PSYCHOLOGICAL SCALING WITHOUT A UNIT OF MEASUREMENT

BY CLYDE H. COOMBS[1]
University of Michigan

I. INTRODUCTION

The concept of measurement has generally meant the assignment of numbers to objects with the condition that these numbers obey the rules of arithmetic (1). This concept of measurement requires a ratio scale—one with a non-arbitrary origin of zero and a constant unit of measurement (3). The scales which are most widely made use of in psychology are regarded as interval scales in that the origin is recognized to be arbitrary and the unit of measurement is assumed to be constant. But this type of scale should be used only if it can be experimentally demonstrated by manipulation of the objects that the numbers assigned to the objects obey the laws of addition. The unit of measurement in psychology, however, is obtained by a combination of definitions and assumptions, which, if regarded as a first approximation and associated with a statistical theory of error, serves many practical purposes. But because we may sometimes question the meaning of the definitions and the validity of the assumptions which lead to a unit of measurement, it is our intent in this paper to develop a new type of scale not involving a unit of measurement. This type of scale is an addition to the types set up by S. S. Stevens (3). Stevens recognized ratio, interval, ordinal, and nominal scales. The type which we shall develop falls logically between an interval scale and an ordinal scale. We shall make no assumption of equality of intervals, or any other assumption which leads to a unit of measurement. We shall find, however, that on the basis of tolerable assumptions and with appropriate technique we are able to *order* the magnitude of the intervals between objects. We have called such a scale an "ordered metric." We shall develop the concepts first in an abstract manner with a hypothetical experiment and then illustrate the ideas with an actual experiment. Under the limitations of a single paper we shall not present the psychological theory underlying some of the concepts and we shall place certain very limiting conditions on our hypothetical data in order to simplify the presentation.

II. THE PROBLEM

When we set up an attitude scale by any of a variety of methods, for example the method of paired comparisons and the law of comparative judgment, we order statements of opinion on the attitude continuum and assign a number to each statement. We recognize in this instance

[1] This paper is a condensation of some of the ideas contained in a chapter of a general theory of psychological scaling developed in 1948–49 under the auspices of the Rand Corporation and while in residence in the Department and the Laboratory of Social Relations, Harvard University. While the author carries the responsibility for the ideas contained herein, their development would not have been possible without the criticism and stimulation of Samuel A. Stouffer, C. Frederick Mosteller, Paul Lazarsfeld, and Benjamin W. White in a joint seminar during that year. Development of the theory before and after the sojourn at Harvard was made possible by the support of the Bureau of Psychological Services, Institute for Human Adjustment, Horace H. Rackham School of Graduate Studies, University of Michigan.

that the origin for the numbers is arbitrary. We then follow one of several possible procedures (determining which statements an individual will indorse, for example) to locate the positions of individuals on this same continuum. Because both individuals and stimuli have positions on this continuum we shall call it a joint distribution, joint continuum, or J scale. In general, with a psychological continuum, we might expect that for one individual the statements of opinion, or stimuli, have different scale positions than for another individual. Thurstone (4) has provided the concept of stimulus dispersion to describe this variability of the scale positions on a psychological continuum. We have recently (2) discussed an equivalent concept for the variability of scale positions which an individual may assume in responding to a group of stimuli. These two concepts have been basic to the development of a general theory of scaling to which this paper is an introduction.

For didactic purposes we shall achieve brevity and simplicity for the presentation of the basic ideas underlying an ordered metric scale if we impose certain extreme limiting conditions on the variability of the positions of stimuli and individuals on the continuum. These conditions are that the dispersions of both stimuli and individuals be zero. In other words these conditions are that each stimulus has one and only one scale position for all individuals and that each individual has one and only one scale position for all stimuli. For purposes of future generalization we shall classify these conditions as Class 1 conditions. Stimuli will be designated by the subscript j and the position of a stimulus will be designated the Q_j value of the stimulus on the continuum. The position of an individual on the continuum will be designated the C_i value of an individual i.

If we conceive of the attribute as being an attitude continuum, the C_i value of an individual is the Q_j value of that statement of opinion which perfectly represents the attitude of that individual. In this case the C_i value of an individual is his ideal or norm. We shall assume that the degree to which a stimulus represents an individual's ideal value is dependent upon the nearness of the Q_j value of the stimulus to the C_i value of the individual.

We shall then make the further assumption that if we ask an individual which of two statements he prefers to indorse he will indorse that statement the position of which is nearer to his own position on the continuum.

Thus if asked to choose between two stimuli j and k, the individual will make the response

$$j > k \tag{1}$$

if

$$|Q_j - C_i| < |Q_k - C_i|$$

where $j > k$ signifies the judgment "stimulus j preferred to stimulus k."

Under the extreme limiting conditions we have imposed on the C_i and Q_j values the method of paired comparisons would yield an internally consistent (transitive) set of judgments from each individual, though not necessarily the same for each, and each different set of such judgments could be represented by a unique rank order for the stimuli for that individual. We shall call the rank order of the stimuli for a particular individual a qualitative I scale, or, in general, an I scale.

Thus if an individual placed four stimuli in the rank order A B C D as representing the descending order in which he would indorse them, then,

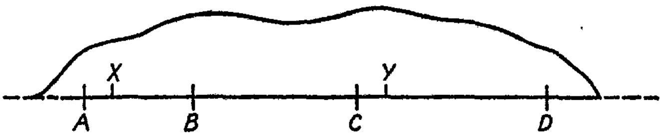

FIG. 1. A joint distribution of stimuli and individuals.

this would be equivalent to the consistent set of judgments A > B, A > C, A > D, B > C, B > D, C > D; where the symbol ">" signifies "prefer to indorse," as before. The order, A B C D, is the qualitative I scale of this individual. Hence for Class 1 conditions it is sufficient to collect the data by the method of rank order; the greater power of the method of paired comparisons would be unnecessary and wasted.

Let us assume now that we have asked each of a group of individuals to place a set of stimuli in rank order with respect to the relative degree to which he would prefer to indorse them. Our understanding of the results that would follow will be clearer if we build a mechanical model which has the appropriate properties. This is very simply done by imagining a hinge located on the J scale at the C_i value of the individual and folding the left side of the J scale over and merging it with the right side. The stimuli on the two sides of the individual will mesh in such a way that the quantity $|C_i - Q_j|$ will be in progressively ascending magnitude from left to right. The order of the stimuli on the folded J scale is the I scale for the individual whose C_i value coincides with the hinge.

It is immediately apparent that there will be classes of individuals whose I scales will be qualitatively identical as to *order* of the stimuli and that these classes will be bounded by the midpoints between pairs of stimuli on the J scale. For example, suppose that there are four stimuli, A B C D, whose Q_j values or positions on a joint continuum are as shown in Fig. 1 and that there is a distribution function of the positions of individuals on this same continuum as indicated.

If we take the individual whose position is at X in Fig. 1, the I scale for that individual is obtained by folding the J scale at that point and we have the scale shown in Fig. 2.

The qualitative I scale for the individual at X is A B C D.

If we take the individual in position Y as shown in Fig. 1 and construct his I scale, we have the scale shown in Fig. 3.

The qualitative I scale for the individual at Y is C D B A.

Consider all individuals to the left of position X on the J scale in Fig. 1. The I scales of all such individuals will be quantitatively different for different positions to the left of X. For every one of them, however, the *order* of the stimuli on the I scale will be the same, A B C D. We shall re-

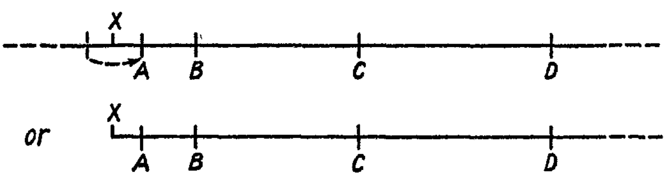

FIG. 2. The I scale of an individual located at X in the joint distribution.

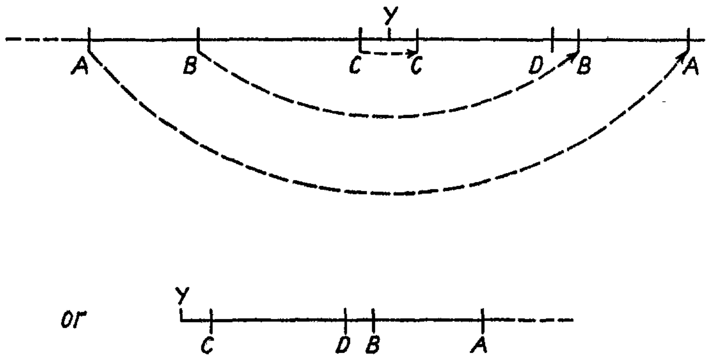

FIG. 3. The I scale of an individual located at Y in the joint distribution.

gard these I scales as being qualitatively the same. As a matter of fact, the I scales of individuals to the right of X continue to be qualitatively the same until we reach the midpoint between stimuli A and B. For an individual immediately to the right of this midpoint the qualitative I scale will be B A C D. I scales immediately to the right of the midpoint AB will continue to be qualitatively the same, B A C D, until we reach the midpoint between stimuli A and C. Immediately past this midpoint the qualitative I scale is B C A D. Continuing beyond this point a complicating factor enters in which we shall discuss in a later section under metric effects.

The distinction which has been made here between quantitative and qualitative I scales is of fundamental importance to the theory of psychological scaling. In almost all existing experimental methods in psychological scaling we do not measure the magnitudes $|C_i - Q_j|$, but only observe their ordinal relations for a fixed C_i.[2]

[2] The ordinal relations of $|C_i - C_j|$ may also be obtained experimentally for a fixed Q_j, over a set of C_i. We call such scales S scales by analogy with I scales. For sake of simplicity they are not treated here but actually the treatment for I scales and S scales is identical if the roles of stimuli and individuals are merely interchanged.

The kind of information which is obtained by the experimenter is essentially qualitative in nature.

As we shall see, data in the form of I scales may tell us certain things:

1) whether there is a latent attribute underlying the preferences or judgments,
2) the order of the stimuli on the joint continuum,
3) something about the relative magnitudes of the distances between stimuli,
4) the intervals which individuals are placed in and the order of the intervals on the continuum, and
5) something about the relative magnitudes of these intervals.

III. A HYPOTHETICAL EXAMPLE

Let us now conduct a hypothetical experiment designed merely to illustrate the technique. Of course this experiment, if actually conducted, would not turn out as we shall construct it, because we shall assume the extreme limiting conditions on Q and C values that were previously imposed.

Let us imagine that we have a number of members of a political party and that we have four individuals who are potential presidential candidates. Let us ask each member of the party to place the four candidates, designated A, B, C, and D, in the rank order in which he would prefer them as President. With four stimuli the potential number of qualitatively different rank orders is 24—the number of permutations of four things taken four at a time. If there were no systematic forces at work among the party members we would get a distribution of occurrences of these 24 I scales which could be fitted by a Poisson or Binomial distribution, everything could be attributed to chance, and the experiment would stop there. Instead, let us imagine for illustrative purposes a different result equally extreme in the opposite direction. Let us imagine that from the N individuals doing the judging only seven qualitatively different I scales were obtained and these were the following:

I_1	A B C D
I_2	B A C D
I_3	B C A D
I_4	C B A D
I_5	C B D A
I_6	C D B A
I_7	D C B A

The significance of the deviation of such results as these from pure chance would be self-evident. Consequently we would look at these seven I scales to see if there was some systematic latent attribute represented by a joint continuum such that individual differences and stimulus differences on such a continuum could account for these manifest data. Studying the set of seven I scales we observe that two of them are identical except in reverse order, A B C D and D C B A. Furthermore we see that in going from one to the next, two adjacent stimuli in the one have changed positions in the next. These are the characteristics of a set of I scales which have been generated from a single J scale. Seven I scales is the maximum number that one can obtain from a single quantitative J scale of four stimuli under the conditions of Class 1. The systematic latent attribute underlying this set of I scales is represented by the J scale which generates them. Our objective, then, is to recover this J scale and discover its properties or characteristics.

To recover the J scale we proceed as follows. Every complete set of I scales has two and only two scales which are identical except in reverse order. These are the I scales which arise from the first and last intervals of the J scale. Consequently these two I scales immediately define the ordinal relations of the stimuli on the J scale, in this case A B C D (the reverse order, of course, is equally acceptable). From the seven I scales we can order on the J scale the six midpoints between all possible pairs of stimuli. In going down the ordered list of I scales as previously determined, the pair of adjacent stimuli in one I scale which have changed places in the next I scale specify the midpoint on the J scale which has been passed.

Thus in the first interval (Fig. 4), we have all the individuals to the left of the midpoint between stimuli A and B. The second I scale is B A C D, and as stimuli A and B have changed places in going from the I_1 scale to the

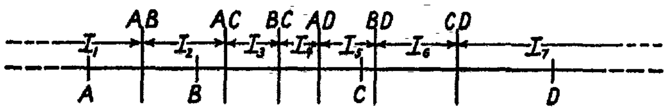

FIG. 4. An example of how the midpoints of four stimuli may section the joint distribution into seven intervals, each characterized by an I scale.

I_2 scale we have passed the midpoint AB. In going from I_2 to I_3 stimuli A and C exchange orders on the two I scales and hence the midpoint between A and C is the boundary between I_2 and I_3. If we continue this process we see that the order of the six midpoints is as follows: AB, AC, BC, AD, BD, CD. These six boundaries section the joint distribution into seven intervals which are ordered as also are the stimuli. From the order of the six midpoints in the case of four stimuli we have one and only one piece of information about metric relations on the joint continuum. Because midpoint BC precedes AD we know that the distance between stimuli C and D is greater than the distance between stimuli A and B. We shall discuss these points and characteristics in more detail in the section on metric effects. There are then an infinite variety of quantitatively different J scales which would yield this same set of seven I scales—but there is only one qualitative J scale. The J scale in Fig. 4 meets the conditions necessary to yield the manifest data.

It must be emphasized that all *metric* magnitudes in Fig. 4 are arbitrary except that the distance from stimulus A to B is less than the distance from stimulus C to D.

With the qualitative information obtained in this experiment about the latent attribute underlying the preferences for presidential candidates the next task is the identification of this attribute. Here all the experimenter can do is to ask himself what it is that these stimuli have and the individuals have to these different degrees as indicated by their ordinal and metric relations. One might find in this hypothetical case that it appears to be a continuum of liberalism, for example, or of isolationism. One would then have to conduct an independent experiment with other criteria to validate one's interpretation.

IV. METRIC EFFECTS

While the data with which we deal in the vast majority of scaling experiments are qualitative and non-numerical there are certain relations between the manifest data and the metric relations of the continuum. These relations have not all been worked out and general expressions have yet to be developed. The complexity of the relations very rapidly increases with the number of stimuli; therefore to illustrate the effect of metric we shall take the simplest case in which its effect is made apparent, the case of four stimuli.

With four stimuli, A, B, C, and D, there are 24 permutations possible. Thus it is possible to find 24 qualitatively different I scales. Also, obviously, each of these 24 orders could occur as a J scale which could give rise to a set of I scales. Half of the J scales may be regarded as merely mirror images of the other half. Thus if we have a J scale with the stimuli ordered B A D C and identify the continuum as liberalism-conservativism, then, in principle, we also have the J scale with the stimuli ordered C D A B and would identify it as conservativism-liberalism. Hence there are only twelve J scales which may be regarded as qualitatively distinguishable on the basis of the order of the four stimuli. I scales which are mirror images of each other, though, are definitely not to be confused. They may well represent entirely different psychological meanings. The direction of an I scale is defined experimentally—the direction of a J scale is a matter of choice.

Each J scale of four stimuli gives rise to a set of seven qualitative I scales. We are interested in knowing, of course, whether the J scale deduced from a set of I scales obtained in an experiment is qualitatively unique. The answer to this appears to be yes and immediately obvious when it is recognized that a set of I scales generated from a J scale has two and only two I scales which are mirror images of each other; that these two I scales must have been generated from the intervals on the opposite ends of the J scale, and that the order of the I scales within a set is unique. These statements are still to be developed as formal mathematical proofs and hence must be regarded as tentative conclusions.

However, a given qualitative J scale does not give a unique *set* of I scales. For example, with four stimuli, we may have the qualitative J scale A B C D. This order of four stimuli on a J scale can yield *two different sets* of I scales as follows:

set 1		set 2
A B C D		A B C D
B A C D		B A C D
B C A D		B C A D
B C D A	———	C B A D
C B D A		C B D A
C D B A		C D B A
D C B A		D C B A

It will be noticed that these two sets of seven I scales from the same qualitative J scale are identical except for the I scale from the middle interval. This arises from the following fact. There are six midpoints for the four stimuli on the J scale. These are as follows: AB, AC, AD, BC, BD, CD. The order and identity of the first two and the last two are immutable; they must be, in order: AB, AC, , , BD, CD. But the order of the remaining two midpoints is not defined by the qualitative J scale but by its quantitative characteristics. If the interval between stimuli A and B is greater than the interval between C and D, then the midpoint AD comes before the midpoint BC and the set of seven I scales will be set 1 listed above. If the quantitative relations on the J scale are the reverse and the midpoint BC comes before AD, then the set of seven I scales which will result are those listed in set 2 above.

Thus we see that in the case of four stimuli, a set of I scales will uniquely determine a qualitative J scale and will provide one piece of information about the metric relations. For five or more stimuli the number of pieces of information about metric relations exceeds the minimum number that are needed for ordering the successive intervals. *However*, the particular pieces of information that are obtained might not be the appropriate ones for doing this. It is interesting to note here that this is a new type of scale not discussed by Stevens. This is a type of scale that falls between what he calls ordinal scales and interval scales. In ordinal scales nothing is known about the intervals. In interval scales the intervals are equal. In this scale, which we call an ordered metric, the intervals are not equal but they may be ordered in magnitude.

As the number of stimuli increases, the variety of different *sets* of I scales from a single qualitative J scale increases rapidly. This means that a great deal of information is being given about metric relations. For example a J scale of five stimuli yields a set of eleven I scales $\left(\text{in general } n \text{ stimuli will provide } \binom{n}{2} + 1 \text{ different I scales from one J scale}\right)$. Depending on the relative magnitudes of the four intervals between the five stimuli on the J scale, the same qualitative J scale may yield *twelve different sets* of I scales. This means that for a given order of five stimuli on a J scale there are twelve experimentally differentiable quantitative J scales. Previously, in the case of four stimuli, we found only two differentiable quantitative J scales for a given qualitative J scale.

The particular set of I scales obtained from five stimuli *may* provide up to five of the independent relations between pairs or intervals. For example, suppose we have the qualitative J scale A B C D E. Among the twelve possible sets of I scales which could arise are the following two, chosen at random:

set 1	set 2
A B C D E	A B C D E
B A C D E	B A C D E
B C A D E	B C A D E
B C D A E	B C D A E
C B D A E	B C D E A
C D B A E	C B D E A
D C B A E	C D B E A
D C B E A	D C B E A
D C E B A	D C E B A
D E C B A	D E C B A
E D C B A	E D C B A

Let us see what information is given by each of these sets about the relative magnitudes of the intervals between the stimuli on the J scales. Consider set 1 first. The order of the ten midpoints of the five stimuli according to set 1 is as follows: AB, AC, AD, BC, BD, CD, AE, BE, CE, DE. We know immediately, from the fact that the midpoint BC comes after AD, that the interval between stimuli A and B ($\overline{AB}$) is greater than the interval between the stimuli C and D ($\overline{CD}$). We have summarized this in the first row of the table below. The other rows contain the other metric relations which can be deduced from set 1.

SET 1

Order of midpoints	Relative magnitude of intervals on J scale
AD, BC	$\overline{CD} < \overline{AB}$
CD, BE	$\overline{BC} < \overline{DE}$
BD, AE	$\overline{AB} < \overline{DE}$
CD, AE	$\overline{AC} < \overline{DE}$

Or, in brief form, the I scales contained in set 1 indicate that the following relations must hold between stimuli on the J scale.

$$\overline{CD} < \overline{AB} < \overline{DE}$$
$$\overline{AB} + \overline{BC} < \overline{DE}$$

In the same manner we may study the implications of set 2 for the metric relations between stimuli on the J scale. The midpoints for this set are in the following order: AB AC AD, AE, BC, BD CD, BE, CE, DE.

SET 2

Order of midpoints	Relative magnitude of intervals on J scale
AD, BC	$\overline{CD} < \overline{AB}$
CD, BE	$\overline{BC} < \overline{DE}$
AE, BD	$\overline{DE} < \overline{AB}$
AE, BC	$\overline{CE} < \overline{AB}$

Or, in brief, the relative magnitudes of the intervals between stimuli on the J scale are known to the following extent.

$$\overline{BC} < \overline{DE} < \overline{AB}$$
$$\overline{DE} + \overline{CD} < \overline{AB}$$

The different implications of these two sets of I scales for the metric relations on the J scale may be illustrated by sketching two quantitative J scales which have the appropriate metric relations (Fig. 5).

The two sets of I scales which are illustrated here were only two of twelve possible different sets which could be generated from a single *qualitative* J scale of five stimuli. Each of the twelve sets of I scales would imply a different set of quantitative relations

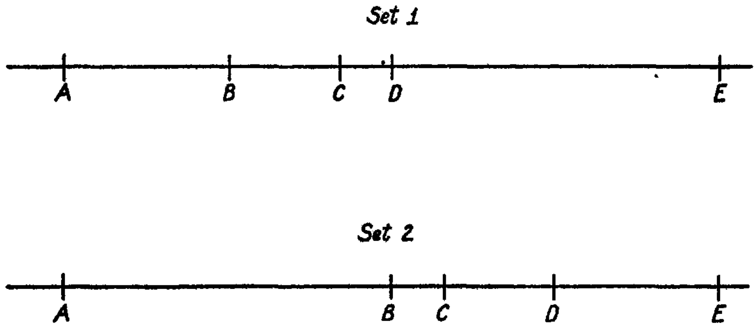

FIG. 5. An example of two joint distributions with the same order of stimuli but different metric relations obtained from different sets of I scales.

among the distances between stimuli on the J scale. The two sets of I scales used here happened to differ from each other in three of their particular members. If we take the twelve potential sets and make a frequency distribution of the number of pairs of sets which have 1, 2, 3, 4, or 5 particular I scales different or 10, 9, 8, 7, or 6 I scales in common, we get the following distribution.

Number of identical ordinal positions in a pair of sets with the same qualitative I scale	Number of such pairs of sets
10	18
9	24
8	17
7	6
6	1
	$66 = \binom{12}{2}$

The surface has not even been scratched on the generalizations which can be developed. Enough has been presented here to provide a general idea of the type of information which can be derived.

V. AN EXPERIMENT

In order to study the feasibility of this unfolding technique and to compare several different psychologica scaling techniques an experiment was conducted in several classes in the Department of Social Relations at Harvard University. A questionnaire was administered pertaining to grade expectations in a course. Data were collected suitable for four different kinds of analyses on the same content area from the same individuals. The four types of analyses for which data were collected were (1) the generation of the joint continuum by the unfolding of I scales obtained by the method of rank order, (2) the generation of the joint continuum by the unfolding of I scales obtained by the method of paired comparisons, (3) the generation of the joint continuum by what we shall call the Guttman triangular analysis, and (4) the generation of the joint continuum by what we shall call the parallelogram analysis.

We shall present here the analysis of only the rank order data. The experiment was first conducted in a graduate course in statistics and then, with a slight change in the wording of some questions, in an undergraduate course in sociology. Despite these differences in subjects and questions the general resuts were practicallyl identical in the two groups and because our primary interest is in the technique and not the content of this experiment we have lumped the data of the two groups.

The questionnaire was arranged as a small booklet with each question on a different page. The questions appropriate to the different techniques were deliberately mixed. The nature of the instructions, the separate paging for the questions, and the mixture of the questions were part of a deliberate effort to induce inconsistency, or at least to minimize a deliberate and artificial consistency. We felt this was accomplished but the high degree of consistency was surprising.

The content of the questionnaire follows.

page 1. *Instructions:* This is an experiment to test certain theoretical aspects of psychological scaling techniques. It is entirely voluntary and you need not answer the questionnaire if you so choose. However, you, as an individual, will not be identified; complete anonymity is preserved. We are interested *only* in certain internal relations in the data. This will become obvious to you because it will appear that we are getting the same information repeatedly in different ways.

You are free, of course, to mark these items entirely at random. It is our hope, however, that enough of you will take a serious attitude toward the experiment and make an effort to respond to each item on the basis of considered judgment.

The questions pertain to your grade expectations in this course. Of course, everyone wants an A or B, but we would like to ask you to give serious consideration to what you really can expect to get. We want you to be neither modest nor self-protective. If you think you will get an A or flunk the course, make your judgments accordingly. Remember: there is complete anonymity.

There is one item or question on each of the following pages. Consider each question or item independently of the others. Answer each one without looking back at previous answers. Treat each item as an item in its own right and do not concern yourself with trying to be logical or consistent. Work quickly.

page 2. In the following list of grades circle the two grades which best represent what you expect to get in this course.
A B C D E

page 3. I expect to get a grade at least as good as a B.
yes no

page 4. Of these two grades, which is nearer the grade you expect to get?
A D

page 5. Of these two grades, which is nearer the grade you expect to get?
B C

page 6. Of these two grades, which is nearer the grade you expect to get?
A B

page 7. I expect to get a grade at least as good as a D.
yes no

page 8. Of these two grades, which is nearer the grade you expect to get?
D E

page 9. Of these two grades, which is nearer the grade you expect to get?
C A

page 10. I expect to get a grade at least as good as an A.
yes no

page 11. Of these two grades, which is nearer the grade you expect to get?
B E

page 12. Of these two grades, which is nearer the grade you expect to get?
E C

page 13. I expect to get a good grade.
yes no

page 14. Of these two grades, which is nearer the grade you expect to get?
D B

page 15. I expect to get a grade at least as good as a C.
yes no

page 16. Of these two grades, which is nearer the grade you expect to get?
E A

page 17. Of these two grades, which is nearer the grade you expect to get?
C D

page 18. Place the five grades in rank order below such that the one on the left is the grade you most expect to get, then in the next space is the grade you next most expect to get, and so on, until finally at the right is the grade you least expect to get.

—	—	—	—	—
1	2	3	4	5
the grade I *most* expect to get			the grade I *least* expect to get	

Page 18 of the questionnaire contained the rank order data. The total number of subjects for whom usable rank data were obtained was 121 (statistics class 40, sociology class 81). The individuals not included in the data which follow were people who introduced new grades (F), plus or minus grades, or left blanks. All individuals who wrote down the five letters A B C D E in some order are contained in the analysis. The I scales obtained by the method of rank order and the number of people in each of the classes who so responded are given in Table I.

TABLE I

NUMBER OF PEOPLE IN TWO CLASSES GIVING EACH OF THE RANK ORDER I SCALES

I scale	Statistics class	Sociology class
A B C D E	14	6
B A C D E	10	22
B C A D E	6	21
C B A D E	1	4
C B D A E	1	11
C B D E A	2	3
C D B E A	1	0
D E C B A	1	0
B C D A E	3	7
B C D E A	0	6
B A C E D	0	1
C A B D E	1	0
Total	40	81

Let us first consider the two I scales in the bottom two rows of Table I. These two scales, B A C E D and C A B D E, were each given by one individual. There is no way that either of these two scales could have arisen from a J scale on which the stimuli are in the order A B C D E regardless of the metric relations on the J scale. All the evidence, both *a priori* and from the other response patterns, indicates that the order of stimuli on the J continuum is A B C D E. So we must regard these two I scales as errors on the part of respondents or assume some esoteric psychologics to explain them, the latter completely unjustified. Hence we shall drop these two individuals from further consideration.

Let us now look at the first eight scales listed in Table 1. From five stimuli we can have eleven different I scales to correspond to the eleven intervals into which the J scale is sectioned by the ten midpoints between stimuli. Consequently these eight I scales constitute a *partial* set. But because one of the missing intervals (interval 8) has two alternative I scales, there are two possibilities for the complete set of which these eight are a partial set, as follows:

	I scale		Total N
	A B C D E		20
	B A C D E		32
	B C A D E		27
	C B A D E		5
	C B D A E		12
	C B D E A		5
	C D B E A		1
D C B E A	—	C D E B A	0
	D C E B A		0
	D E C B A		1
	E D C B A		0

There were three intervals toward the low end of the J scale which were not occupied by any students. Apparently not very many students expected to get a low grade. The fact that one of the intervals on the J scale (interval 8) is blank and could be represented by two alternative I scales means that one of the metric characteristics of the intervals between stimuli on the J scale is not experimentally given. But from the remaining I scales the order on the J scale and some of the metric effects are determined.

The indications are that the order of stimuli on the joint continuum is A B C D E. We know this, of course, from *a priori* grounds, but the point is that this fact need not be known beforehand. Secondly, the individuals are placed in intervals and the intervals ordered on the continuum. The number of ordered intervals is eleven but individuals occupy only eight of the eleven intervals. Thirdly, we know certain metric relations among the intervals between stimuli. These are obtained as follows. From the order of the I scales within the set, the successive midpoints between stimuli are: AB, AC, BC, AD, AE, BD, $\begin{pmatrix} DC \\ BE \end{pmatrix}$, $\begin{pmatrix} BE \\ DC \end{pmatrix}$, CE, DE. Because interval 8 was unoccupied and there are two alternative I scales which satisfy it, it is not known whether the midpoint DC or BE comes first. Hence one piece of metric information is lacking. In the table below are the metric relations which may be deduced and the basis for the deduction.

Order of midpoints	Metric relations
BC, AD	$\overline{AB} < \overline{CD}$
AE, BD	$\overline{DE} < \overline{AB}$

Or, in brief, $\overline{DE} < \overline{AB} < \overline{CD}$. The psychological distance between the grades D and E is the least and the distance between the grades C and D the largest, with the distance from A to B in between. No information is given of the relative magnitude of the distance between the grades B and C.

But now for another portion of the students there is a different interpretation. There are two I scales in Table I, B C D A E and B C D E A, which have not yet been considered. They are also members of a set that is as valid as the first set. If we remove I scales 4 and 5 from the partial set of 8 and substitute these two we have the following:

	I scale		Total N
	A B C D E		20
	B A C D E		32
	B C A D E		27
	B C D A E		10
	B C D E A		6
	C B D E A		5
	C D B E A		1
D C B E A	—	C D E B A	0
	D C E B A		0
	D E C B A		1
	E D C B A		0

This set differs from the preceding only in I scales 4 and 5. If we analyze the significance of this set to the metric relations we have the following:

Order of midpoints	Metric relations
AE, BC	$\overline{AB} > \overline{CE}$

It appears from this that for the 16 individuals who gave the two I scales B C A D E and B C D E A, if they are treated as members of the same set, the psychological distance between the grades A and B is greater than either the distance from C to D or from D to E or, in fact, is greater than the sum of these two distances. This is in contrast to the 17 individuals in the first set who gave the I scales C B A D E and C B D A E in positions 4 and 5 for whom the relative distances between grades was in the order $\overline{DE}$, $\overline{AB}$, $\overline{CD}$.

The reader must be aware of the fact that it is only these 33 individuals for whom these metric relations are deduced. There is no way of knowing, for example, that the 20 individuals who gave the I scale A B C D E did so from the first interval on one of these two possible quantitative J scales or from any one of the many more differing in metric relations. An individual yielding the I scale A B C D E is known to be to the left of the midpoint AB and the relative distances $\overline{BC}$, $\overline{CD}$, and $\overline{DE}$ do not affect his I scale. It is just the critical I scales which provide information about metric and this information is valid only for the individuals who yield these I scales. By putting them together we can construct a total picture, but our only evidence that they go together is one of internal consistency.

The general picture of metric relations among grades given by these two sets of I scales is the following:

1) For 17 individuals the metric relations are $DE < \overline{AB} < \overline{CD}$
2) For 16 individuals the metric relations are $\overline{AB} > \overline{CE}$

Thus we find from the data that some of these individuals are simply on a different continuum from other individuals. To somehow compute scale values for the stimuli which will be assumed to hold for all these individuals is to do violence to the experimental evidence. From the data we have learned where an individual is on the continuum in relation to the stimuli, and in addition something about how the whole continuum looks to him.

VI. SUMMARY

We have presented a new type of scale called an ordered metric and have presented the experimental procedures required under certain limiting conditions to secure such a scale.

We have pointed out that the information which could be obtained under these conditions is as follows:

1) the discovery of a latent attribute underlying preferences,
2) the order of the stimuli on the attribute continuum,
3) something about the relative magnitudes of the distances between pairs of stimuli,
4) the sectioning of the continuum into intervals, the placing of people in these intervals, and the ordering of these intervals on this attribute continuum,
5) something about the relative magnitudes of these intervals.

These were illustrated with a hypothetical example and an experiment.

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