = 0

where m is the numeric constant specified in the MULT= option, or 1 if MULT= option is not specified.

### **INITIAL=**method

specifies the method of computing initial estimates for the A-estimates (ABW, AWAVE, and AHUBER). The following methods are not allowed for the INITIAL= option: ABW, AHUBER, AWAVE, and IN.

The default value is INITIAL=MAD.

### **METHOD**=*method*

specifies the method of computing proximity measures.

For use in PROC CLUSTER, distance or dissimilarity measures such as METHOD=EUCLID or METHOD=DGOWER should be chosen.

The following six tables outline the proximity measures available for the METHOD= option. These tables are classified by levels of measurement accepted by each method. Each table contains four or five columns: the Method column shows the proximity measures, one or two Range columns show the upper and lower bounds, and the TYPE= column shows the type of proximity. The TYPE= column contains SIMILAR if a method generates similarity measures or DISTANCE if a method generates distance or dissimilarity measures. The output data set is of the type shown. For more information about the output data set, see the OUT= option.

For formulas and descriptions of these methods, see the section "Details: DISTANCE Procedure" on page 2309.

Table 36.2 shows the range and output matrix type of the GOWER and DGOWER methods. These two methods accept all measurement levels including ratio, interval, ordinal, nominal, and asymmetric nominal. METHOD=GOWER or METHOD=DGOWER always implies standardization. Assuming all the numeric (ordinal, interval, and ratio) variables are standardized by their corresponding default methods, the possible range values for both methods are from 0 and 1, inclusive. For more information about the default methods of standardization for METHOD=GOWER or METHOD=DGOWER, see the STD= option in the section "VAR Statement" on page 2302.

Table 36.2 Methods That Accept All Measurement Levels

Method	Description	Range	TYPE=
GOWER	Gower and Legendre (1986) similarity	0 to 1	SIMILAR
<b>DGOWER</b>	1 minus GOWER	0 to 1	DISTANCE

Table 36.3 shows *methods* that accept ratio, interval, and ordinal variables.

Table 36.3 Methods That Accept Ratio, Interval, and Ordinal Variables

Method	Description	Range	TYPE=
EUCLID	Euclidean distance	$\geq 0$	DISTANCE
SQEUCLID	Squared Euclidean distance	$\geq 0$	DISTANCE
SIZE	Size distance	$\geq 0$	DISTANCE
SHAPE	Shape distance	$\geq 0$	DISTANCE
COV	Covariance	$\geq 0$	SIMILAR
CORR	Correlation	-1 to 1	SIMILAR
DCORR	Correlation transformed to Euclidean distance	0 to 2	DISTANCE
SQCORR	Squared correlation	0 to 1	SIMILAR
DSQCORR	One minus squared correlation	0 to 1	DISTANCE
L(p)	Minkowski ( $L_p$ ) distance, where $p$ is a positive nu-	$\geq 0$	DISTANCE
	meric value		
CITYBLOCK	L <sub>1</sub> , city-block, or Manhattan distance	$\geq 0$	DISTANCE
CHEBYCHEV	$L_{\infty}$	$\geq 0$	DISTANCE
POWER(p, r)	Generalized Euclidean distance where $p$ is a positive	$\geq 0$	DISTANCE
	numeric value and $r$ is a nonnegative numeric value.		
	The distance between two observations is the $r$ th root		
	of sum of the absolute differences to the pth power		
	between the values for the observations.		

Table 36.4 shows *methods* that accept ratio variables. Notice that all possible range values are nonnegative, because ratio variables are assumed to be positive.

Table 36.4 Methods That Accept Ratio Variables

Method	Description	Range	TYPE=
SIMRATIO	Similarity ratio (if variables are binary, this is the	0 to 1	SIMILAR
	Jaccard coefficient)		
DISRATIO	One minus similarity ratio	0 to 1	DISTANCE
<b>NONMETRIC</b>	Lance and Williams nonmetric coefficient	0 to 1	DISTANCE
<b>CANBERRA</b>	Canberra metric distance coefficient	0 to 1	DISTANCE
COSINE	Cosine coefficient	0 to 1	SIMILAR
DOT	Dot (inner) product coefficient	$\geq 0$	SIMILAR
OVERLAP	Overlap similarity	$\geq 0$	SIMILAR
<b>DOVERLAP</b>	Overlap dissimilarity	$\geq 0$	DISTANCE
CHISQ	Chi-square coefficient	$\geq 0$	DISTANCE
CHI	Square root of chi-square coefficient	$\geq 0$	DISTANCE
PHISQ	Phi-square coefficient	$\geq 0$	DISTANCE
PHI	Square root of phi-square coefficient	≥ 0	DISTANCE

Table 36.5 shows *methods* that accept nominal variables.

Table 36.5 Methods That Accept Nominal Variables

Method	Description	Range	TYPE=
HAMMING	Hamming distance	0 to v	DISTANCE
MATCH	Simple matching coefficient	0 to 1	SIMILAR
<b>DMATCH</b>	Simple matching coefficient transformed to Eu-	0 to 1	<b>DISTANCE</b>
	clidean distance		
DSQMATCH	Simple matching coefficient transformed to	0 to 1	<b>DISTANCE</b>
	squared Euclidean distance		
HAMANN	Hamann coefficient	-1 to 1	SIMILAR
RT	Roger and Tanimoto	0 to 1	SIMILAR
SS1	Sokal and Sneath 1	0 to 1	SIMILAR
SS3	Sokal and Sneath 3	0 to 1	SIMILAR

Note that v denotes the number of variables (dimensionality).

Table 36.6 shows *methods* that accept asymmetric nominal variables. Use the ABSENT= option to create a value to be considered absent.

Table 36.6 Methods That Accept Asymmetric Nominal Variables

Method	Description	Range	TYPE=
DICE	Dice coefficient or Czekanowski/Sorensen similarity	0 to 1	SIMILAR
	coefficient		
RR	Russell and Rao	0 to 1	SIMILAR
BLWNM	Binary Lance and Williams nonmetric, or Bray-Curtis	0 to 1	DISTANCE
	coefficient		
K1	Kulcynski 1	$\geq 0$	SIMILAR

Table 36.7 shows *methods* that accept asymmetric nominal and ratio variables. Use the ABSENT= option to create a value to be considered absent. The table contains five columns. The third column contains possible range values if only one level of measurement (either ratio or asymmetric nominal but not both) is specified; the fourth column contains possible range values if both levels are specified.

The JACCARD method is equivalent to the SIMRATIO method if there is no asymmetric nominal variable; if both ratio and asymmetric nominal variables are present, the coefficient is computed as the sum of the coefficient from the ratio variables and the coefficient from the asymmetric nominal variables. See "Proximity Measures" in the section "Details: DISTANCE Procedure" on page 2309 for the formula and descriptions of the JACCARD method.

Method	Description	Range (One Level)	Range (Two Levels)	TYPE=
JACCARD	Jaccard similarity coefficient	0 to 1	0 to 2	SIMILAR
DJACCARD	Jaccard dissimilarity coefficient	0 to 1	0 to 2	DISTANCE

Table 36.7 Methods That Accept Asymmetric Nominal and Ratio Variables

### MULT=c

specifies a numeric constant, c, by which to multiply each value after standardizing. The default value is 1.

### **NOMISS**

omits observations with missing values from computation of the location and scale measures when standardizing; generates undefined (missing) distances for observations with missing values when computing distances. Use the UNDEF= option to specify the undefined values.

If a distance matrix is created to be used as an input to PROC CLUSTER, the NOMISS option should not be used because PROC CLUSTER does not accept distance matrices with missing values.

### **NORM**

normalizes the scale estimator to be consistent for the standard deviation of a normal distribution when you specify the option STD=AGK, STD=IQR, STD=MAD, or STD=SPACING in the VAR statement.

### **NOSTD**

suppresses standardization of the variables. The NOSTD option should not be specified with the STDONLY option or with the REPLACE option.

### OUT=SAS-data-set

specifies the name of the SAS data set created by PROC DISTANCE. The output data set contains the BY variables, the ID variable, computed distance variables, the COPY variables, the FREQ variable, and the WEIGHT variables.

If you omit the OUT= option, PROC DISTANCE creates an output data set named according to the DATAn convention.

The output data set is of type TYPE=DISTANCE or TYPE=SIMILAR. See the METHOD= option for more information about the association between the method and the output data set type. Data set types do not persist when you copy or modify a data set. You must specify the TYPE= data set option for the new data set, as in the following example:

```
data dist2(type=distance);
    set dist;
run;
```

If you do not specify the TYPE=DISTANCE data set option, the new data set is the default TYPE=DATA. If you use the new data set in a procedure that accepts both TYPE=DATA or TYPE=DISTANCE data sets (such as PROC CLUSTER or PROC MODECLUS), the results will be incorrect.

### **OUTSDZ=**SAS-data-set

specifies the name of the SAS data set containing the standardized scores. The output data set contains a copy of the DATA= data set, except that the analyzed variables have been standardized. Analyzed variables are those listed in the VAR statement.

### PREFIX=name

specifies a prefix for naming the distance variables in the OUT= data set. By default, the names are Dist1, Dist2, ..., Distn. If you specify PREFIX=ABC, the variables are named ABC1, ABC2, ..., ABCn. If the ID statement is also specified, the variables are named by appending the value of the ID variable to the prefix.

### RANKSCORE=MIDRANK | INDEX

specifies the method of assigning scores to ordinal variables. The available methods are listed as follows:

MIDRANK assigns consecutive integers to each category with consideration of the frequency

value. This is the default method.

**INDEX** assigns consecutive integers to each category regardless of frequencies.

The following example explains how each method assigns the rank scores. Suppose the data contain an ordinal variable ABC with values A, B, C. There are two ways to assign numbers. One is to use midranks, which depend on the frequencies of each category. Another is to assign consecutive integers to each category, regardless of frequencies.

**Table 36.8** Example of Assigning Rank Scores

ABC	MIDRANK	INDEX
A	1.5	1
A	1.5	1
В	4	2
В	4	2
В	4	2
C	6	3

### **REPLACE**

replaces missing data with zero in the standardized data (to correspond to the location measure before standardizing). To replace missing data with something else, use the MISSING= option in the VAR statement. The REPLACE option implies standardization.

You cannot specify the following options together:

- both the REPLACE and the REPONLY options
- both the REPLACE and the NOSTD options

### REPONLY

measure specified by the MISSING= option or the STD= option (if the MISSING= option is not specified), but does not standardize the data. If the MISSING= option is not specified and METHOD=GOWER is specified, missing values are replaced by the location measure from the RANGE method (the minimum value), no matter what the value of the STD= option is.

You cannot specify both the REPLACE and the REPONLY options.

### SHAPE=TRIANGLE | TRI | SQUARE | SQU | SQR

specifies the shape of the proximity matrix to be stored in the OUT= data set. SHAPE=TRIANGLE requests the matrix to be stored as a lower triangular matrix; SHAPE=SQUARE requests that the matrix be stored as a square matrix. Use SHAPE=SQUARE if the output data set is to be used as input to the MODECLUS procedures. The default is TRIANGLE.

### **SNORM**

normalizes the scale estimator to have an expectation of approximately 1 for a standard normal distribution when the STD=SPACING option is specified.

### **STDONLY**

standardizes variables only and computes no distance matrix. You must use the OUTSDZ= option to save the standardized scores. You cannot specify both the STDONLY option and the NOSTD option.

### **UNDEF**=n

specifies the numeric constant used to replace undefined distances, such as when an observation has all missing values, or if a divisor is zero.

### VARDEF=DF | N | WDF | WEIGHT | WGT

specifies the divisor to be used in the calculation of distance, dissimilarity, or similarity measures, and for standardizing variables whenever a variance or covariance is computed. By default, VARDEF=DF. The values and associated divisors are as follows:

Value	Divisor	Formula
DF	degrees of freedom	n-1
N	number of observations	n
WDF	sum of weights minus 1	$(\sum_i w_i) - 1$
WEIGHT   WGT	sum of weights	$\sum_i w_i$

### **VAR Statement**

VAR level (variables < / options >) < level (variables < / options >)... > ;

where the syntax for the *options* is as follows:

ABSENT=value MISSING=miss-method | value **ORDER**=order-option STD=std-method WEIGHTS=weight-list

The VAR statement lists variables from which distances are to be computed. The VAR statement is required. The variables can be numeric or character depending on their measurement levels. A variable cannot appear more than once in either the same list or a different list.

*level* is required. It declares the levels of measurement for those variables specified within the parentheses. Available values for *level* are as follows:

**ANOMINAL** variables are asymmetric nominal and can be either numeric or character. **NOMINAL** variables are symmetric nominal and can be either numeric or character.

**ORDINAL** variables are ordinal and can be either numeric or character. Values of ordinal variables

are replaced by their corresponding rank scores. If standardization is required, the standardized rank scores are output to the data set specified in the OUTSDZ= option. See the RANKSCORE= option in the PROC DISTANCE statement for methods available for assigning rank scores to ordinal variables. After being replaced by scores, ordinal

variables are considered interval.

**INTERVAL** variables are interval and numeric.

**RATIO** variables are ratio and numeric. Ratio variables should always contain positive measure-

ments.

Each variable list can be followed by an *option* list. Use "/" after the list of variables to start the option list. An *option* list contains options that are applied to the variables. The following *options* are available in the option list:

**ABSENT=** specifies the value to be used as an absence value in an irrelevant absent-absent match for

asymmetric nominal variables.

**MISSING=** specifies the method (or numeric value) with which to replace missing data.

**ORDER=** selects the order for assigning scores to ordinal variables.

**STD=** selects the standardization method.

**WEIGHTS=** assigns weights to the variables in the list.

If an *option* is missing from the current attribute list, PROC DISTANCE provides default values for all the variables in the current list.

For example, in the VAR statement

```
var ratio(x1-x4/std= mad weights= .5 .5 .1 .5 missing= -99)
interval(x5/std= range)
ordinal(x6/order= desc);
```

the first option list defines x1-x4 as ratio variables to be standardized by the MAD method. Also, any missing values in x1-x4 should be replaced by -99. x1 is given a weight of 0.5, x2 is given a weight of 0.5, x3 is given a weight of 0.1, and x4 is given a weight of 0.5.

The second option list defines x5 as an interval variable to be standardized by the RANGE method. If the REPLACE option is specified in the PROC DISTANCE statement, missing values in x5 are replaced by the location estimate from the RANGE method. By default, x5 is given a weight of 1.

The last option list defines x6 as an ordinal variable. The scores are assigned from highest to lowest by its unformatted values. Although the STD= option is not specified, x6 is standardized by the default method

(STD) because there is more than one level of measurements (ratio, interval, and ordinal) in the VAR statement. Again, if the REPLACE option is specified, missing values in x6 are replaced by the location estimate from the STD method. Finally, by default, x6 is given a weight of 1.

More details for the options are explained as follows.

### **STD**=*std*-*method*

specifies the standardization method. Valid values for *std-method* are MEAN, MEDIAN, SUM, EUCLEN, USTD, STD, RANGE, MIDRANGE, MAXABS, IQR, MAD, ABW, AHUBER, AWAVE, AGK, SPACING, and L. Table 36.9 lists available methods of standardization as well as their corresponding location and scale measures.

Method	Scale	Location
MEAN	1	mean
MEDIAN	1	median
SUM	sum	0
<b>EUCLEN</b>	Euclidean length	0
USTD	standard deviation about origin	0
STD	standard deviation	mean
RANGE	range	minimum
<b>MIDRANGE</b>	range/2	midrange
MAXABS	maximum absolute value	0
IQR	interval quartile range	median
MAD	median absolute deviation from	median
	median	
ABW(c)	biweight A-estimate	biweight 1-step M-estimate
AHUBER(c)	Huber A-estimate	Huber 1-step M-estimate
AWAVE(c)	Wave 1-step M-estimate	Wave A-estimate
AGK(p)	AGK estimate (ACECLUS)	mean
SPACING(p)	minimum spacing	mid minimum-spacing
L(p)	$\mathrm{L}_p$	$\mathrm{L}_p$

**Table 36.9** Available Standardization Methods

These standardization methods are further documented in the section on the METHOD= option in the PROC STDIZE statement of the STDIZE procedure (see the section "Standardization Methods" on page 8709 in Chapter 106, "The STDIZE Procedure").

Standardization is not required if there is only one level of measurement, or if only asymmetric nominal and nominal levels are specified; otherwise, standardization is mandatory. When standardization is mandatory, a default method is provided when the STD= option is not specified. You can suppress the mandatory standardization by using the NOSTD option in the PROC DISTANCE statement. See the NOSTD option in the section "PROC DISTANCE Statement" on page 2296 and the section "Mandatory Standardization" on page 2290 for details.

The default method is STD for standardizing interval variables and MAXABS for standardizing ratio variables unless METHOD=GOWER or METHOD=DGOWER is specified. If METHOD=GOWER is specified, interval variables are standardized by the RANGE method, and whatever is specified in the STD= option is ignored; if METHOD=DGOWER is specified, the RANGE method is the default

standardization method for interval variables. The MAXABS method is the default standardization method for ratio variables for both the GOWER and DGOWER methods.

Notice that a ratio variable should always be positive.

Table 36.10 lists standardization methods and the levels of measurement that can be accepted by each method. For example, the SUM method can be used to standardize ratio variables but not interval or ordinal variables. Also, the AGK and SPACING methods should not be used to standardize ordinal variables. If you apply AGK and SPACING to ranks, the results are degenerate because all the spacings of a given order are equal.

Standardization Legitimate Method **Levels of Measurement MEAN** ratio, interval, ordinal ratio, interval, ordinal **MEDIAN** SUM ratio **EUCLEN** ratio **USTD** ratio STD ratio, interval, ordinal ratio, interval, ordinal RANGE MIDRANGE ratio, interval, ordinal **MAXABS** ratio ratio, interval, ordinal IQR MAD ratio, interval, ordinal ratio, interval, ordinal ABW(c)AHUBER(c)ratio, interval, ordinal AWAVE(c)ratio, interval, ordinal AGK(p)ratio, interval SPACING(p)ratio, interval ratio, interval, ordinal L(p)

Table 36.10 Legitimate Levels of Measurements for Each Method

### ABSENT=numner | qs

specifies the value to be used as an absence value in an irrelevant absent-absent match for asymmetric nominal variables. The absence value specified here overwrites the absence value specified through the ABSENT= option in the PROC DISTANCE statement for those variables in the current variable list.

An absence value for a variable can be either a numeric value or a quoted string consisting of combinations of characters. For instance, ., –999, "NA" are legal values for the ABSENT= option.

The default for an absence value for a character variable is "NONE" (notice that a blank value is considered a missing value), and the default for an absence value for a numeric variable is 0.

### MISSING=miss-method | value

specifies the method or a numeric value for replacing missing values. If you omit the MISSING= option, the REPLACE option replaces missing values with the location measure given by the STD= option. Specify the MISSING= option when you want to replace missing values with a different value. You can specify any method that is valid in the STD= option. The corresponding location measure is used to replace missing values.

If a numeric value is given, the value replaces missing values after standardizing the data. However, when standardization is not mandatory, you can specify the REPONLY option with the MISSING= option to suppress standardization for cases in which you want only to replace missing values.

If the NOSTD option is specified, there is no standardization, but missing values are replaced by the corresponding location measures or by the numeric value of the MISSING= option. See the section "Missing Values" on page 2315 for details about missing values replacement with and without standardization.

# ORDER=ASCENDING | DESCENDING | ASCFORMATTED | DESFORMATTED | DSORDER ORDER=ASC | DESC | ASCFMT | DESFMT | DATA

specifies the order for assigning score to ordinal variables. The value for the ORDER= option can be one of the following:

**ASCENDING** scores are assigned in lowest-to-highest order of unformatted values. **DESCENDING** scores are assigned in highest-to-lowest order of unformatted values.

**ASCFORMATTED** scores are assigned in ascending order by their formatted values. This

option can be applied to character variables only, since unformatted values

are always used for numeric variables.

**DESFORMATTED** scores are assigned in descending order by their formatted values. This

option can be applied to character variables only, since unformatted values

are always used for numeric variables.

**DSORDER** scores are assigned according to the order of their appearance in the input

data set.

The default value is ASCENDING.

### **WEIGHTS**=*weight-list*

specifies a list of values for weighting individual variables while computing the proximity. Values in this list can be separated by blanks or commas. You can include one or more items of the form *start* TO *stop* BY *increment*. This list should contain at least one weight. The maximum number of weights you can list is equal to the number of variables. If the number of weights is less than the number of variables, the last value in the *weight-list* is used for the rest of the variables; conversely, if the number of weights is greater than the number of variables, the trailing weights are discarded.

The default value is 1.

### **ID Statement**

### ID variable;

The ID statement specifies a single variable to be copied to the OUT= data set and used to generate names for the distance variables. The ID variable must be character.

Typically, each ID value occurs only once in the input data set or, if you use a BY statement, only once within a BY group.

If you specify both the ID and BY statements, the ID variable must have the same values in the same order in each BY group.

### **COPY Statement**

**COPY** variables;

The COPY statement specifies a list of additional variables to be copied to the OUT= data set.

### **BY Statement**

BY variables;

You can specify a BY statement with PROC DISTANCE to obtain separate analyses of observations in groups that are defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables. If you specify more than one BY statement, only the last one specified is used.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the NOTSORTED or DESCENDING option in the BY statement for the DISTANCE procedure. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure (in Base SAS software).

For more information about BY-group processing, see the discussion in SAS Language Reference: Concepts. For more information about the DATASETS procedure, see the discussion in the Base SAS Procedures Guide.

### FREQ Statement

FREQ | FREQUENCY variable;

The frequency variable is used for either standardizing variables or assigning rank scores to the ordinal variables. It has no direct effect on computing the distances.

For standardizing variables and assigning rank scores, PROC DISTANCE treats the data set as if each observation appeared *n* times, where *n* is the value of the FREQ variable for the observation. Nonintegral values of the FREQ variable are truncated to the largest integer less than the FREQ value. If the FREQ variable has a value that is less than 1 or is missing, the observation is not used in the analysis.

### **WEIGHT Statement**

WGT | WEIGHT variable;

The WEIGHT statement specifies a numeric variable in the input data set with values that are used to weight each observation. This weight variable is used for standardizing variables rather than computing the distances. Only one variable can be specified.

The WEIGHT variable values can be nonintegers. An observation is used in the analysis only if the value of the WEIGHT variable is greater than zero. The WEIGHT variable applies to variables that are standardized by the following *options*: STD=MEAN, STD=SUM, STD=EUCLEN, STD=USTD, STD=STD, STD=AGK, or STD=L.

PROC DISTANCE uses the value of the WEIGHT variable  $w_i$  to compute the sample mean, uncorrected sample variances, and sample variances as follows:

$$\overline{x}_w = \sum_i w_i x_i / \sum_i w_i$$

$$u_w^2 = \sum_i w_i x_i^2 / d$$

$$s_w^2 = \sum_i w_i (x_i - \overline{x}_w)^2 / d$$

 $w_i$  is the weight value of the *i*th observation,  $x_i$  is the value of the *i*th observation, and d is the divisor controlled by the VARDEF= option (see the VARDEF= option in the PROC DISTANCE statement for details).

PROC DISTANCE uses the value of the WEIGHT variable to calculate the following statistics for standardization:

MEAN the weighted mean,  $\overline{x}_w$ 

SUM the weighted sum,  $\sum_i w_i x_i$ 

USTD the weighted uncorrected standard deviation,  $\sqrt{u_w^2}$ 

STD the weighted standard deviation,  $\sqrt{s_w^2}$ 

EUCLEN the weighted Euclidean length, computed as the square root of the weighted uncorrected

sum of squares:

$$\sqrt{\sum_{i} w_{i} x_{i}^{2}}$$

AGK the AGK estimate. This estimate is documented further in the ACECLUS procedure as the

METHOD=COUNT option. See the discussion of the WEIGHT statement in Chapter 24, "The ACECLUS Procedure," for information about how the WEIGHT variable is applied

to the AGK estimate.

L the  $L_p$  estimate. This estimate is documented further in the FASTCLUS procedure as the LEAST= option. See the discussion of the WEIGHT statement in Chapter 38, "The FASTCLUS Procedure," for information about how the WEIGHT variable is used to compute weighted cluster means. Note that the number of clusters is always 1.

# **Details: DISTANCE Procedure**

### **Proximity Measures**

The following notation is used in this section:

v the number of variables or the dimensionality

 $x_j$  data for observation x and the jth variable, where j = 1 to v

 $y_j$  data for observation y and the jth variable, where j = 1 to v

 $w_j$  weight for the jth variable from the WEIGHTS= option in the VAR statement.  $w_j = 0$ 

when either  $x_j$  or  $y_j$  is missing.

W the sum of total weights. No matter if the observation is missing or not, its weight is

added to this metric.

 $\bar{x}$  mean for observation x

 $\bar{x} = \sum_{j=1}^{v} w_j x_j / \sum_{j=1}^{v} w_j$ 

 $\bar{y}$  mean for observation y

 $\bar{y} = \sum_{j=1}^{v} w_j y_j / \sum_{j=1}^{v} w_j$ 

d(x, y) the distance or dissimilarity between observations x and y

s(x, y) the similarity between observations x and y

The factor  $W/\sum_{j=1}^{v} w_j$  is used to adjust some of the proximity measures for missing values.

# **Methods That Accept All Measurement Levels**

**GOWER** 

Gower's similarity

$$s_1(x, y) = \sum_{j=1}^{v} w_j \delta_{x,y}^j d_{x,y}^j / \sum_{j=1}^{v} w_j \delta_{x,y}^j$$

 $\delta_{x,y}^{j}$  is computed as follows:

For nominal, ordinal, interval, or ratio variable,

$$\delta_{x,y}^j = 1$$

For asymmetric nominal variable,

$$\delta_{x,y}^{j} = 1$$
, if either  $x_{j}$  or  $y_{j}$  is present

$$\delta_{x,y}^{j} = 0$$
, if both  $x_{j}$  and  $y_{j}$  are absent

For nominal or asymmetric nominal variable,

$$d_{x,y}^j = 1$$
, if  $x_j = y_j$ 

$$d_{x,y}^j = 0$$
, if  $x_j \neq y_j$ 

For ordinal, interval, or ratio variable,

$$d_{x,y}^j = 1 - |x_j - y_j|$$

**DGOWER** 1 minus Gower

$$d_2(x, y) = 1 - s_1(x, y)$$

### Methods That Accept Ratio, Interval, and Ordinal Variables

**EUCLID** 

Euclidean distance 
$$d_3(x, y) = \sqrt{\left(\sum_{j=1}^v w_j (x_j - y_j)^2\right) W / \left(\sum_{j=1}^v w_j\right)}$$

squared Euclidean distance **SQEUCLID** 

$$d_4(x, y) = (\sum_{j=1}^{v} w_j (x_j - y_j)^2) W / (\sum_{j=1}^{v} w_j)$$

**SIZE** size distance

$$d_5(x, y) = |\sum_{j=1}^{v} w_j(x_j - y_j)| \sqrt{W} / (\sum_{j=1}^{v} w_j)$$

**SHAPE** shape distance

$$d_6(x,y) = \sqrt{(\sum_{j=1}^v w_j [(x_j - \bar{x}) - (y_j - \bar{y})]^2) W/(\sum_{j=1}^v w_j)}$$

NOTE: squared shape distance plus squared size distance equals squared Euclidean distance.

**COV** covariance similarity coefficient

$$s_7(x, y) = \sum_{j=1}^{v} w_j (x_j - \bar{x})(y_j - \bar{y}) / vardiv$$
, where

$$vardiv = v \text{ if VARDEF} = N$$
  
 $= v - 1 \text{ if VARDEF} = DF$   
 $= \sum_{j=1}^{v} w_j \text{ if VARDEF} = \text{WEIGHT}$   
 $= \sum_{j=1}^{v} w_j - 1 \text{ if VARDEF} = \text{WDF}$ 

**CORR** 

correlation similarity coefficient
$$s_8(x, y) = \frac{\sum_{j=1}^{v} w_j (x_j - \bar{x})(y_j - \bar{y})}{\sqrt{\sum_{j=1}^{v} w_j (x_j - \bar{x})^2 \sum_{j=1}^{v} w_j (y_j - \bar{y})^2}}$$

**DCORR** correlation transformed to Euclidean distance as sqrt(1-CORR)

$$d_9(x, y) = \sqrt{1 - s_8(x, y)}$$

**SQCORR** 

squared correlation 
$$s_{10}(x, y) = \frac{\left[\sum_{j=1}^{v} w_{j}(x_{j} - \bar{x})(y_{j} - \bar{y})\right]^{2}}{\sum_{j=1}^{v} w_{j}(x_{j} - \bar{x})^{2} \sum_{j=1}^{v} w_{j}(y_{j} - \bar{y})^{2}}$$

**DSQCORR** squared correlation transformed to squared Euclidean distance as (1–SQCORR)

$$d_{11}(x, y) = 1 - s_{10}(x, y)$$

L(p)Minkowski ( $L_p$ ) distance, where p is a positive numeric value

$$d_{12}(x,y) = \left[ \left( \sum_{j=1}^{v} w_j |x_j - y_j|^p \right) W / \left( \sum_{j=1}^{v} w_j \right) \right]^{1/p}$$

$$d_{13}(x,y) = (\sum_{j=1}^{v} w_j | x_j - y_j |) W / (\sum_{j=1}^{v} w_j)$$

$$d_{14}(x, y) = \max_{j=1}^{v} w_j |x_j - y_j|$$

generalized Euclidean distance, where p is a nonnegative numeric value and r is a positive numeric value. The distance between two observations is the rth root of sum of the absolute differences to the pth power between the values for the observations:

$$d_{15}(x, y) = \left[ \left( \sum_{i=1}^{v} w_i | x_i - y_i |^p \right) W / \left( \sum_{i=1}^{v} w_i \right) \right]^{1/r}$$

### **Methods That Accept Ratio Variables**

similarity ratio
$$s_{16}(x, y) = \frac{\sum_{j=1}^{v} w_{j}(x_{j} y_{j})}{\sum_{j=1}^{v} w_{j}(x_{j} y_{j}) + \sum_{j=1}^{v} w_{j}(x_{j} - y_{j})^{2}}$$

$$d_{17}(x, y) = 1 - s_{16}(x, y)$$

Lance-Williams nonmetric coefficient 
$$d_{18}(x, y) = \frac{\sum_{j=1}^{v} w_j |x_j - y_j|}{\sum_{j=1}^{v} w_j (x_j + y_j)}$$

CANBERRA Canberra metric coefficient. See Sneath and Sokal (1973, pp. 125–126) 
$$d_{19}(x,y) = \sum_{j=1}^{v} \frac{w_j |x_j - y_j|}{w_j (x_j + y_j)}$$

$$d_{19}(x, y) = \sum_{j=1}^{v} \frac{w_j |x_j - y_j|}{w_j (x_j + y_j)}$$

cosine coefficient 
$$s_{20}(x, y) = \frac{\sum_{j=1}^{v} w_{j}(x_{j} y_{j})}{\sqrt{\sum_{j=1}^{v} w_{j} x_{j}^{2} \sum_{j=1}^{v} w_{j} y_{j}^{2}}}$$

$$s_{21}(x, y) = \left[\sum_{j=1}^{v} w_j(x_j y_j)\right] / \sum_{j=1}^{v} w_j$$

$$s_{22}(x, y) = \sum_{j=1}^{v} w_j [\min(x_j, y_j)]$$

DOVERLAP maximum of the sum of the 
$$x$$
 and the sum of  $y$  minus overlap

$$d_{23}(x,y) = \max(\sum_{j=1}^{v} w_j x_j, \sum_{j=1}^{v} w_j y_j) - s_{22}(x,y)$$

If the data represent the frequency counts, chi-square dissimilarity between two sets of

frequencies can be computed. A 2-by-v contingency table is illustrated to explain how the chi-square dissimilarity is computed as follows:

		Row		
Observation	Var 1	Var 2	 Var v	Sum
X	$x_1$	<i>x</i> <sub>2</sub>	 $x_v$	$r_{\chi}$
Y	У1	<i>y</i> <sub>2</sub>	 $y_v$	$r_y$
Column Sum	$c_1$	$c_2$	 $c_v$	T

where

$$r_x = \sum_{j=1}^{v} w_j x_j$$

$$r_y = \sum_{j=1}^{v} w_j y_j$$

$$c_j = w_j (x_j + y_j)$$

$$T = r_x + r_y = \sum_{j=1}^{v} c_j$$

The chi-square measure is computed as follows:

$$d_{24}(x,y) = \left(\sum_{j=1}^{v} \frac{(w_j x_j - E(x_j))^2}{E(x_j)} + \sum_{j=1}^{v} \frac{(w_j y_j - E(y_j))^2}{E(y_j)}\right) W / \left(\sum_{j=1}^{v} w_j\right)$$
 where for  $j = 1, 2, ..., v$ 

$$E(x_j) = r_x c_j / T$$
  
$$E(y_j) = r_y c_j / T$$

CHI square root of chi-square  $d_{25}(x, y) = \sqrt{d_{23}(x, y)}$ 

PHISQ phi-square

This is the CHISQ dissimilarity normalized by the sum of weights

 $d_{26}(x, y) = d_{24}(x, y) / (\sum_{j=1}^{v} w_j)$ 

PHI square root of phi-square  $d_{27}(x, y) = \sqrt{d_{25}(x, y)}$ 

### **Methods That Accept Symmetric Nominal Variables**

The following notation is used for computing  $d_{28}(x, y)$  to  $s_{35}(x, y)$ . Notice that only the nonmissing pairs are discussed below; all the pairs with at least one missing value will be excluded from any of the computations in the following section because  $w_i = 0$ , if either  $x_i$  or  $y_i$  is missing.

M nonmissing matches 
$$M = \sum_{j=1}^{v} w_j \delta_{x,y}^j, \text{ where }$$
 
$$\delta_{x,y}^j = 1, \text{ if } x_j = y_j$$
 
$$\delta_{x,y}^j = 0, \text{ otherwise }$$

$$X = \sum_{j=1}^{v} w_j \delta_{x,y}^j$$
, where

$$\delta_{x,y}^j = 1$$
, if  $x_j \neq y_j$ 

$$\delta_{x,y}^{j} = 0$$
, otherwise

N total nonmissing pairs

$$N = \sum_{j=1}^{v} w_j$$

$$d_{28}(x,y) = X$$

$$s_{29}(x, y) = M/N$$

$$d_{30}(x, y) = \sqrt{1 - M/N} = \sqrt{(X/N)}$$

$$d_{31}(x, y) = 1 - M/N = X/N$$

$$s_{32}(x, y) = (M - X)/N$$

$$s_{33}(x, y) = M/(M + 2X)$$

$$s_{34}(x, y) = 2M/(2M + X)$$

minate (missing) since there is no mismatch.

$$s_{35}(x, y) = M/X$$

The following notation is used for computing  $s_{36}(x, y)$  to  $d_{41}(x, y)$ . Notice that only the nonmissing pairs are discussed in the following section; all the pairs with at least one missing value are excluded from any of the computations in the following section because  $w_j = 0$ , if either  $x_j$  or  $y_j$  is missing.

Also, the observed nonmissing data of an asymmetric binary variable can have only two possible outcomes: presence or absence. Therefore, the notation, PX (present mismatches), always has a value of zero for an asymmetric binary variable.

The following methods distinguish between the presence and absence of attributes.

X mismatches with at least one present

$$X = \sum_{j=1}^{v} w_j \delta_{x,y}^j$$
, where

$$\delta_{x,y}^{j} = 1$$
, if  $x_{j} \neq y_{j}$  and not both  $x_{j}$  and  $y_{j}$  are absent

$$\delta_{x,y}^{j} = 0$$
, otherwise

*PM* present matches

$$PM = \sum_{j=1}^{v} w_j \delta_{x,y}^j$$
, where

$$\delta_{x,y}^{j} = 1$$
, if  $x_{j} = y_{j}$  and both  $x_{j}$  and  $y_{j}$  are present

$$\delta_{x,y}^{j} = 0$$
, otherwise

PX present mismatches

$$PX = \sum_{j=1}^{v} w_j \delta_{x,y}^j$$
, where

$$\delta_{x,y}^{j} = 1$$
, if  $x_{j} \neq y_{j}$  and both  $x_{j}$  and  $y_{j}$  are present

$$\delta_{x,y}^j = 0$$
, otherwise

PP both present = PM + PX

P at least one present = PM + X

PAX present-absent mismatches

$$PAX = \sum_{j=1}^{v} w_j \delta_{x,y}^j$$
, where

 $\delta_{x,y}^{j} = 1$ , if  $x_{j} \neq y_{j}$  and either  $x_{j}$  is present and  $y_{j}$  is absent or  $x_{j}$  is absent and  $y_{j}$  is present

 $\delta_{x,y}^{j} = 0$  otherwise

N total nonmissing pairs

$$N = \sum_{j=1}^{v} w_j$$

### **Methods That Accept Asymmetric Nominal and Ratio Variables**

JACCARD Jaccard similarity coefficient

The JACCARD method is equivalent to the SIMRATIO method if there are only ratio variables; if there are both ratio and asymmetric nominal variables, the coefficient is computed as sum of the coefficient from the ratio variables (SIMRATIO) and the coefficient from the asymmetric nominal variables.

$$s_{36}(x, y) = s_{16}(x, y) + PM/P$$

DJACCARD Jaccard dissimilarity coefficient

The DJACCARD method is equivalent to the DISRATIO method if there are only ratio variables; if there are both ratio and asymmetric nominal variables, the coefficient is computed as sum of the coefficient from the ratio variables (DISRATIO) and the coefficient from the asymmetric nominal variables.

$$d_{37}(x, y) = d_{17}(x, y) + X/P$$

### **Methods That Accept Asymmetric Nominal Variables**

DICE Dice coefficient or Czekanowski/Sorensen similarity coefficient

$$s_{38}(x, y) = 2PM/(P + PM)$$

RR Russell and Rao. This is the binary equivalent of the dot product coefficient.

$$s_{39}(x, y) = PM/N$$

BLWNM | BRAYCURTIS Binary Lance and Williams, also known as Bray and Curtis coefficient

$$d_{40}(x, y) = X/(PAX + 2PP)$$

K1 Kulcynski 1. The coefficient between an observation and itself is always indeterminate

(missing) since there is no mismatch.

$$d_{41}(x, y) = PM/X$$

# **Missing Values**

### Standardization versus No Standardization

You can replace the missing values with or without standardization. Missing values are replaced after standardization by specifying either the REPLACE option in the PROC DISTANCE statement or the MISSING= option in the VAR statement.

To replace missing values without standardization, use the following two options:

- the NOSTD option in the PROC DISTANCE statement. The NOSTD option suppresses standardization but still replaces the missing values with the location of the method or the numeric value specified in the MISSING= option in the VAR statement.
- the REPONLY option in the PROC DISTANCE statement. PROC DISTANCE replaces missing values with the location of the standardization method or with the numeric value specified in the MISSING= option in the VAR statement. This approach assumes that standardization is not mandatory (see the section "Standardization" on page 2290).

### **Eliminating Observations with Missing Values**

If you specify the NOMISS option, PROC DISTANCE omits observations with any missing values in the analyzed variables from computation of the location and scale measures.

### **Distance Measures**

If you specify the NOMISS option, PROC DISTANCE generates missing distance for observations with missing values. If the NOMISS option is not specified, the sum of total weights, no matter if an observation is missing or not, is incorporated into the computation of some of the proximity measures. See the section "Details: DISTANCE Procedure" on page 2309 for the formulas and descriptions.

### Formatted versus Unformatted Values

PROC DISTANCE uses the formatted values from a character variable, if the variable has a format—for example, one assigned by a format statement. PROC DISTANCE uses the unformatted values from a numeric variable, even if it has a format.

### **Output Data Sets**

### **OUT= Data Set**

The DISTANCE procedure always produces an output data set, regardless of whether you specify the OUT= option in the PROC DISTANCE statement. PROC DISTANCE displays no output. Use PROC PRINT to display the output data set.

The output data set contains the following variables:

- the ID variable, if any
- the BY variables, if any
- the COPY variables, if any
- the FREQ variable, if any
- the WEIGHT variable, if any
- the new distance variables, named from PREFIX= options along with the ID values, or from the default values

The output data set is of type TYPE=DISTANCE or TYPE=SIMILAR. See the METHOD= option for more information about the output data set types. Data set types do not persist when you copy or modify a data set. You must specify the TYPE= data set option for the new data set, as in the following example:

```
data dist2(type=distance);
    set dist;
run;
```

See the OUT= option for more information about data set type persistence.

### **OUTSDZ= Data Set**

The output data set is a copy of the DATA= data set except that the analyzed variables have been standardized. Analyzed variables are those listed in the VAR statement.

# **Examples: DISTANCE Procedure**

### **Example 36.1: Divorce Grounds – the Jaccard Coefficient**

A wide variety of distance and similarity measures are used in cluster analysis (Anderberg 1973; Sneath and Sokal 1973). If your data are in coordinate form and you want to use a non-Euclidean distance for clustering, you can compute a distance matrix by using the DISTANCE procedure.

Similarity measures must be converted to dissimilarities before being used in PROC CLUSTER. Such conversion can be done in a variety of ways, such as taking reciprocals or subtracting from a large value. The choice of conversion method depends on the application and the similarity measure. If applicable, PROC DISTANCE provides a corresponding dissimilarity measure for each similarity measure.

In the following example, the observations are states. Binary-valued variables correspond to various grounds for divorce and indicate whether the grounds for divorce apply in each of the U.S. states. A value of "1" indicates that the ground for divorce applies, and a value of "0" indicates the opposite. The 0-0 matches are treated as totally irrelevant; therefore, each variable has an asymmetric nominal level of measurement. The absence value is 0.

The DISTANCE procedure is used to compute the Jaccard coefficient (Anderberg 1973, pp. 89, 115, and 117) between each pair of states. The Jaccard coefficient is defined as the number of variables that are coded as 1 for both states divided by the number of variables that are coded as 1 for either or both states. Since dissimilarity measures are required by PROC CLUSTER, the DJACCARD coefficient is selected. Output 36.1.1 displays the distance matrix between the first 10 states.

The CENTROID method is used to perform the cluster analysis, and the resulting tree diagram from PROC CLUSTER is saved into the tree output data set. Output 36.1.2 displays the cluster history.

The TREE procedure generates nine clusters in the output data set out. After being sorted by the state, the out data set is then merged with the input data set divorce. After being sorted by the state, the merged data set is printed to display the cluster membership as shown in Output 36.1.3.

The following statements produce Output 36.1.1 through Output 36.1.3:

```
data divorce;
  length State $ 15;
  input State &$
        Incompatibility Cruelty Desertion Non_Support Alcohol
        Felony Impotence Insanity Separation @@;
  datalines;
               1 1 1 1 1 1 1 1 1
                                                   1 1 1 0 1 1 1 1 0
Alabama
                                   Alaska
               1 0 0 0 0 0 0 0 0
                                                   0 1 1 1 1 1 1 1 1
Arizona
                                   Arkansas
              100000010
                                                   1 0 0 0 0 0 0 0 0
California
                                   Colorado
Connecticut
               111111011
                                   Delaware
                                                   1 0 0 0 0 0 0 0 1
```

```
... more lines ...
                                              10000011
            1 0 0 0 0 0 0 0 1
                                     Wyoming
title 'Grounds for Divorce';
proc distance data=divorce method=djaccard absent=0 out=distjacc;
   var anominal(Incompatibility--Separation);
   id state;
run;
proc print data=distjacc(obs=10);
   id state; var alabama--georgia;
   title2 'First 10 States';
run;
title2;
proc cluster data=distjacc method=centroid
            pseudo outtree=tree;
   id state;
   var alabama--wyoming;
run;
proc tree data=tree noprint n=9 out=out;
   id state;
run;
proc sort;
  by state;
run;
data clus;
   merge divorce out;
   by state;
run;
proc sort;
  by cluster;
run;
proc print;
   id state;
   var Incompatibility--Separation;
   by cluster;
run;
```

### Grounds for Divorce First 10 States

State	Alabama	Alaska	Arizona	Arkansas	California	Colorado	Connecticut	Delaware	Florida	Georgia
Alabama	0.00000									
Alaska	0.22222	0.00000								•
Arizona	0.88889	0.85714	0.00000			•				
Arkansas	0.11111	0.33333	1.00000	0.00000						•
California	0.77778	0.71429	0.50000	0.88889	0.00000	•				
Colorado	0.88889	0.85714	0.00000	1.00000	0.50000	0.00000				
Connecticut	0.11111	0.33333	0.87500	0.22222	0.75000	0.87500	0.00000			
Delaware	0.77778	0.87500	0.50000	0.88889	0.66667	0.50000	0.75000	0.00000		•
Florida	0.77778	0.71429	0.50000	0.88889	0.00000	0.50000	0.75000	0.66667	0.00000	
Georgia	0.22222	0.00000	0.85714	0.33333	0.71429	0.85714	0.33333	0.87500	0.71429	0

Output 36.1.2 Clustering History

**Grounds for Divorce** 

The CLUSTER Procedure Centroid Hierarchical Cluster Analysis

Root-Mean-Square Distance Between Observations 0.694873

Output 36.1.2 continued

Cluster History									
Number		0.0010.		,		Norm			
of				Pseudo F		Centroid			
Clusters	Clusters		Freq	Statistic	t-Squared	Distance Tie			
	Arizona	Colorado	2	•	•	0 T			
	California	Florida	2	•	•	0 T			
	Alaska	Georgia	2	•	•	0 T			
	Delaware	Hawaii	2	•	•	0 T			
	Connecticut	Idaho	2	•	•	0 T			
	CL49	lowa	3			0 T			
	CL47	Kansas	3			0 T			
	CL44	Kentucky	4 5			0 T 0 T			
	CL42	Michigan		•	•	0 T			
	CL41 CL43	Minnesota Mississippi	6 4	•	•	0 T			
		Mississippi	7	•	•				
	CL40	Missouri	•		•	0 T 0 T			
	CL38	Montana	8	•	•				
	CL37	Nebraska	9			0 T			
	North Dakota	Oklahoma	2			0 T			
	CL36	Oregon	10			0 T			
	Massachusetts New Hampshire		2			0 T			
	•		2		•	0 T			
	CL46	Washington	3			0 T			
	CL31	Wisconsin	4			0 T			
	Nevada	Wyoming	2	. 1501	•	0 0.1500 T			
	Alabama	Arkansas	2	1561		0.1599 T			
	CL33	CL32	4	479		0.1799 T			
	CL39	CL35	6	265	•	0.1799 T			
	CL45	West Virginia	3	231		0.1799			
	Maryland	Pennsylvania	2	199		0.2399			
	CL28	Utah	3	167	3.2	0.2468			
	CL27	Ohio	5	136	5.4	0.2698			
	CL26	Maine	7	111	8.9	0.2998			
	CL23	CL21	10	75.2	8.7	0.3004			
	CL25	New Jersey	4	71.8	6.5	0.3053 T			
	CL19	Texas	5	69.1	2.5	0.3077			
	CL20	CL22	15	48.7	9.9	0.3219			
	New York	Virginia	2	50.1		0.3598			
	CL18	Vermont	6	49.4	2.9	0.3797			
	CL17	Illinois	16	47.0	3.2	0.4425			
	CL14	CL15	22	29.2	15.3	0.4722 0.4707 T			
	CL48	CL29	4	29.5		0.4797 T			
	CL13	CL24 South Dakota	24	27.6	4.5	0.5042			
	CL11 Louisiana		25 3	28.4 30.3	2.4	0.5449			
		CL16		30.3	3.5	0.5844			
	CL34	CL30	14	23.3	15.0	0.7196			
	CL10	CL12	18	19.3	15.0	0.7175			
	CL10	South Carolina	26	21.4	4.2	0.7384			
	CL6	New Mexico	27	24.0	4.7	0.8303			
4	CL5	Indiana	28	28.9	4.1	0.8343			

#### **Cluster History** Number Norm Pseudo F Pseudo Centroid Clusters **Clusters Joined** Freq Statistic t-Squared Distance Tie **3** CL4 CL9 31.7 10.9 0.8472 **2** CL3 North Carolina 32 55.1 4.1 1.0017 50 1 CL2 CL7 55.1 1.0663

## Output 36.1.3 Cluster Membership

#### **Grounds for Divorce**

#### CLUSTER=1

State	Incompatibility	Cruelty	Desertion	Non_Support	Alcohol	Felony	Impotence	Insanity	Separation
Arizona	1	0	0	0	0	0	0	0	0
Colorado	1	0	0	0	0	0	0	0	0
Iowa	1	0	0	0	0	0	0	0	0
Kentucky	1	0	0	0	0	0	0	0	0
Michigan	1	0	0	0	0	0	0	0	0
Minnesota	1	0	0	0	0	0	0	0	0
Missouri	1	0	0	0	0	0	0	0	0
Montana	1	0	0	0	0	0	0	0	0
Nebraska	1	0	0	0	0	0	0	0	0
Oregon	1	0	0	0	0	0	0	0	0

#### CLUSTER=2

State	Incompatibility	Cruelty	Desertion	Non_Support	Alcohol	Felony	Impotence	Insanity	Separation
California	1	0	0	0	0	0	0	1	0
Florida	1	0	0	0	0	0	0	1	0
Nevada	1	0	0	0	0	0	0	1	1
Wyoming	1	0	0	0	0	0	0	1	1

## Output 36.1.3 continued

#### CLUSTER=3

State	Incompatibility	Cruelty	Desertion	Non_Support	Alcohol	Felony	Impotence	Insanity	Separation
Alabama	1	1	1	1	1	1	1	1	1
Alaska	1	1	1	0	1	1	1	1	0
Arkansas	0	1	1	1	1	1	1	1	1
Connecticut	1	1	1	1	1	1	0	1	1
Georgia	1	1	1	0	1	1	1	1	0
Idaho	1	1	1	1	1	1	0	1	1
Illinois	0	1	1	0	1	1	1	0	0
Kansas	1	1	1	0	1	1	1	1	0
Maine	1	1	1	1	1	0	1	1	0
Maryland	0	1	1	0	0	1	1	1	1
Massachusetts	1	1	1	1	1	1	1	0	1
Mississippi	1	1	1	0	1	1	1	1	0
<b>New Hampshire</b>	1	1	1	1	1	1	1	0	0
New Jersey	0	1	1	0	1	1	0	1	1
North Dakota	1	1	1	1	1	1	1	1	0
Ohio	1	1	1	0	1	1	1	0	1
Oklahoma	1	1	1	1	1	1	1	1	0
Pennsylvania	0	1	1	0	0	1	1	1	0
Rhode Island	1	1	1	1	1	1	1	0	1
South Dakota	0	1	1	1	1	1	0	0	0
Tennessee	1	1	1	1	1	1	1	0	0
Texas	1	1	1	0	0	1	0	1	1
Utah	0	1	1	1	1	1	1	1	0
Vermont	0	1	1	1	0	1	0	1	1
West Virginia	1	1	1	0	1	1	0	1	1

#### CLUSTER=4

State	Incompatibility	Cruelty	Desertion	Non_Support	Alcohol	Felony	Impotence	Insanity	Separation
Delaware	1	0	0	0	0	0	0	0	1
Hawaii	1	0	0	0	0	0	0	0	1
Washington	1	0	0	0	0	0	0	0	1
Wisconsin	1	0	0	0	0	0	0	0	1

#### CLUSTER=5

State	Incompatibility	Cruelty	Desertion	Non_Support	Alcohol	Felony	Impotence	Insanity	Separation
Louisiana	0	0	0	0	0	1	0	0	1
New York	0	1	1	0	0	1	0	0	1
Virginia	0	1	0	0	0	1	0	0	1

#### CLUSTER=6

State	Incompatibility	Cruelty	Desertion	Non_Support	Alcohol	Felony	Impotence	Insanity	Separation
South Carolina	0	1	1	0	1	0	0	0	1

## Output 36.1.3 continued

#### CLUSTER=7

State	Incompatibilit	y Cruelt	y Desertio	n Non_s	Support	Alcohol	Felony	Impotence	Insanit	/ Separatio
New Mexico		1	1	1	0	0	0	0	(	)
				CLU	STER=8					
State In	compatibility	Cruelty	Desertion	Non_Su	pport Ale	cohol Fe	lony In	npotence li	nsanity S	Separation
Indiana	1	0	0		0	0	1	1	1	0
				CLU:	STER=9					
tate	Incompatibil	ity Crue	lty Desert	ion Non	_Support	Alcoho	l Felon	y Impotenc	e Insan	ty Separati
orth Carolina	1	0	0	0	0	) (	)	0	1	1

#### **Example 36.2: Financial Data – Stock Dividends**

The following data set contains the average dividend yields for 15 utility stocks in the United States. The observations are names of the companies, and the variables correspond to the annual dividend yields for the period 1986–1990. The objective is to group similar stocks into clusters.

Before the cluster analysis is performed, the correlation similarity is chosen for measuring the closeness between each observation. Since distance type of measures are required by PROC CLUSTER, METHOD=DCORR is used in the PROC DISTANCE statement to transform the correlation measures to the distance measures. Notice that in Output 36.2.1, all the values in the distance matrix are between 0 and 2.

PROC CLUSTER performs hierarchical clustering by using agglomerative methods based on the distance data created from the previous PROC DISTANCE statement. Since the cubic clustering criterion is not suitable for distance data, only the pseudo F statistic is requested to identify the number of clusters.

The two clustering methods are Ward's and the average linkage methods. Since the results of the pseudo  $t^2$  statistic from both Ward's and the average linkage methods contain many missing values, only the plot of the pseudo F statistic versus the number of clusters is requested along with the dendrogram by specifying PLOTS(ONLY)=(PSF DENDROGRAM) in the PROC CLUSTER statement.

Both Output 36.2.2 and Output 36.2.3 suggest four clusters. Both methods produce the same clustering result, as shown in Output 36.2.4 and Output 36.2.5. The four clusters are as follows:

- Cincinnati G&E and Detroit Edison
- Texas Utilities and Pennsylvania Power & Light
- Union Electric, Iowa-Ill Gas & Electric, Oklahoma Gas & Electric, and Wisconsin Energy
- Orange & Rockland Utilities, Kentucky Utilities, Kansas Power & Light, Allegheny Power, Green Mountain Power, Dominion Resources, and Minnesota Power & Light

```
title 'Stock Dividends';
data stock;
  length Company $ 27;
  input Company &$ Div_1986 Div_1987 Div_1988 Div_1989 Div_1990;
  datalines;
Cincinnati G&E
                         8.4
                              8.2
                                     8.4
                                           8.1
                                                  8.0
Texas Utilities
                                         8.9
                         7.9
                             8.9 10.4
                                                  8.3
Detroit Edison
                         9.7 10.7 11.4 7.8
                                                 6.5
Orange & Rockland Utilities 6.5 7.2
                                     7.3
                                                 7.9
                                           7.7
                                         7.2
                             6.9
                                    7.0
Kentucky Utilities 6.5
                                                 7.5
Kansas Power & Light 5.9 6.4 6.9 7.4 8.0
                        7.1 7.5 8.4 7.8
Union Electric
                                                 7.7
Dominion Resources
                             6.9 7.0
                     6.7
                                           7.0 7.4
                                     7.8 7.9
                        6.7 7.3
Allegheny Power
                                                 8.3
Minnesota Power & Light 5.6 6.1 7.2 7.0
                                                 7.5
Iowa-Ill Gas & Electric
                       7.1 7.5 8.5 7.8
                                                 8.0
Pennsylvania Power & Light 7.2
                             7.6 7.7 7.4
                                                 7.1
Oklahoma Gas & Electric 6.1 6.7 7.4 6.7 6.8
Wisconsin Energy
                        5.1 5.7 6.0 5.7 5.9
Green Mountain Power 7.1 7.4 7.8 7.8 8.3
proc distance data=stock method=dcorr out=distdcorr;
  var interval(div_1986 div_1987 div_1988 div_1989 div_1990);
  id company;
run;
proc print data=distdcorr;
  id company;
  title2 'Distance Matrix for 15 Utility Stocks';
run;
title2;
ods graphics on;
/\star compute pseudo statistic versus number of clusters and create plot \star/
proc cluster data=distdcorr method=ward pseudo plots(only)=(psf dendrogram);
  id company;
run;
/* compute pseudo statistic versus number of clusters and create plot */
proc cluster data=distdcorr method=average pseudo plots(only)=(psf dendrogram);
  id company;
run;
ods graphics off;
```

Output 36.2.1 Distance Matrix Based on the DCORR Coefficient

## Stock Dividends Distance Matrix for 15 Utility Stocks

Company	Cincinnati_G_E	Texas_Utili	ties Detroit_	Edison	OrangeRock	land_Utilities	Kentucky_Utilities
Cincinnati G&E	0.00000						
Texas Utilities	0.82056	0.00	000	•			
Detroit Edison	0.40511	0.65	453	0.00000			
Orange & Rockland Utilities	1.35380	0.88	583	1.27306		0.00000	
Kentucky Utilities	1.35581	0.92	539	1.29382		0.12268	0.00000
Kansas Power & Light	1.34227	0.94	371	1.31696		0.19905	0.12874
Union Electric	0.98516	0.29	043	0.89048		0.68798	0.71824
<b>Dominion Resources</b>	1.32945	0.96	853	1.29016		0.33290	0.21510
Allegheny Power	1.30492	0.81	666	1.24565		0.17844	0.15759
Company	Kansas_Power_	Light Un	ion_Electric	Dominio	on_Resources A	.llegheny_Pov	ver
Cincinnati G&E					•		•
Texas Utilities					•		•
Detroit Edison					•		•
Orange & Rockland Utilities		•	٠				
Kentucky Utilities					•		
Kansas Power & Light		0.00000			•		
Union Electric		0.72082	0.00000				
Dominion Resources		0.24189	0.76587		0.00000		
Allegheny Power		0.17029	0.58452		0.27819	0.000	000
Company	Minnesota_Pow	erLight	lowa_III_Gas	Elect	tric Pennsylvani	a_PowerL	ight
Cincinnati G&E		•			•		•
Texas Utilities		•			•		
Detroit Edison		•			•		
Orange & Rockland Utilities					•		•
Kentucky Utilities							•
Kansas Power & Light							•
Union Electric							•
Dominion Resources		•			•		•
Allegheny Power							<u>.</u>
Company	Oklahoma_Gas_	Electric	Wisconsin_E	nergy G	Green_Mountain_	_Power	
Cincinnati G&E							
Texas Utilities		•				•	
Detroit Edison		· ·					
Detroit Edison Orange & Rockland Utilities						•	
Orange & Rockland		· · ·					
Orange & Rockland Utilities		· · · · · · ·					
Orange & Rockland Utilities Kentucky Utilities		· · ·					
Orange & Rockland Utilities Kentucky Utilities Kansas Power & Light							

#### Output 36.2.1 continued

#### **Stock Dividends** Distance Matrix for 15 Utility Stocks

Company	Cincinnati G F	Texas Utilitie	s Detroit_Edison	Orange	Rockland Utilities	Kentucky_Utilities
Minnesota Power & Light	1.24069	0.74082			0.32581	0.30462
Iowa-III Gas & Electric	1.04924	0.43100			0.61166	0.61760
Pennsylvania Power & Light	0.74931	0.3782			1.03566	1.08878
Oklahoma Gas & Electric	1.00604	0.3014	0.86200		0.68021	0.70259
Wisconsin Energy	1.17988	0.54830	1.03081		0.45013	0.47184
Green Mountain Power	1.30397	0.88063	3 1.27176		0.26948	0.17909
						_
Company	Kansas_Power_		_	_	es Allegheny_Pow	
Minnesota Power & Light		0.27231	0.48372	0.357		
Iowa-III Gas & Electric		0.61736	0.16923	0.635	45 0.479	00
Pennsylvania Power & Light		1.12876	0.63285	1.143	54 1.023	58
Oklahoma Gas & Electric		0.73158	0.17122	0.729	77 0.583	91
Wisconsin Energy		0.53381	0.37405	0.519	69 0.375	22
Green Mountain Power		0.15377	0.64869	0.173	60 0.139	58
Company	Minnesota_Pow	erLight lov	va_III_GasElec	ctric Pennsy	lvania_PowerLi	ight
Minnesota Power & Light		0.00000				
Iowa-III Gas & Electric		0.36368	0.00	000		
Pennsylvania Power & Light		0.99384	0.75	596	0.00	000
Oklahoma Gas & Electric		0.50744	0.19	673	0.60	216
Wisconsin Energy		0.36319	0.30	259	0.76	085
Green Mountain Power		0.19370	0.52	083	1.09	269
Company	Oklahoma_Gas_	Electric Wi	sconsin_Energy	Green_Mour	ntain_Power	
Minnesota Power & Light					•	
Iowa-III Gas & Electric						
Pennsylvania Power & Light						
Oklahoma Gas & Electric		0.00000				
Wisconsin Energy		0.28070	0.00000			

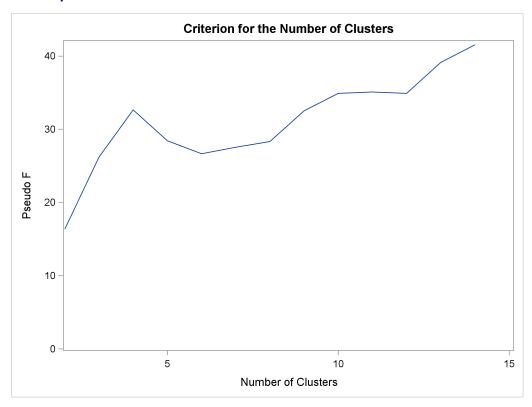
0.44814

0.64175

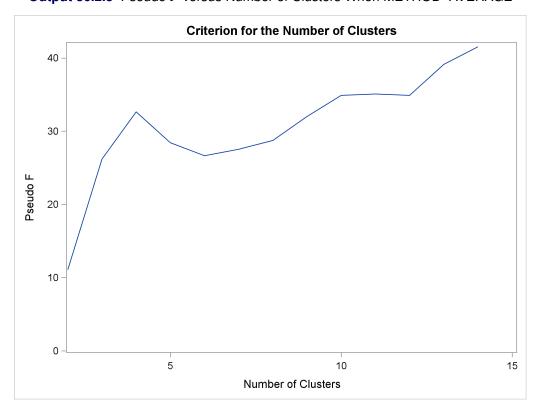
**Green Mountain Power** 

0

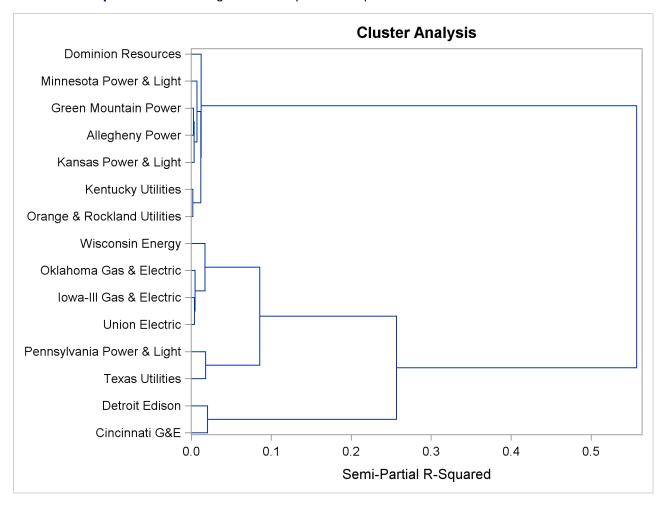
Output 36.2.2 Pseudo F versus Number of Clusters When METHOD=WARD

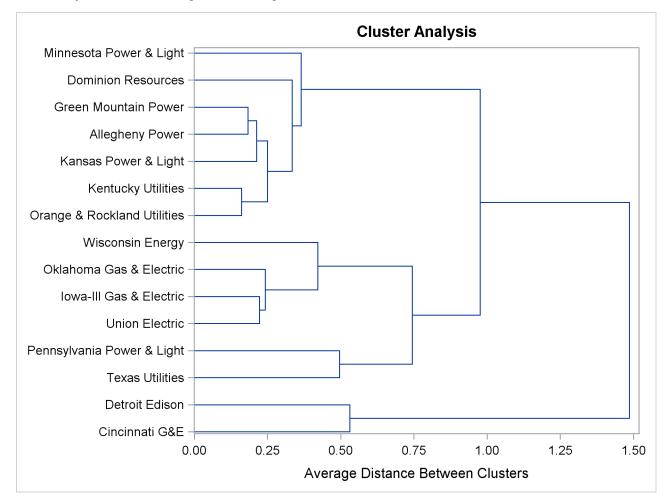


 $\begin{tabular}{lll} \textbf{Output 36.2.3} & \textbf{Pseudo} & F & \textbf{versus Number of Clusters When METHOD=AVERAGE} \\ \end{tabular}$ 



Output 36.2.4 Dendrogram of Semipartial R-Square Values When METHOD=WARD





Output 36.2.5 Dendrogram of Average Distance between Clusters When METHOD=AVERAGE

## References

Anderberg, M. R. (1973). Cluster Analysis for Applications. New York: Academic Press.

Gower, J. C., and Legendre, P. (1986). "Metric and Euclidean Properties of Dissimilarity Coefficients." *Journal of Classification* 3:5–48.

Hand, D. J., Daly, F., Lunn, A. D., McConway, K. J., and Ostrowski, E. (1994). *A Handbook of Small Data Sets*. London: Chapman & Hall.

Sneath, P. H. A., and Sokal, R. R. (1973). Numerical Taxonomy. San Francisco: W. H. Freeman.

# Subject Index

absolute level of measurement, definition	Bray and Curtis coefficient, 2315
DISTANCE procedure, 2289	Canberra metric coefficient, 2311
asymmetric binary variable	Chebychev distance coefficient, 2311
DISTANCE procedure, 2289	chi-square coefficient, 2311, 2312
•	City-block distance coefficient, 2311
Binary Lance and Williams nonmetric coefficient	computing distances with weights, 2306
DISTANCE procedure, 2315	correlation dissimilarity coefficient, 2310
Bray and Curtis coefficient	correlation similarity coefficient, 2310
DISTANCE procedure, 2315	cosine coefficient, 2311
	covariance similarity coefficient, 2310
Canberra metric coefficient	Czekanowski/Sorensen similarity coefficient,
DISTANCE procedure, 2311	2315
Chebychev distance coefficient	Dice coefficient, 2315
DISTANCE procedure, 2311	dot Product coefficient, 2311
chi-square coefficient	Euclidean distance coefficient, 2310
DISTANCE procedure, 2311, 2312	examples, 2290, 2317, 2323
City-block distance coefficient	extension of binary variable, 2289
DISTANCE procedure, 2311	formatted values, 2316
clustering and computing distance matrix	formulas for proximity measures, 2309
Correlation coefficients, example, 2323	frequencies, 2307
Jaccard coefficients, example, 2317	functional summary, 2296
clustering and scaling	fuzz factor, 2297
DISTANCE procedure, example, 2292	generalized Euclidean distance coefficient, 231
correlation dissimilarity coefficient	Gower's dissimilarity coefficient, 2310
DISTANCE procedure, 2310	Gower's similarity coefficient, 2319
correlation similarity coefficient	Hamann coefficient, 2313
DISTANCE procedure, 2310	Hamming distance coefficient, 2313
cosine coefficient	identity transformation, 2289
DISTANCE procedure, 2311	initial estimates for A-estimates, 2297
covariance similarity coefficient	interval level of measurement, 2288
DISTANCE procedure, 2310	Jaccard dissimilarity coefficient, 2315
Czekanowski/Sorensen similarity coefficient	Jaccard similarity coefficient, 2314
DISTANCE procedure, 2315	Kulcynski 1 coefficient, 2315
	Lance-Williams nonmetric coefficient, 2311
Dice coefficient	levels of measurement, 2288
DISTANCE procedure, 2315	linear transformation, 2289
distance measures available in DISTANCE procedure,	log-interval level of measurement, 2289
see proximity measures	many-to-one transformation, 2288
DISTANCE procedure	Minkowski $L_p$ distance coefficient, 2310
absent-absent match, asymmetric binary variable, 2289	missing values, 2300–2302, 2305, 2315
absent-absent match, example, 2317	monotone increasing transformation, 2288
absolute level of measurement, definition, 2289	nominal level of measurement, 2288
affine transformation, 2288	nominal variable, 2289
asymmetric binary variable, 2289	normalization, 2300, 2302
available levels of measurement, 2303	one-to-one transformation, 2288
available options for the option list, 2303	ordinal level of measurement, 2288
Binary Lance and Williams nonmetric coefficient,	output data sets, 2300, 2301, 2316
2315	Overlap dissimilarity coefficient, 2311

Overlap similarity coefficient, 2311	interval level of measurement
phi-square coefficient, 2312	DISTANCE procedure, 2288
power distance coefficient, 2311	
power transformation, 2289	Jaccard dissimilarity coefficient
ratio level of measurement, 2289	DISTANCE procedure, 2315
Roger and Tanimoto coefficient, 2313	Jaccard similarity coefficient
Russell and Rao similarity coefficient, 2315	DISTANCE procedure, 2314
scaling variables, 2290	Y 1 114 00 1
shape distance coefficient, 2310	Kulcynski 1 coefficient
similarity Ratio coefficient, 2311	DISTANCE procedure, 2315
similarity ratio coefficient, 2311	Lance-Williams nonmetric coefficient
simple Matching coefficient, 2313	DISTANCE procedure, 2311
simple matching dissimilarity coefficient, 2313	levels of measurement
size distance coefficient, 2310	
Sokal and Sneath 1 coefficient, 2313	DISTANCE procedure, 2288
Sokal and Sneath 3 coefficient, 2313	log-interval level of measurement
squared correlation dissimilarity coefficient, 2310	DISTANCE procedure, 2289
squared correlation similarity coefficient, 2310	Minkowski L <sub>p</sub> distance coefficient
squared Euclidean distance coefficient, 2310	DISTANCE procedure, 2310
squared simple matching dissimilarity coefficient,	missing values
2313	DISTANCE procedure, 2300–2302, 2305, 2315
standardization methods, 2304	DISTAINCE procedure, 2500–2502, 2505, 2515
standardization suppression, 2300	nominal level of measurement
standardization with frequencies, 2307	DISTANCE procedure, 2288
standardization with weights, 2307	nominal variable
standardization, default methods, 2290, 2304	DISTANCE procedure, 2289
standardization, example, 2292	r
standardization, mandatory, 2290	ordinal level of measurement
strictly increasing transformation, 2288	DISTANCE procedure, 2288
summary of options, 2296	ordinal variables
symmetric binary variable, 2289	transformed to interval (RANKSCORE=), 2301
transforming ordinal variables to interval, 2289,	Overlap dissimilarity coefficient
2301	DISTANCE procedure, 2311
weights, 2306, 2307	Overlap similarity coefficient
dot Product coefficient	DISTANCE procedure, 2311
DISTANCE procedure, 2311	
r	phi-square coefficient
Euclidean distance coefficient	DISTANCE procedure, 2312
DISTANCE procedure, 2310	power distance coefficient
	DISTANCE procedure, 2311
formatted values	proximity measures
DISTANCE procedure, 2316	available methods for computing (DISTANCE),
	2297
generalized Euclidean distance coefficient	formulas(DISTANCE), 2309
DISTANCE procedure, 2311	
Gower's dissimilarity coefficient	ratio level of measurement
DISTANCE procedure, 2310	DISTANCE procedure, 2289
Gower's similarity coefficient	Roger and Tanimoto coefficient
DISTANCE procedure, 2309	DISTANCE procedure, 2313
II CC	Russell and Rao similarity coefficient
Hamann coefficient	DISTANCE procedure, 2315
DISTANCE procedure, 2313	casling variables
Hamming distance coefficient	scaling variables
DISTANCE procedure, 2313	DISTANCE procedure, 2290
	shape distance coefficient

```
DISTANCE procedure, 2310
similarity Ratio coefficient
    DISTANCE procedure, 2311
similarity ratio coefficient
    DISTANCE procedure, 2311
simple Matching coefficient
    DISTANCE procedure, 2313
simple matching dissimilarity coefficient
    DISTANCE procedure, 2313
size distance coefficient
    DISTANCE procedure, 2310
Sokal and Sneath 1 coefficient
    DISTANCE procedure, 2313
Sokal and Sneath 3 coefficient
    DISTANCE procedure, 2313
squared correlation dissimilarity coefficient
    DISTANCE procedure, 2310
squared correlation similarity coefficient
    DISTANCE procedure, 2310
squared Euclidean distance coefficient
    DISTANCE procedure, 2310
squared simple matching dissimilarity coefficient
    DISTANCE procedure, 2313
standardization
    comparisons between DISTANCE and STDIZE
        procedures, 2290
standardization suppression
    DISTANCE procedure, 2300
STD= option (DISTANCE), 2304
symmetric binary variable
    DISTANCE procedure, 2289
transformations
    affine(DISTANCE), 2288
    identity(DISTANCE), 2289
    linear(DISTANCE), 2289
    many-to-one(DISTANCE), 2288
    monotone increasing(DISTANCE), 2288
    one-to-one(DISTANCE), 2288
    power(DISTANCE), 2289
    strictly increasing(DISTANCE), 2288
transforming ordinal variables to interval
    DISTANCE procedure, 2289
```

# Syntax Index

ABSENT= option	DISTANCE procedure, 2307
PROC DISTANCE statement, 2297	FUZZ= option
VAR statement, 2305	PROC DISTANCE statement, 2297
ADD= option	
PROC DISTANCE statement, 2297	ID statement
	DISTANCE procedure, 2306
BY statement	INITIAL= option
DISTANCE procedure, 2307	PROC DISTANCE statement, 2297
COPY statement	METHOD= option
DISTANCE procedure, 2307	PROC DISTANCE statement, 2297
•	MISSING= option
DATA= option	VAR statement, 2305
PROC DISTANCE statement, 2297	MULT= option
DISTANCE procedure, BY statement, 2307	PROC DISTANCE statement, 2300
DISTANCE procedure, COPY statement, 2307	
DISTANCE procedure, FREQ statement, 2307	NOMISS option
DISTANCE procedure, ID statement, 2306	PROC DISTANCE statement, 2300
DISTANCE procedure, PROC DISTANCE statement,	NORM option
2296	PROC DISTANCE statement, 2300
ABSENT= option, 2297	NOSTD option
ADD= option, 2297	PROC DISTANCE statement, 2300
DATA= option, 2297	
FUZZ= option, 2297	ORDER= option
INITIAL= option, 2297	VAR statement, 2306
METHOD= option, 2297	OUT= option
MULT= option, 2300	PROC DISTANCE statement, 2300
NOMISS, 2300	OUTSDZ= option
NORM option, 2300	PROC DISTANCE statement, 2301
NOSTD, 2300	
OUT= option, 2300	PREFIX= option
OUTSDZ= option, 2301	PROC DISTANCE statement, 2301
PREFIX= option, 2301	PROC DISTANCE statement, see DISTANCE
RANKSCORE= option, 2301	procedure
REPLACE, 2301	DANIZCODE antique
REPONLY, 2302	RANKSCORE= option
SHAPE= option, 2302	PROC DISTANCE statement, 2301
SNORM option, 2302	REPLACE option
STDONLY option, 2302	PROC DISTANCE statement, 2301
UNDEF= option, 2302	REPONLY option
VARDEF= option, 2302	PROC DISTANCE statement, 2302
DISTANCE procedure, VAR statement	SHAPE= option
ABSENT= option, 2305	PROC DISTANCE statement, 2302
MISSING= option, 2305	SNORM option
ORDER= option, 2306	PROC DISTANCE statement, 2302
WEIGHTS= option, 2306	STDONLY option
DISTANCE procedure, WGT statement, 2307	*
•	PROC DISTANCE statement, 2302
FREQ statement	UNDEF= ontion

PROC DISTANCE statement, 2302

VARDEF= option PROC DISTANCE statement, 2302

WEIGHTS= option VAR statement, 2306 WGT statement DISTANCE procedure, 2307