## Review: The Sampling Design Process

Fig. 11.1
Framing vs. Validating
How to define? Based on what?


Select Sampling Technique(s)

Determine the Sample Size

Execute the Sampling Process


1979: 45\% in Taiwan
PepsiCo global sales: $\$ 6$ billion


Half empty or half full?

## Review: Multivariate Methods



## Analysis of Variance and Covariance



## Review: F-test

- In honor of Sir Ronald A. Fisher
- Fisher initially developed the statistic as the variance ratio in the 1920s, where he analyzed its immense data from crop experiments since the 1840s, and developed the analysis of variance (ANOVA).
- He is also known as one of the three principal founders of population genetics, Fisher's principle, the Fisherian runaway and sexy son hypothesis theories of sexual selection, and important contributions to statistics, including the maximum likelihood, fiducial inference, the derivation of various sampling distributions.



## Review: F(isher) Test

- Randomized Block Design
- Decompose error term into

$$
\varepsilon_{\mathrm{ij}}=\mathrm{b}_{\mathrm{i}}+\varepsilon_{\mathrm{ij}}^{\prime}
$$



- Latin Square Design
- Decompose error term into

$$
\varepsilon_{\mathrm{ij}}=\mathrm{b}_{1}+\mathrm{b}_{2}+\varepsilon^{\prime \prime}{ }_{\mathrm{ij}}
$$

- Factorial Design ( N -way


## ANOVA)

- Decompose treatment term into

$$
\mathrm{T}_{\mathrm{j}}=\mathrm{a}_{\mathrm{i}}+\beta_{\mathrm{j}}+(\mathrm{a})_{\mathrm{ij}}
$$



Exercise（悲喜交加對決策的影响）
－Use the example in p． 515 to identify eight different interaction patterns for a two factors ANOVA，both $\mathrm{X}_{1}$ and $\mathrm{X}_{2}$ have two levels．




## Review: ANOVA-Family

- One way ANOVA: one grouping (independent) variable, one dependent variable
- Within-Subjects ANOVA: scores are obtained from the same subject measured on separate occasions
- Factorial (n-way) ANOVA: more than one grouping (independent) variable, one dependent variable
- Analysis of Covariance (ANCOVA): scores are obtained both before and after a treatment intervention, and pre-treatment scores are used to adjust post-treatment scores
- Multivariate Analysis of Variance (MANOVA): two or more dependent variables
- MANCOVA


17-14

## Analysis of Covariance (ANCOVA)

- Two group (treatment vs. control) quasi-experimental design
- $M_{\mathrm{X} . \mathrm{T}}$, is higher than the mean pretest score of the control group, $M_{\text {x.C }}$.
- ANCOVA: If we hold constant the pretest scores (i.e., the slopes of the regression lines are equivalent), is there a significant differences between the posttest scores for the two groups?


Analysis of Covariance (ANCOVA).


Pretest (X)


Pretest (X)



Pretest (X)


## Extension：DID（difference－in－Difference）

WIKIPEDIA
The Free Encyclopedia
－Difference in differences is a statistical technique used in the social sciences that attempts to mimic an experimental research design using observational study data，by studying the differential effect of a treatment on a＇treatment group＇versus a＇control group＇in a natural experiment．
－It calculates the effect of a treatment（i．e．，an explanatory variable or an independent variable）on an outcome（i．e．，a response variable or dependent variable）by comparing the average change over time in the outcome variable for the treatment group，compared to the average change over time for the control group．
－由配對t 統計可知，實驗組與對照組都個別有進步幅度（也就是兩組都有一個平均值，Mex and Mcon），如果組別在進步幅度達顯著差異（ $P<0.05$ ），而且又是實驗組的進步幅度顯著高於對照組（Mex $>M c o n)$ ，我們即可宣稱此差異即為實驗的淨效果。

| Summary |  |  |
| :--- | :--- | :--- |
| Types of Explanatory <br> Variables | Number of Dependent <br> Variables | Technique |
| Metric | One | Multiple Regression |
| Categorical | One | ANOVA |
| Both | One | ANCOVA |
| Metric | m | Multivariate Multiple <br> Regression |
| Categorical | m | MANOVA |
| Both | m | MANCOVA |

## Review: The GLM Procedure

| Specification | Kind of Model |
| :---: | :---: |
| Model $\mathrm{y}=\mathrm{x} 1$; <br> Model $y=x 1$ x2; <br> Model $\mathrm{y}=\mathrm{x} 1 \mathrm{x} 1^{*} \mathrm{x} 1$; <br> Model y1 y2= x1 x2; | Simple regression <br> Multiple regression <br> Polynomial regression <br> Multivariate regression |
| Model y=a; <br> Model $y=a \mathrm{~b} c$; <br> Model $\mathrm{y}=\mathrm{a} \mathrm{b} \mathrm{a}$ *b; <br> Model $y=a b(a) c(b a) ;$ <br> Model y1 y2=a b; | One-way ANOVA <br> Main effects model Factorial model (with interaction) Nested model (hierarchical) Multivariate analysis of variance (MANOVA) |
| Model $\mathrm{y}=\mathrm{a} \mathrm{x} 1$; <br> Model $\mathrm{y}=\mathrm{a}$ x1(a); <br> Model $\mathrm{y}=\mathrm{a}$ x1 x1*a; | Analysis-of-covariance model (ANCOVA) <br> Separate-slopes model <br> Homogeneity-of-slopes model |

## Chapter Seventeen

## Correlation and Regression

## Linear regression

is the most widely used of all statistical techniques
$>$ is the fitting of straight lines to data
$>$ was so-named by Sir Francis Galton, a 19th century amateur scientist \& adventurer who was famous for his explorations and wrote a best-selling book on "the art of travel" (still in print) that introduced the sleeping bag \& other wilderness gear to the Western world

## Product Moment Correlation

- The product moment correlation, $r_{\text {, }}$ summarizes the strength of association between two metric (interval or ratio scaled) variables, say $X$ and $Y$.
- It is an index used to determine whether a linear or straight-line relationship exists between $X$ and $Y$.
- As it was originally proposed by Karl Pearson, it is also known as the Pearson correlation coefficient. It is also referred to as simple correlation, bivariate correlation, or merely the correlation coefficient.


## Product Moment Correlation

From a sample of $n$ observations, $X$ and $Y$, the product moment correlation, $r$, can be calculated as:

$$
r=\frac{\sum_{i=1}^{n}\left(X_{i}-X\right)\left(Y_{i}-Y\right)}{\sqrt{\sum_{i=1}^{n}\left(X_{i}-X\right)^{2} \sum_{i=1}^{n}\left(Y_{i}-Y\right)^{2}}}
$$

Division of the numerator and denominator by $(n-1)$ gives

$$
\begin{aligned}
r & =\frac{\sum_{i=1}^{n} \frac{\left(X_{i}-\bar{X}\right)\left(Y_{i}-\bar{Y}\right)}{n-1}}{\sqrt{\sum_{i=1}^{n} \frac{\left(X_{i}-X\right)^{2}}{n-1} \sum_{i=1}^{n} \frac{\left(Y_{i}-\bar{Y}\right)^{2}}{n-1}}} \\
& =\frac{C O V_{x y}}{S_{x} S_{y}}
\end{aligned}
$$

## Regression Analysis

Regression analysis examines associative relationships between a metric dependent variable and one or more independent variables in the following ways:

- Determine whether the independent variables explain a significant variation in the dependent variable: whether a relationship exists.
- Determine how much of the variation in the dependent variable can be explained by the independent variables: strength of the relationship.
- Determine the structure or form of the relationship: the mathematical equation relating the independent and dependent variables.
- Predict the values of the dependent variable.
- Control for other independent variables when evaluating the contributions of a specific variable or set of variables.
- Regression analysis is concerned with the nature and degree of association between variables and does not imply or assume any causality.


## Regression Analysis

- Underlying assumptions
- Linear relationship
- Constant error variance (homoscedasticity)
- Normally distributed errors
- Independent observations
- Stationary process


## Statistics Associated with Bivariate Regression Analysis

- Bivariate regression model. The basic regression equation is $Y_{i}=\beta_{0}+\beta_{1} X_{i}+e_{i}$, where $Y=$ dependent or criterion variable, $X=$ independent or predictor variable, $\beta_{0}=$ intercept of the line, $\beta_{1}=$ slope of the line, and $e_{i}$ is the error term associated with the $i$ th observation.
- Coefficient of determination. The strength of association is measured by the coefficient of determination, $r^{2}$. It varies between 0 and 1 and signifies the proportion of the total variation in $Y$ that is accounted for by the variation in $X$.
- Estimated or predicted value. The estimated or predicted value of $Y_{i}$ is $\hat{Y}_{i}=a+b x$, where $\hat{Y}_{i}$ is the predicted value of $Y_{i}$, and $a$ and $b$ are estimators of $\beta_{0}$ and $\beta_{1}$, respectively.


## Statistics Associated with Bivariate Regression Analysis

- Regression coefficient. The estimated parameter $b$ is usually referred to as the non-standardized regression coefficient.
- Scattergram. A scatter diagram, or scattergram, is a plot of the values of two variables for all the cases or observations.
- Standard error of estimate. This statistic, SEE, is the standard deviation of the actual $Y$ values from the predicted $\hat{Y}$ values.
- Standard error. The standard deviation of $b, S E_{b}$, is called the standard error.


## Statistics Associated with Bivariate Regression Analysis

- Standardized regression coefficient. Also termed the beta coefficient or beta weight, this is the slope obtained by the regression of $Y$ on $X$ when the data are standardized.
- Sum of squared errors. The distances of all the points from the regression line are squared and added together to arrive at the sum of squared errors, which is a measure of total error, $\Sigma e^{2}{ }_{j}$.
- tstatistic. A $t$ statistic with $n-2$ degrees of freedom can be used to test the null hypothesis that no linear relationship exists between $X$ and $Y$, or $\mathrm{H}_{\mathbf{0}}: \beta_{1}=0$, where $t=\frac{b}{S E_{b}}$


## Conducting Bivariate Regression Analysis

 Plot the Scatter Diagram- A scatter diagram, or scattergram, is a plot of the values of two variables for all the cases or observations.
- The most commonly used technique for fitting a straight line to a scattergram is the least-squares procedure.
- In fitting the line, the least-squares procedure minimizes the sum of squared errors (MSE), $\Sigma e^{2}{ }_{j}$.
- For every value of $X$, the errors $\left(\varepsilon_{i}\right)$ have identical distributions with mean 0 and equal variances; Errors are independent, unrelated to each other; Errors are normally distributed


## Plot of Attitude with Duration

Figure 17.3


Duration of Residence



Decomposition of the Total
Variation in Bivariate Regression
Figure 17.5


The strength of association may then be calculated as follows:

$$
\begin{aligned}
r^{2} & =\frac{S S_{r e g}}{S S_{y}} \\
& =\frac{S S_{y}-S S_{r e s}}{S S_{y}}
\end{aligned}
$$

To illustrate the calculations of $r^{2}$, let us consider again the effect of attitude toward the city on the duration of residence. It may be recalled from earlier calculations of the simple correlation coefficient that:

$$
\begin{aligned}
S S_{y} & =\sum_{i=1}^{n}\left(Y_{i}-\overline{)^{2}}\right. \\
& =120.9168
\end{aligned}
$$

## Conducting Bivariate Regression Analysis

Determine the Strength and Significance of Association
The predicted values $(\widehat{Y})$ can be calculated using the regression equation:

Attitude $(\widehat{Y})=1.0793+0.5897$ (Duration of residence)

For the first observation in Table 17.1, this value is:
$(\widehat{Y})=1.0793+0.5897 \times 10=6.9763$.

For each successive observation, the predicted values are, in order, 8.1557, 8.1557, 3.4381, 8.1557, 4.6175, 5.7969, 2.2587, 11.6939, $6.3866,11.1042$, and 2.2587.

## Multiple Regression

The general form of the multiple regression model is as follows:

$$
Y=\beta_{0}+\beta_{1} X_{1}+\beta_{2} X_{2}+\beta_{3} X_{3}+\ldots+\beta_{\mathrm{k}} X_{\mathrm{k}}+\mathbf{e}
$$

which is estimated by the following equation:
$\hat{Y}=a+b_{1} X_{1}+b_{2} X_{2}+b_{3} X_{3}+\ldots+b_{k} X_{k}$
As before, the coefficient a represents the intercept, but the $b$ 's are now the partial regression coefficients.

## How "Good" is the Model?

- T-stats, P-values for the Regression Coefficients
- Standard Error of the Regression (Standard Deviation of Residuals)
- R Square (Coefficient of Determination)
- Interpretation
- Adjusted R Square


## Analysis of the Residuals

- What are the residuals?
- Why are the residuals of such interest?
- Tables, plots of residuals

1. Histogram of residuals
2. Residuals vs. predicted values
3. Residuals vs. the independent variable
4. Ordered plot of residuals (time series)


Do the residuals look like $N\left(0, \sigma_{\varepsilon}^{2}\right)$ ?




Predicted Y Values

## Comparing Models

## How can we compare models? <br> - In terms of descriptive "fit" (LSE)

- In terms of underlying inferential assumptions (Xs in theoretical construct, MLE)
- In terms of quality of forecasts (predicted value)


## Stepwise Regression

The purpose of stepwise regression is to select, from a large number of predictor variables, a small subset of variables that account for most of the variation in the dependent or criterion variable. In this procedure, the predictor variables enter or are removed from the regression equation one at a time. There are several approaches to stepwise regression.

- Forward inclusion. Initially, there are no predictor variables in the regression equation. Predictor variables are entered one at a time, only if they meet certain criteria specified in terms of Fratio. The order in which the variables are included is based on the contribution to the explained variance.
- Backward elimination. Initially, all the predictor variables are included in the regression equation. Predictors are then removed one at a time based on the Fratio for removal.
- Stepwise solution. Forward inclusion is combined with the removal of predictors that no longer meet the specified criterion at each step.


## Multicollinearity

- Multicollinearity arises when intercorrelations among the predictors are very high.
- Multicollinearity can result in several problems, including:
- The partial regression coefficients may not be estimated precisely. The standard errors are likely to be high.
- The magnitudes as well as the signs of the partial regression coefficients may change from sample to sample.
- It becomes difficult to assess the relative importance of the independent variables in explaining the variation in the dependent variable.
- Predictor variables may be incorrectly included or removed in stepwise regression.



## Solution 2: Residual Centering (Burrill 1997)

- X1X2 = X1*X2
- /* this step output the residuals of the interaction term*/
- PROC REG DATA=DATA1;
- MODEL X1X2 = X1 X2;
- OUTPUT OUT=DATA2 R=R_X1X2;
- /* this step uses the residual as an orthogonalized variable */
- PROC REG DATA=DATA2;
- MODEL Y = X1 X2 R_X1X2;


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Table I Fit measure of brand extensions

| Extension products | Average score | Extension products | Average score |
| :--- | :--- | :--- | :--- |
| Hon Hai (Foxconn) |  | Media Tek |  |
| Notebook computer | 4.16 | DVD recorder | 4.21 |
| MP3 personal stereo | 3.27 | Liquid crystal television | 3.71 |
| Laser printer | 2.43 | MP3 personal stereo | 3.31 |
| Stereophonic | 2.01 | Digital set-top box | 2.96 |
| Digital camera | 1.65 | Compact disc | 1.79 |
| AUO |  | Quanta |  |
| Liquid crystal television | 4.68 | Personal computer | 4.45 |
| Digital camera | 3.11 | PDA | 3.52 |
| Intelligent mobile | 2.63 | WEB CAM | 3.06 |
| Scanner | 2.32 | Base station for wireless | 2.54 |
| GPS satellite positioning system | 1.52 | network |  |

Bold: Selected extension products with high, medium and low scores.


Table 3 Regression results for individual industrial brands

| Independent variable | Standardised regression coefficient |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | AUO | Hon Hai | Media Tek | Quanta |
| $Q$ (perceived quality of parent brand) | 0.129** | 0.207** | 0.064 | 0.121** |
| $I$ (perceived ability in product innovation) | 0.056 | 0.121 | 0.091** | $0.235^{* * *}$ |
| $E$ (commitment to environment protection) | 0.025 | 0.017 | -0.010 | 0.002 |
| $D$ (difficulty of making extension) | $-0.152^{* 1 \%}$ | $-0.011^{\text {*** }}$ | -0.077 | $-0.176^{* 10}$ |
| $C$ (brand concept consistency between the parent brand and the extensions) | 0.597*** | 0.423*** | 0.712*** | 0.473*** |
| $T$ (transfer of skills/assets from parent to extension product class) | $0.155^{* * *}$ | $0.205^{* * *}$ | 0.096* | 0.094* |
| QC (interaction term between quality perception with brand concept consistency) | -0.002 | 0.022 | 0.115* | $0.123^{* *}$ |
| QT (interaction term between quality perception with transfer) | 0.015 | 0.019 | $-0.042^{* * *}$ | 0.037 |
| Sample size $=378$ |  |  |  |  |
| Adjusted $\mathrm{r}^{2}=0.63$ | 0.69 | 0.67 | 0.74 | 0.50 |

*p<0.05; **p<0.01; ***p<0.001
Bold values represent highest influential factors.

## Relative Importance of Predictors

Unfortunately, because the predictors are correlated, there is no unambiguous measure of relative importance of the predictors in regression analysis. However, several approaches are commonly used to assess the relative importance of predictor variables.

- Statistical significance. If the partial regression coefficient of a variable is not significant, as determined by an incremental $F$ test, that variable is judged to be unimportant. An exception to this rule is made if there are strong theoretical reasons for believing that the variable is important.
- Square of the simple correlation coefficient. This measure, $r^{2}$, represents the proportion of the variation in the dependent variable explained by the independent variable in a bivariate relationship.


## Relative Importance of Predictors

- Square of the partial correlation coefficient. This measure, $R^{2}{ }_{v x i x j x k}$ is the coefficient of determination between the dependent variable and the independent variable, controlling for the effects of the other independent variables.
- Square of the part correlation coefficient. This coefficient represents an increase in $R^{2}$ when a variable is entered into a regression equation that already contains the other independent variables.
- Measures based on standardized coefficients or beta weights. The most commonly used measures are the absolute values of the beta weights, $\left|B_{i}\right|$, or the squared values, $B_{i}{ }^{2}$.
- Stepwise regression. The order in which the predictors enter or are removed from the regression equation is used to infer their relative importance.


## General Issues in Regression Modeling

- What should we be thinking about when building a regression model or interpreting a model built by someone else?
- What is our goal?
- Describing relationships in data?
- Making inferences about relationships?
- Building theories?
- Forecasting future values?

Choosing independent variables

- Which variables to consider?
- How to represent ordinal or categorical variables?
- Are transformations helpful?
- Is multicollinearity an issue?
- Which combination of variables works best?
- What is a good strategy for considering different combinations of variables?
- How can we compare and choose regression models?


## General Issues in Regression Modeling

- The Dangers of Overfitting
- Fitting vs. forecasting revisited
- Tradeoffs: More information vs. simpler model
- Sample size vs. number of independent variables
- A single model vs. multiple models
- Dealing with outliers
- Data transformation (z-score or log, etc.)



## 4.1 研究架畨

本研究探討單一品類不同型躆規格產品的㵋格，對於其它產品相對吸引力之影響－臸於特定品牌產品相對吸引力的衡量，是以市場的交易資料為基硔，利用台灣某通路商的資料庫系統中，實際 MP3 的交易買賣，共計 19 個月的銷售週期，錄有 8,038 筆下單觀察值，總共售出 43,880 台產品，分析領導品牌於台㸺市場所推出的十種不同規格產品之間的㵋格親爭，以特定產品橮格到於其它產品銷售量的影響，來反映陔品牌產品的相對吸引力•此外，透過產品進化週期之觀點，從單一品牌的產品層級，分四個階段討論特定產品如何影響其它產品的銷售量，並分析其市場競爭結構，研究概念如圖 1 所示。


園1 研究概念圆
4.2 資料庫措述

領導品牌公司所銷售的 MP3 音樂播放器，在 19 個月的觀察期間當中，總共推出 10 種不同的產品規格，整理於表 1 。

表1 領導品牌 MP3 產品之記憶體容量，型號，和䯪色

| 記滰䯈容量 | 128MB |  |  | 256MB |  |  |  |  |  |  | 512 MB |  |  | IGB | 5GB |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 型戱 | 110 | 120 | 150 | 102 | 110 | 120 | 130 | 150 | 180 | 210 | 130 | 200 | 220 | 130 | 720 |
| 皟色 | $\begin{array}{\|l\|l\|} \hline \text { 紫 } \\ \text { 植 } \\ \text { 管 } \end{array}$ | $\begin{array}{\|l\|l\|} \hline \text { 然 } \\ \text { 准 } \\ \text { 栍 } \end{array}$ |  |  | $\begin{array}{\|l\|l} \hline \text { 等 } \\ \text { 箓 } \end{array}$ |  | $\begin{array}{\|l\|l\|} \hline \text { 莢 } \\ \text { 薄 } \\ \text { 棈 } \end{array}$ | $\begin{array}{\|l\|l\|} \hline \text { 等 } \\ \text { 震 } \end{array}$ | $\begin{array}{\|l\|l\|} \hline \text { 得 } \\ \text { 彩 } \\ \text { 或 } \end{array}$ | $\begin{array}{\|l\|l\|} \hline \text { 红 } \\ \text { 噵 } \\ \text { 水 } \end{array}$ | $\begin{array}{\|l\|l\|} \hline \text { 灰 } \\ \text { 衁 } \\ \text { 椦 } \end{array}$ | 泉 |  |  | 黑 |



表2 10 種型號產品在市場上的存繒時間

| 型跖 | 110 | 120 | 150 | 102 | 180 | 130 | 200 | 210 | 720 | 220 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 推出市场 <br> （年／月） | $2004 / 03$ | $2004 / 03$ | $2004 / 03$ | $2004 / 07$ | $2004 / 07$ | $2004 / 11$ | $2004 / 11$ | $2004 / 11$ | $2005 / 02$ | $2005 / 06$ |
| 退出市場 <br> （年／月） | $2004 / 10$ | $2004 / 10$ | $2005 / 03$ | $2005 / 02$ | $2005 / 04$ | $2006 / 01$ | $2006 / 02$ | $2006 / 02$ | $2005 / 08$ | $2006 / 02$ |
| 存活時間 <br> （月） | 8 | 8 | 13 | 8 | 10 | 15 | 16 | 16 | 7 | 9 |




$$
\begin{aligned}
& \text { 假詋一: 產品進化迥期第一階段, 各型號產品的销售量興其它 } \\
& \text { 型諕童品的平均销售供格有顕著闕昲。 } \\
& \text { 假説二: 産品進化迥期第二階段, 各型虎産品的销售量與其它 } \\
& \text { 型就産品的平均邹售傮格有顔著闑昲。 } \\
& \text { 报説三: 産品谁化過期第三階段, 各型㖸品的销售量興其它 } \\
& \text { 型娏產品的平均梢售僓格有熼著䦔渄。 }
\end{aligned}
$$

㕁品進化週期第四階段，即高崞憶體的世代，齐品規格為130， 200•210，220，和 720 •推論型號 130 的銷售量與型號 200 •型號 210 ，型號 220 ，以及型躆 720 的平均銷售傊格間，依此類推共五條關係式具有顯著關係，提出整體的研究假說四。

假竞四：産品進化週期第四階段，各型胧産品的销售量拲其它型虎産品的平均销售曊格有顕著關昲。


$$
1
$$

## Modeling

We begin by using a linear regression model to test the influence of promotions on sales and the interactive impacts of retail outlets, package sizes and product categories. Most sales response models tend to follow the autoregressive process:

$$
Y_{t_{i j k}}=\beta_{0}+\beta_{1} Y_{t-1_{i j k}}+\beta_{2} X_{1_{i}}+\varepsilon_{i_{1}} \varepsilon_{i_{1}} \sim \ddot{i d} d
$$

where $Y_{t_{j k}}$ represents the sales revenue in period $t ; Y_{t-1_{i j k}}$ refers to the sales revenue in period $t-1$; and $X_{1_{i}}$ is the total amount of promotional expenditure (see Appendix, Table AI).

| Brand | Category | Package | Outlet | Sales promotion |
| :--- | :--- | :--- | :--- | :--- |
| One brand | Bouillon | Jar | PX Mart | Coupon |
|  |  |  | KA Mart | Training expenditure |
|  |  | Gube | GT (distributors) | Display expenditure |
|  |  | PX Mart | Coupon |  |
|  |  | KA Mart | Training expenditure |  |
|  |  | GT (distributors) | Display expenditure |  |
|  |  |  | Pottle | PX Mart |
|  |  | KA Mart | Trapon |  |
|  |  |  | GT (distributors) | Display expenditure |
|  |  |  |  |  |

Following Cosslett and Lee (1985), Hamilton (1996) has developed a general non-liner transfer function. Starting from the unconditional probability of state 1 at time $t=1$, given by the well-known formula:

$$
\theta=\theta\left(P_{11}, P_{22}\right)=\frac{\left(1-P_{11}\right)}{\left(2-P_{11}-P_{22}\right)}
$$

The ergodic Markov switching regime model has two features. First, it allows the promotion activities to switch across regimes following a first order Markov chain. The unconditional probability for state $1, \theta$ can be referred to as the frequency of promotional activities a firm apply. Second, the autoregressive parameters are also allowed to change as the expected demand shift, and hence the promoted demands are regime-varying. We set the expected revenue, $R$, for the manufacturer as the product of

|  | Bouillons |  | Seasonings <br> Sottle |  |
| :--- | :---: | :---: | :---: | :---: |
| Sales promotion | Jar | Cube | Sum of $\theta$ |  |
| Coupon | 0.09 | $0.00^{\mathrm{a}}$ | 0.14 | 0.23 |
| Training expenditure | 0.05 | $0.00^{\mathrm{a}}$ | 0.20 | 0.25 |
| Display expenditure | $0.00^{\mathrm{a}}$ | 0.33 | $0.00^{\mathrm{a}}$ | 0.33 |

Note: ${ }^{\text {a }}$ Where the value of $\theta$ was found to be less than 0 , it was counted as 0 promotion strategy in maxımızing revenue:

$$
\begin{equation*}
\theta^{*}=\frac{\bar{P}}{2 k \mu_{2}}\left(\mu_{2}-\mu_{1}\right) \tag{3}
\end{equation*}
$$

## Chapter 85 <br> The REG Procedure <br> Overview: REG Procedure

The REG procedure is one of many regression procedures in the SAS System. It is a general-purpose procedure for regression, while other SAS regression procedures provide more specialized applications.
Other SAS/STAT procedures that perform at least one type of regression analysis are the CATMOD, GEN MOD, GLM, LOGISTIC, MIXED, NLIN, ORTHOREG, PROBIT, RSREG, and TRANSREG procedures. SAS/ETS procedures are specialized for applications in time series or simultaneous systems. These other SAS/STAT regression procedures are summarized in Chapter 4, "Introduction to Regression Procedures," which also contains an overview of regression techniques and defines many of the statistics computed by PROC REG and other regression procedures

PROC REG provides the following capabilities:

- multiple MODEL statements
- nine model-selection methods
- interactive changes both in the model and the data used to fit the model
- linear equality restrictions on parameters
- tests of linear hypotheses and multivariate hypotheses
- collinearity diagnostics
- predicted values, residuals, studentized residuals, confidence limits, and influence statistics
- correlation or crossproduct inpu
- requested statistics available for output through output data sets
- ODS Graphics. For more information, see the section "ODS Graphics" on page 7106.

