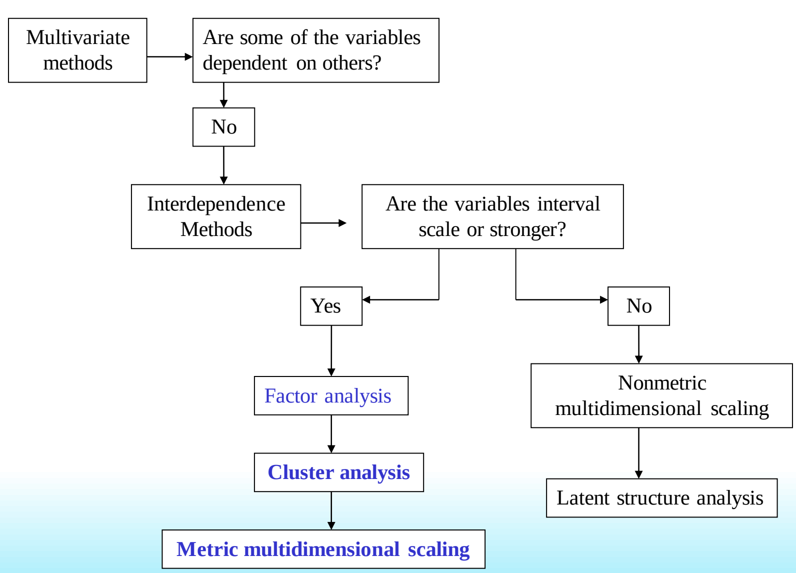
Chapter Nineteen

Factor Analysis

Review: Multivariate Methods in Marketing



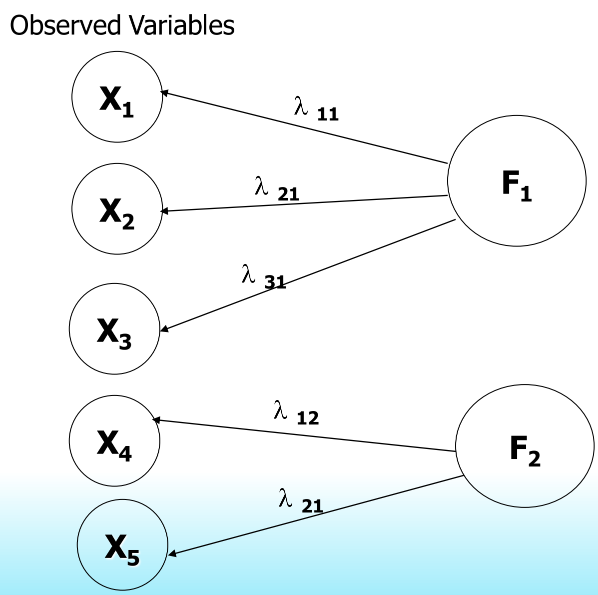
Chapter Outline

1) Overview
2) Basic Concept
3) Factor Analysis Model
4) Statistics Associated with Factor Analysis
5) Conducting Factor Analysis
6) Applications of Common Factor Analysis
7) Summary

Chapter Outline

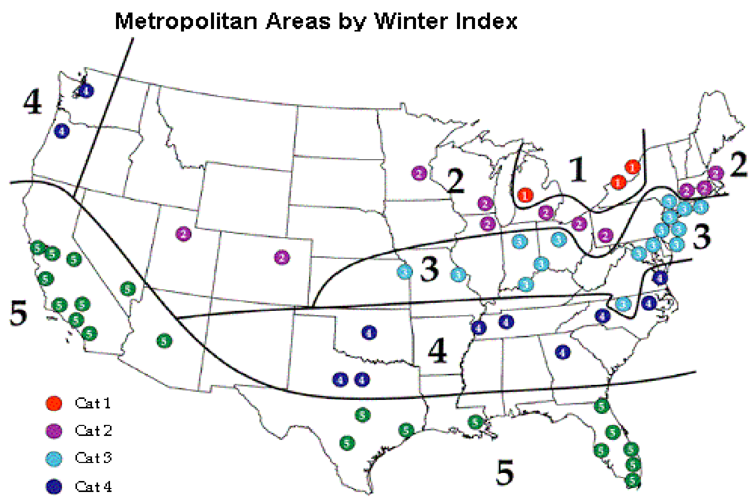
5) Conducting Factor Analysis
   i. Problem Formulation
   ii. Construction of the Correlation Matrix
   iii. Method of Factor Analysis
   iv. Number of Factors
   v. Rotation of Factors
   vi. Interpretation of Factors
   vii. Factor Scores
   viii. Selection of Surrogate Variables
   ix. Model Fit

Factor Analysis



Factor Analysis

- **Factor analysis** is a general name denoting a class of procedures primarily used for data reduction and summarization.
- Factor analysis is an **interdependence technique** in that an entire set of interdependent relationships is examined without making the distinction between dependent and independent variables.
- Factor analysis is used in the following circumstances:
  - To identify underlying dimensions, or **factors**, that explain the correlations among a set of variables.
  - To identify a new, smaller, set of uncorrelated variables to replace the original set of correlated variables in subsequent multivariate analysis (regression or discriminant analysis).
  - To identify a smaller set of salient variables from a larger set for use in subsequent multivariate analysis (canonical analysis)



Average number of days the daily **temperature** falls to or below freezing (
Average number of days where measurable **precipitation** occurs (winter)
Average amount of **snow** (annual)
Average number of times peak **wind** speeds were > 50 mph (annual)
Average number of hours that **ice** occurred per year

供應鏈採購決策因素與電子業赴大陸投資意願之研究
Investigating the Relationship between Supply Chain Procurement Decision Factors and the Investment Intention of Electronics Industry in China

吳敏華、唐瓔璋、戴君芸、張春媛
Min-Hua Wu
Ying-Chan Tang
Chung-Yung Tai
C. Y. Kedy Chang

表1 採購決策條件之因素分析結果摘要

題項和內容	構面	因素負荷量	特徵值	累積解釋變異(%)	Cronbach's α值
1.供應商提供產品與服務的價格一致性	價值分析	0.916	10.48	34.94	0.706
2.透過標準化和加工程序使成本降低		0.898			
3.供應商的成本管理計畫之有效性		0.884			
4.對有可使成本降低機會之認知的反應活動		0.863			
5.對供應商之整體價值態度		0.851			
6.從供應商收到產品的品質	品質追求	0.910	5.50	53.27	0.670
7.對供應商之整體品質態度		0.898			
8.從供應商得到產品的可信度		0.879			
9.供應商對達到產品品質與績效的一致性		0.870			
10.要求與執行錯誤更正活動的供應商品質系統		0.807			
11.對供應商顧客服務與銷售行為表現超過期望	專業期待	0.869	3.12	63.68	0.696
12.供應商對顧客服務與銷售表現之認知程度		0.861			
13.供應商的顧客服務與銷售行為反應時間		0.851			
14.與供應商顧客服務與銷售之合作性		0.824			
15.對供應商之整體服務與銷售表現態度		0.815			

DOES FIRM PERFORMANCE REVEAL ITS OWN CAUSES? THE ROLE OF BAYESIAN INFERENCE

Y.-E. Tang and F.-M. Liou

Table 1. Principal component analysis of financial indicators and the resulting resource configurations

Financial indicators	Resource configuration		
	Factor1: Relationship advantage	Factor2: Management ability	Factor3: Knowledge management
Accounts receivable turnover	**0.578**	−0.085	0.338
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Fixed assets turnover	0.017	**−0.793**	0.101
Eigen value	2.36	1.56	1.45
Accumulated variance (%)	0.26	0.43	0.60

Bold numbers indicate a high correlation between the common factor and the corresponding financial indicator (greater than 0.5).

The concept of Interdependence

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & g \\ f & h \end{pmatrix} = \begin{pmatrix} a+e & b+g \\ c+f & d+h \end{pmatrix}$$

$$\alpha \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \alpha a \\ \alpha b \end{pmatrix}$$

$$\alpha \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \alpha a & \alpha b \\ \alpha c & \alpha d \end{pmatrix}$$

$$(a, b, c) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = ax + by + cz$$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ax + by + cz \\ dx + ey + fz \\ gx + hy + iz \end{pmatrix}$$

Linear Dependent

$$A = \begin{bmatrix} 1 & 2 & 4 & 1 \\ 2 & 1 & 8 & 6 \\ 3 & 6 & 12 & 3 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & -21 & -14 & -29 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Interdependence and Multicollinearity

- **MATRIX MULTIPLICATION: Let A be an mxn matrix and B be an nxm matrix, then AB is an mxm matrix**

$$A = \begin{bmatrix} a_{11} \cdots a_{1n} \\ \vdots \quad\quad \vdots \\ a_{m1} \cdots a_{mn} \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} b_{11} \cdots b_{1m} \\ \vdots \quad\quad \vdots \\ b_{n1} \cdots b_{nm} \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11}b_{11} + \cdots + a_{1n}b_{n1} & \cdots & a_{11}b_{1m} + \cdots + a_{1n}b_{nm} \\ \vdots & & \vdots \\ a_{m1}b_{m1} + \cdots + a_{mn}b_{nm} & \cdots & a_{m1}b_{1m} + \cdots + a_{mn}b_{nm} \end{bmatrix}$$

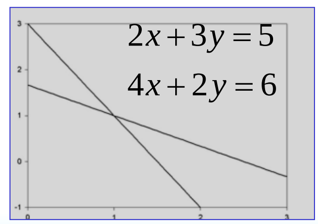
$$b = \frac{\operatorname{cov}(x,y)}{\sigma_x^2} = \frac{\mathrm{ss}_{xy}}{\mathrm{ss}_{xx}}, = (X'X)^{-1} X'Y$$

Unique solution

- **INDEPENDENCE OF VECTORS**

$$x^1 = \begin{bmatrix} x_1^1 \\ \vdots \\ x_n^1 \end{bmatrix} \quad x^2 = \begin{bmatrix} x_1^2 \\ \vdots \\ x_n^2 \end{bmatrix} \quad \ldots \quad x^m = \begin{bmatrix} x_1^m \\ \vdots \\ x_n^m \end{bmatrix}$$

If $\{x_1, x_2, \ldots, x_m\}$ are independent then there exists no set of non-zero Scalars $\{a_1, a_2, \ldots, a_m\}$ such that $a_1x_1 + a_2x_2 + \ldots + a_mx_m = 0$



DETERMINANTS

- **DETERMINANTS**: Finding the determinant of a matrix is a rule for finding a single valued representation of that matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

Then, $|A| = a_{11}a_{22} - a_{21}a_{12}$

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$

RULES ABOUT DETERMINANTS

1. $|A| = |A'|$
2. $|AB| = |A||B|$
3. If the rows or columns of A are linearly dependent, then $|A| = 0$ and A is said to be singular.

RANK (Dimensions, constructs, **factors**)

- **RANK** OF A MATRIX: Let A be an mxn matrix. The row rank of A is the largest number of linearly independent rows. If all the rows of A are linearly independent then A is said to be of full row rank. Column rank is the largest number of linearly independent columns.
- if $|A_{mxm}| = 0$ then $r(A) < m$

$$\mathbf{A} = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix} \qquad \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix Trace

- Matrix Algebra
  - Scalar: a
  - Vector: **a**
  - Matrix: **A**

$$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

- TRACE OF A MATRIX

1. $\text{Tr } A = \sum_{i=1}^{n} a_{ii}$
2. $tr(I_n) = n$
3. tr kA = k tr A, where k is a scalar
4. tr(AB) = tr(BA)

Multiple Regression in matrix form

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

$$\text{tolerance} = 1 - R_j^2,$$

$$\text{VIF} = \frac{1}{\text{tolerance}},$$

$$Y = X\delta + \varepsilon$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_{11} & x_{12} & x_{13} \\ 1 & x_{21} & x_{22} & x_{23} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} & x_{n3} \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

Matrix Trace (Dimensions, constructs, factors)

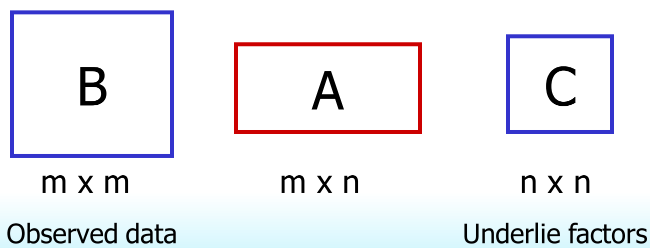
- Matrix Algebra
  - Scalar: a
  - Vector: **a**
  - Matrix: **A**

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Transpose matrix: **A'**
- Symmetric Matrix: **A = A'**
- Identity matrix: **I**
- Null matrix: **0**
- Inverse Matrix: $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$
- Orthogonal Matrix: **PP' = P'P = I**

Matrix Trace (Dimensions, constructs, factors)

- Theorem: Diagonal Reduction of a Matrix
  - Consider A_{mxn}, r(A) = r. Then there exists an mxm matrix B and an nxn matrix C, both nonsingular, such that



Eigenvalue特徵值 (Dimensions, constructs, factors)

- Eigenvalues
  - The purpose of eigenvalues is data reduction. You have the matrices of correlations (e.g., multicollinearity), and you want to distill it into something simpler.
  - λ is an eigenvalue of **A** iff there is a nonzero vector x such that **A**x = λx (The matrix **A** - λ**I** is singular, Det(**A**-λ**I**) = 0).
  - The set of scalars that make this true are known as latent roots, eigen values, or characteristic roots.
- In an n-dimensional system, we would have n eigenvalues with associated eigenvectors.

Matrix Algebra Overview

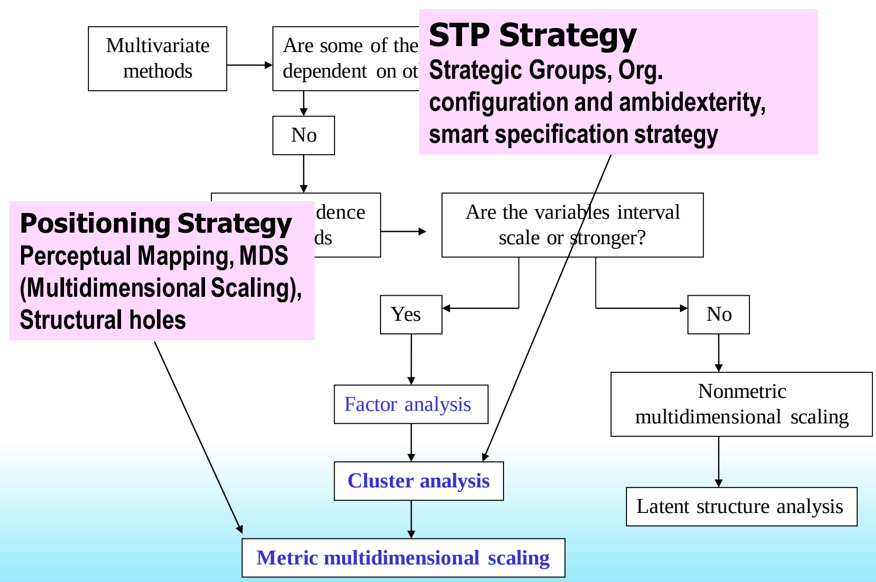
- Linear dependent
- Determinant
- Rank
- Trace (I_n)
- Number of Eigenvalue >1
- For any matrix B, we can always find A and C, such that

Idempotent Matrix 冪等矩陣

- **M** is said to be **idempotent** if **M**^{2}**=M**
- the sum of the square products **B'B** of a square **matrix B** can be written as (**MB**)'**MB** = **B'MB**
- Using two different matrices **B** and **C**， it is possible to write the deviations from the mean of the sum of square products (**B'C**) as follows: (**MB**)'**MC** = **B'MC**.
- tr **A** = rank **A** if **A** is **idempotent.**

$$A^2 = A.A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} . \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = A.$$

Review: Multivariate Methods in Marketing



Factor Analysis Model

Each variable (X) is expressed as a linear combination of underlying factors (F). The covariation among the variables is described in terms of a small number of common factors plus a unique factor for each variable. If the variables are standardized, the factor model may be represented as:

$$X_i = \lambda_{i1}F_1 + \lambda_{i2}F_2 + \lambda_{i3}F_3 + \ldots + \lambda_{im}F_m + V_iU_i,$$

where

X_i = ith standardized variable

λ_{ij} = standardized multiple regression coefficient of variable i on common factor j

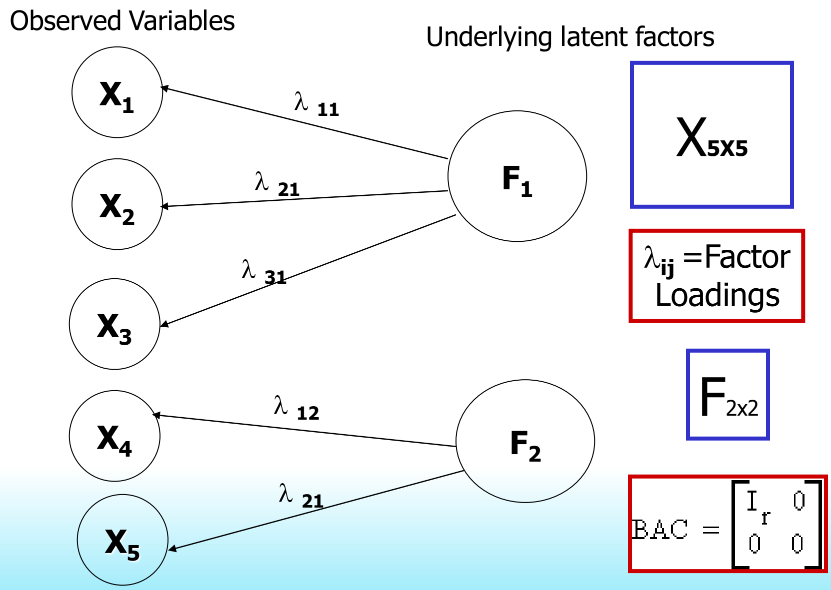
F = hypothetical, unobservable random variables in linearly generating each X_i **(unknown)**

V_i = standardized regression coefficient of variable i on unique factor i

U_i = the unique factor for variable i

m = number of common factors

Factor Analysis



Factor Analysis Model

The unique factors are uncorrelated with each other and with the common factors. The common factors themselves can be expressed as linear combinations of the observed variables.

$F_i = W_{i1}X_1 + W_{i2}X_2 + W_{i3}X_3 + \ldots + W_{ik}X_k$

where

F_i	=	estimate of *i* th factor
W_i	=	weight or **factor score coefficient**
k	=	number of variables

Factor Analysis Model

- It is possible to select weights or factor score coefficients so that the first factor explains the largest portion of the total variance. Stepwise concept in regression
- Then a second set of weights can be selected, so that the second factor accounts for most of the residual variance, subject to being uncorrelated with the first factor.
- This same principle could be applied to selecting additional weights for the additional factors.

Statistics Associated with Factor Analysis

- **Bartlett's test of sphericity**. Bartlett's test of sphericity is a test statistic used to examine the hypothesis that the variables are uncorrelated in the population. In other words, the population correlation matrix is an identity matrix; each variable correlates perfectly with itself ($r = 1$) but has no correlation with the other variables ($r = 0$).
- **Correlation matrix**. A correlation matrix is a lower triangle matrix showing the simple correlations, *r*, between all possible pairs of variables included in the analysis. The diagonal elements, which are all 1, are usually omitted.

Statistics Associated with Factor Analysis

- **Communality.** Communality is the amount of variance a variable shares with all the other variables being considered. This is also the proportion of variance explained by the common factors.
- **Eigenvalue.** The eigenvalue represents the total variance explained by each factor.
- **Factor loadings.** Factor loadings are simple correlations between the variables and the factors.
- **Factor loading plot.** A factor loading plot is a plot of the original variables using the factor loadings as coordinates.
- **Factor matrix.** A factor matrix contains the factor loadings of all the variables on all the factors extracted.

Statistics Associated with Factor Analysis

- **Factor scores.** Factor scores are composite scores estimated for each respondent on the derived factors.
- **Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy.** The Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy is an index used to examine the appropriateness of factor analysis. High values (between 0.5 and 1.0) indicate factor analysis is appropriate. Values below 0.5 imply that factor analysis may not be appropriate.
- **Percentage of variance.** The percentage of the total variance attributed to each factor.
- **Residuals** are the differences between the observed correlations, as given in the input correlation matrix, and the reproduced correlations, as estimated from the factor matrix.
- **Scree plot.** A scree plot is a plot of the Eigenvalues against the number of factors in order of extraction.

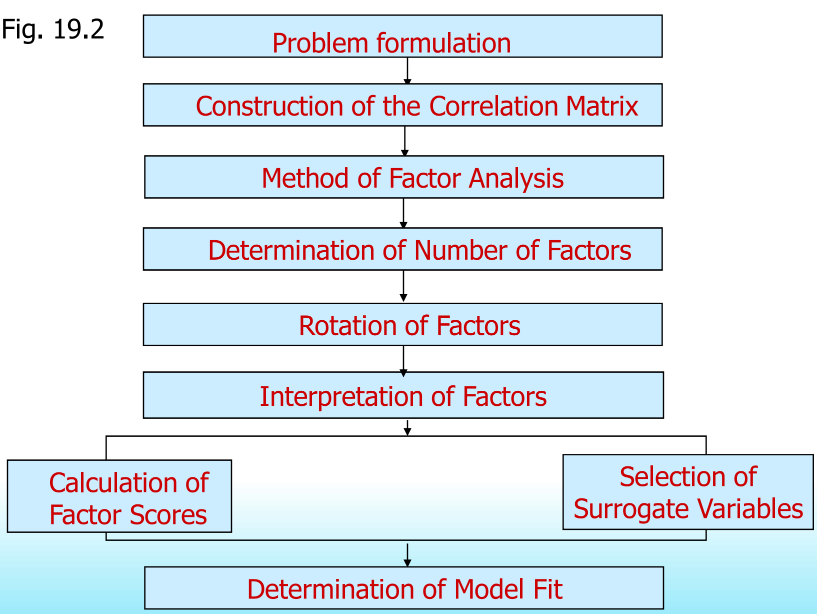
Conducting Factor Analysis

Table 19.1

RESPONDENT NUMBER	V1	V2	V3	V4	V5	V6
1	7.00	3.00	6.00	4.00	2.00	4.00
2	1.00	3.00	2.00	4.00	5.00	4.00
3	6.00	2.00	7.00	4.00	1.00	3.00
4	4.00	5.00	4.00	6.00	2.00	5.00
5	1.00	2.00	2.00	3.00	6.00	2.00
6	6.00	3.00	6.00	4.00	2.00	4.00
7	5.00	3.00	6.00	3.00	4.00	3.00
8	6.00	4.00	7.00	4.00	1.00	4.00
9	3.00	4.00	2.00	3.00	6.00	3.00
10	2.00	6.00	2.00	6.00	7.00	6.00
11	6.00	4.00	7.00	3.00	2.00	3.00
12	2.00	3.00	1.00	4.00	5.00	4.00
13	7.00	2.00	6.00	4.00	1.00	3.00
14	4.00	6.00	4.00	5.00	3.00	6.00
15	1.00	3.00	2.00	2.00	6.00	4.00
16	6.00	4.00	6.00	3.00	3.00	4.00
17	5.00	3.00	6.00	3.00	3.00	4.00
18	7.00	3.00	7.00	4.00	1.00	4.00
19	2.00	4.00	3.00	3.00	6.00	3.00
20	3.00	5.00	3.00	6.00	4.00	6.00
21	1.00	3.00	2.00	3.00	5.00	3.00
22	5.00	4.00	5.00	4.00	2.00	4.00
23	2.00	2.00	1.00	5.00	4.00	4.00
24	4.00	6.00	4.00	6.00	4.00	7.00
25	6.00	5.00	4.00	2.00	1.00	4.00
26	3.00	5.00	4.00	6.00	4.00	7.00
27	4.00	4.00	7.00	2.00	2.00	5.00
28	3.00	7.00	2.00	6.00	4.00	3.00
29	4.00	6.00	3.00	7.00	2.00	7.00
30	2.00	3.00	2.00	4.00	7.00	2.00

Conducting Factor Analysis

Fig. 19.2



Conducting Factor Analysis
Formulate the Problem

- The objectives of factor analysis should be identified.
- The variables to be included in the factor analysis should be specified based on past research, theory, and judgment of the researcher. It is important that the variables be appropriately measured on an interval or ratio scale.
- An appropriate sample size should be used. As a rough guideline, there should be at least four or five times as many observations (sample size) as there are variables.

Correlation Matrix

Table 19.2

Variables	V1	V2	V3	V4	V5	V6
V1	1.000					
V2	-0.530	1.000				
V3	0.873	-0.155	1.000			
V4	-0.086	0.572	-0.248	1.000		
V5	-0.858	0.020	-0.778	-0.007	1.000	
V6	0.004	0.640	-0.018	0.640	-0.136	1.000

V1: prevention of cavities
V3: Strong gum
V5: prevention of decay is not important

V2: shiny teeth
V4: Fresh teeth
V6: attractive teeth

Conducting Factor Analysis
Construct the Correlation Matrix

- The analytical process is based on a matrix of correlations between the variables.
- Bartlett's test of sphericity can be used to test the null hypothesis that the variables are uncorrelated in the population: in other words, the population correlation matrix is an identity matrix. If this hypothesis cannot be rejected, then the appropriateness of factor analysis should be questioned.
- Another useful statistic is the Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy. Small values of the KMO statistic indicate that the correlations between pairs of variables cannot be explained by other variables and that factor analysis may not be appropriate.

Conducting Factor Analysis
Determine the Method of Factor Analysis

- In **principal components analysis**, the total variance in the data is considered. The diagonal of the correlation matrix consists of unities, and full variance is brought into the factor matrix. Principal components analysis is recommended when the primary concern is to determine the minimum number of factors that will account for maximum variance in the data for use in subsequent multivariate analysis. The factors are called *principal components*.
- In **common factor analysis**, the factors are estimated based only on the common variance. Communalities are inserted in the diagonal of the correlation matrix. This method is appropriate when the primary concern is to identify the underlying dimensions and the common variance is of interest. This method is also known as *principal axis factoring*.

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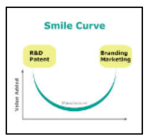
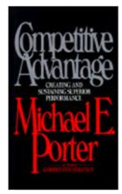
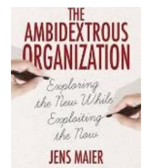






Results of Principal Components Analysis

Table 19.3

Communalities

Variables	Initial	Extraction
V1	1.000	0.926
V2	1.000	0.723
V3	1.000	0.894
V4	1.000	0.739
V5	1.000	0.878
V6	1.000	0.790

Initial Eigen values

Factor	Eigen value	% of variance	Cumulat. %
1	2.731	45.520	45.520
2	2.218	36.969	82.488
3	0.442	7.360	89.848
4	0.341	5.688	95.536
5	0.183	3.044	98.580
6	0.085	1.420	100.000

Results of Principal Components Analysis

Table 19.3, cont.

Extraction Sums of Squared Loadings

Factor	Eigen value	% of variance	Cumulat. %
1	2.731	45.520	45.520
2	2.218	36.969	82.488

Factor Matrix

Variables	Factor 1	Factor 2
V1	0.928	0.253
V2	-0.301	0.795
V3	0.936	0.131
V4	-0.342	0.789
V5	-0.869	-0.351
V6	-0.177	0.871

Rotation Sums of Squared Loadings

Factor	Eigenvalue	% of variance	Cumulat. %
1	2.688	44.802	44.802
2	2.261	37.687	82.488

Results of Principal Components Analysis

Table 19.3, cont.

Rotated Factor Matrix

Variables	Factor 1	Factor 2
V1	0.962	-0.027
V2	-0.057	0.848
V3	0.934	-0.146
V4	-0.098	0.845
V5	-0.933	-0.084
V6	0.083	0.885

Factor Score Coefficient Matrix

Variables	Factor 1	Factor 2
V1	0.358	0.011
V2	-0.001	0.375
V3	0.345	-0.043
V4	-0.017	0.377
V5	-0.350	-0.059
V6	0.052	0.395

Results of Principal Components Analysis

Table 19.3, cont.

The lower-left triangle contains the reproduced correlation matrix; the diagonal, the communalities; the upper-right triangle, the residuals between the observed correlations and the reproduced (rotated) correlations.

Factor Score Coefficient Matrix

Variables	V1	V2	V3	V4	V5	V6
V1	0.926	0.024	-0.029	0.031	0.038	-0.053
V2	-0.078	0.723	0.022	-0.158	0.038	-0.105
V3	0.902	-0.177	0.894	-0.031	0.081	0.033
V4	-0.117	0.730	-0.217	0.739	-0.027	-0.107
V5	-0.895	-0.018	-0.859	0.020	0.878	0.016
V6	0.057	0.746	-0.051	0.748	-0.152	0.790

Conducting Factor Analysis
Determine the Number of Factors

- **A Priori Determination.** Sometimes, because of prior knowledge, the researcher knows how many factors to expect and thus can specify the number of factors to be extracted beforehand.

- **Determination Based on Eigenvalues.** In this approach, only factors with Eigenvalues greater than 1.0 are retained. An Eigenvalue represents the amount of variance associated with the factor. Hence, only factors with a variance greater than 1.0 are included. Factors with variance less than 1.0 are no better than a single variable, since, due to standardization, each variable has a variance of 1.0. If the number of variables is less than 20, this approach will result in a conservative number of factors.

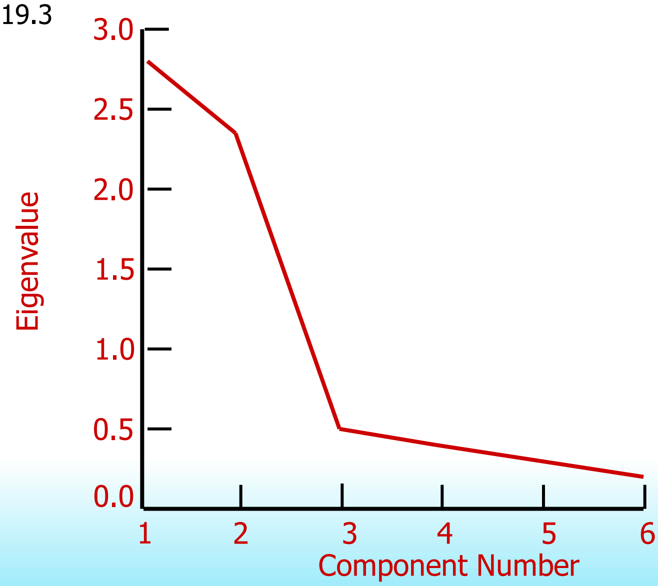
Conducting Factor Analysis
Determine the Number of Factors

- **Determination Based on Scree Plot.** A scree plot is a plot of the Eigenvalues against the number of factors in order of extraction. Experimental evidence indicates that the point at which the scree begins denotes the true number of factors. Generally, the number of factors determined by a scree plot will be one or a few more than that determined by the Eigenvalue criterion.

- **Determination Based on Percentage of Variance.** In this approach the number of factors extracted is determined so that the cumulative percentage of variance extracted by the factors reaches a satisfactory level. It is recommended that the factors extracted should account for at least 60% of the variance.

Scree Plot

Fig. 19.3



Conducting Factor Analysis
Determine the Number of Factors

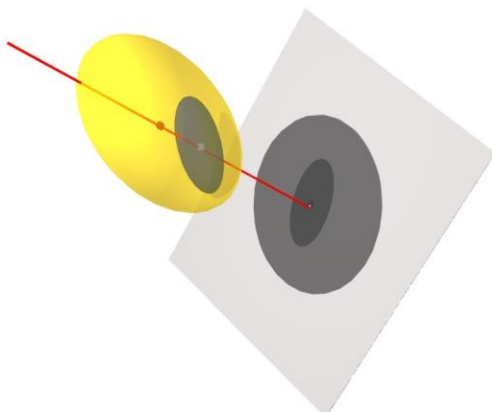
- **Determination Based on Split-Half Reliability.** The sample is split in half and factor analysis is performed on each half. Only factors with high correspondence of factor loadings across the two subsamples are retained.

- **Determination Based on Significance Tests.** It is possible to determine the statistical significance of the separate Eigenvalues and retain only those factors that are statistically significant. A drawback is that with large samples (size greater than 200), many factors are likely to be statistically significant, although from a practical viewpoint many of these account for only a small proportion of the total variance.

Conducting Factor Analysis
Rotate Factors

- Although the initial or unrotated factor matrix indicates the relationship between the factors and individual variables, it seldom results in factors that can be interpreted, because the factors are correlated with many variables. Therefore, through rotation the factor matrix is transformed into a simpler one that is easier to interpret.
- In rotating the factors, we would like each factor to have nonzero, or significant, loadings or coefficients for only some of the variables. Likewise, we would like each variable to have nonzero or significant loadings with only a few factors, if possible with only one.
- The rotation is called **orthogonal rotation** if the axes are maintained at right angles.

Rotation (transformation)



Let **T** be any orthogonal matrix such that **B = ΛT, B B'= ΛT (ΛT)' = Λ Λ'.**

Conducting Factor Analysis Rotate Factors

- The most commonly used method for rotation is the **varimax procedure**. This is an orthogonal method of rotation that minimizes the number of variables with high loadings on a factor, thereby enhancing the interpretability of the factors. Orthogonal rotation results in factors that are uncorrelated. (e.g., big five personality traits)
- The rotation is called **oblique rotation** when the axes are not maintained at right angles, and the factors are correlated. Sometimes, allowing for correlations among factors can simplify the factor pattern matrix. Oblique rotation should be used when factors (constructs) in the population are likely to be strongly correlated. (e.g., GDP and population size)

Factor Matrix Before and After Rotation

Fig. 19.4

Factors

Variables	1	2
1	X	
2	X	X
3	X	
4	X	X
5	X	X
6		X

(a)
High Loadings
Before Rotation

Factors

Variables	1	2
1	X	
2		X
3	X	
4		X
5	X	
6		X

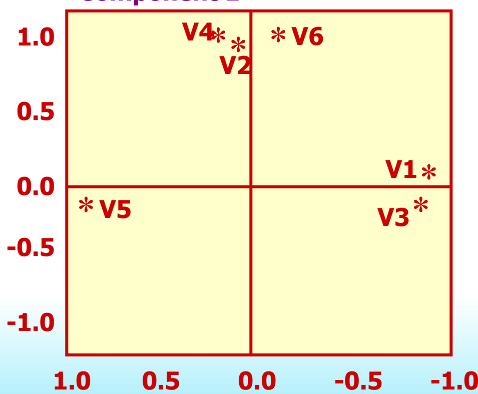
(b)
High Loadings
After Rotation

Conducting Factor Analysis
Interpret Factors

- A factor can then be interpreted in terms of the variables that load high on it.
- Another useful aid in interpretation is to plot the variables, using the factor loadings as coordinates. Variables at the end of an axis are those that have high loadings on only that factor, and hence describe the factor.

Factor Loading Plot

Fig. 19.5



Rotated Component Matrix
Component 2

Variable	Component 1	Component 2
V1	0.962	-2.66E-02
V2	-5.72E-02	0.848
V3	0.934	-0.146
V4	-9.83E-02	0.854
V5	-0.933	-8.40E-02
V6	8.337E-02	0.885

Conducting Factor Analysis Calculate Factor Scores

The **factor scores** for the *i*th factor may be estimated
as follows:

$$F_i = W_{i1}X_1 + W_{i2}X_2 + W_{i3}X_3 + \ldots + W_{ik}X_k$$

Conducting Factor Analysis
Select Surrogate Variables

- By examining the factor matrix, one could select for each factor the variable with the highest loading on that factor. That variable could then be used as a surrogate variable 代理 for the associated factor.
- However, the choice is not as easy if two or more variables have similarly high loadings. In such a case, the choice between these variables should be based on theoretical and measurement considerations.

Conducting Factor Analysis
Determine the Model Fit

- The correlations between the variables can be deduced or reproduced from the estimated correlations between the variables and the factors.
- The differences between the observed correlations (as given in the input correlation matrix) and the reproduced correlations (as estimated from the factor matrix) can be examined to determine model fit. These differences are called *residuals*.

Results of Common Factor Analysis

Table 19.4

Communalities

Variables	Initial	Extraction
V1	0.859	0.928
V2	0.480	0.562
V3	0.814	0.836
V4	0.543	0.600
V5	0.763	0.789
V6	0.587	0.723

Barlett test of sphericity

- Approx. Chi-Square = 111.314
- df = 15
- Significance = 0.00000
- Kaiser-Meyer-Olkin measure of sampling adequacy = 0.660

Initial Eigenvalues

Factor	Eigenvalue	% of variance	Cumulat. %
1	2.731	45.520	45.520
2	2.218	36.969	82.488
3	0.442	7.360	89.848
4	0.341	5.688	95.536
5	0.183	3.044	98.580
6	0.085	1.420	100.000

Results of Common Factor Analysis

Table 19.4, cont.

Extraction Sums of Squared Loadings

Factor	Eigenvalue	% of variance	Cumulat. %
1	2.570	42.837	42.837
2	1.868	31.126	73.964

Factor Matrix

Variables	Factor 1	Factor 2
V1	0.949	0.168
V2	-0.206	0.720
V3	0.914	0.038
V4	-0.246	0.734
V5	-0.850	-0.259
V6	-0.101	0.844

Rotation Sums of Squared Loadings

Factor	Eigenvalue	% of variance	Cumulat. %
1	2.541	42.343	42.343
2	1.897	31.621	73.964

Results of Common Factor Analysis

Table 19.4, cont.

Rotated Factor Matrix

Variables	Factor 1	Factor 2
V1	0.963	-0.030
V2	-0.054	0.747
V3	0.902	-0.150
V4	-0.090	0.769
V5	-0.885	-0.079
V6	0.075	0.847

Factor Score Coefficient Matrix

Variables	Factor 1	Factor 2
V1	0.628	0.101
V2	-0.024	0.253
V3	0.217	-0.169
V4	-0.023	0.271
V5	-0.166	-0.059
V6	0.083	0.500

Results of Common Factor Analysis

Table 19.4, cont.

The lower-left triangle contains the reproduced correlation matrix; the diagonal, the communalities; the upper-right triangle, the residuals between the observed correlations and the reproduced correlations.

Factor Score Coefficient Matrix

Variables	V1	V2	V3	V4	V5	V6
V1	0.928	0.022	-0.000	0.024	-0.008	-0.042
V2	-0.075	0.562	0.006	-0.008	0.031	0.012
V3	0.873	-0.161	0.836	-0.005	0.008	0.042
V4	-0.110	0.580	-0.197	0.600	-0.025	-0.004
V5	-0.850	-0.012	-0.786	0.019	0.789	0.003
V6	0.046	0.629	-0.060	0.645	-0.133	0.723

SAS example: Job Ratings

```
options validvarname=any;
data jobratings;
   input ('Communication Skills'n
          'Problem Solving'n
          'Learning Ability'n
          'Judgment Under Pressure'n
          'Observational Skills'n
          'Willingness to Confront Problems'n
          'Interest in People'n
          'Interpersonal Sensitivity'n
          'Desire for Self-Improvement'n
          'Appearance'n
          'Dependability'n
          'Physical Ability'n
          'Integrity'n
          'Overall Rating'n) (1.);
   datalines;
26838853879867
74758876857667
56757863775875
67869777988997
```

```
proc factor data=jobratings(drop='Overall Rating'n) priors=smc
   rotate=varimax;
run;
```

The FACTOR Procedure
Initial Factor Method: Principal Factors

Prior Communality Estimates: SMC

Communication Skills	Problem Solving	Learning Ability	Judgment Under Pressure	Observational Skills
0.62981394	0.58657431	0.61009871	0.63766021	0.67187583

Willingness to Confront Problems	Interest in People	Interpersonal Sensitivity	Desire for Self-Improvement
0.64779805	0.75641519	0.75584891	0.57460176

Appearance	Dependability	Physical Ability	Integrity
0.45505304	0.63449045	0.42245324	0.68195454

Eigenvalues of the Reduced Correlation Matrix:
Total = 8.06463816   Average = 0.62035678

	Eigenvalue	Difference	Proportion	Cumulative
1	6.17760549	4.71531946	0.7660	0.7660
2	1.46228602	0.90183348	0.1813	0.9473
3	0.56045254	0.28093933	0.0695	1.0168
4	0.27951322	0.04766016	0.0347	1.0515
5	0.23185305	0.16113428	0.0287	1.0802
6	0.07071877	0.07489624	0.0088	1.0890
7	-.00417747	0.03387533	-0.0005	1.0885
8	-.03805279	0.04776534	-0.0047	1.0838
9	-.08581814	0.02438060	-0.0106	1.0731
10	-.11019874	0.01452741	-0.0137	1.0595
11	-.12472615	0.02356465	-0.0155	1.0440
12	-.14829080	0.05823605	-0.0184	1.0256
13	-.20652684		-0.0256	1.0000

3 factors will be retained by the PROPORTION criterion.

$$X_i = \lambda_{i1}F1 + \lambda_{i2}F2 + \lambda_{i3}F3 + \ldots + \lambda_{im}Fm$$

Initial Factor Method: Principal Factors

Factor Pattern

	Factor1	Factor2	Factor3
Communication Skills	0.75441	0.07707	-0.25551
Problem Solving	0.68590	0.08026	-0.34788
Learning Ability	0.65904	0.34808	-0.25249
Judgment Under Pressure	0.73391	-0.21405	-0.23513
Observational Skills	0.69039	0.45292	0.10298
Willingness to Confront Problems	0.66458	0.47460	0.09210
Interest in People	0.70770	-0.53427	0.10979
Interpersonal Sensitivity	0.64668	-0.61284	-0.07582
Desire for Self-Improvement	0.73820	0.12506	0.09062
Appearance	0.57188	0.20052	0.16367
Dependability	0.79475	-0.04516	0.16400
Physical Ability	0.51285	0.10251	0.34860
Integrity	0.74906	-0.35091	0.18656

The pattern matrix suggests that Factor1 represents general ability. All loadings for Factor1 in the Factor Pattern are at least 0.5. Factor2 consists of high positive loadings on certain task-related skills (Willingness to Confront Problems, Observational Skills, and Learning Ability) and high negative loadings on some interpersonal skills (Interpersonal Sensitivity, Interest in People, and Integrity).

After Rotate=Varimax

The rotated factor pattern matrix is somewhat simpler to interpret. If a magnitude of at least 0.5 is required to indicate a salient variable-factor relationship, Factor1 now represents interpersonal skills (Interpersonal Sensitivity, Interest in People, Integrity, Judgment Under Pressure, and Dependability). Factor2 measures physical skills and job enthusiasm (Observational Skills, Willingness to Confront Problems, Physical Ability, Desire for Self-Improvement, Dependability, and Appearance). Factor3 measures cognitive skills (Communication Skills, Problem Solving, Learning Ability, and Judgment Under Pressure).

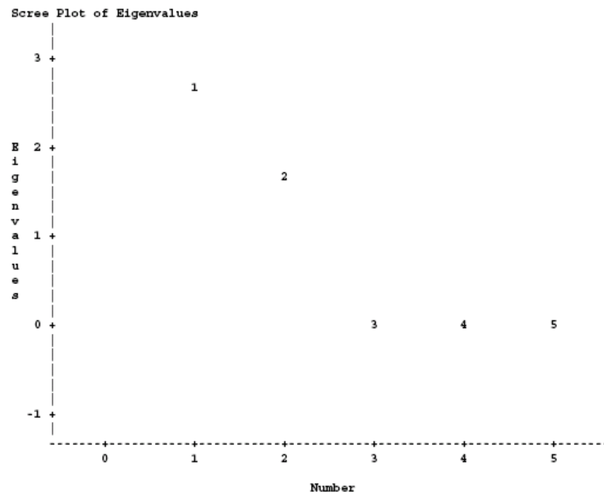
	Factor1	Factor2	Factor3
Communication Skills	0.35991	0.32744	0.63530
Problem Solving	0.30802	0.23102	0.67058
Learning Ability	0.08679	0.41149	0.66512
Judgment Under Pressure	0.58287	0.17901	0.51764
Observational Skills	0.05533	0.70488	0.43870
Willingness to Confront Problems	0.02168	0.69391	0.43978
Interest in People	0.85677	0.21422	0.13562
Interpersonal Sensitivity	0.86587	0.02239	0.22200
Desire for Self-Improvement	0.34498	0.55775	0.37242
Appearance	0.19319	0.54327	0.24814
Dependability	0.52174	0.54981	0.29337
Physical Ability	0.25445	0.57321	0.04165
Integrity	0.74172	0.38033	0.15567

SCREE PLOT

Output 27.2.2. Scree Plot

Principal Factor Analysis with Promax Rotation

Initial Factor Method: Principal Factors



Example 33.1: Principal Component Analysis

This example analyzes socioeconomic data provided by Harman (1976). The five variables represent total population, median school years, total employment, miscellaneous professional services and median house value. Each observation represents one of twelve census tracts in the Los Angeles Standard Metropolitan Statistical Area.

The first analysis is a principal component analysis. Simple descriptive statistics and correlations are also displayed. The following statements produce Output 33.1.1:

```
data SocioEconomics;
   title 'Five Socioeconomic Variables';
   title2 'See Page 14 of Harman: Modern Factor Analysis, 3rd Ed';
   input Population School Employment Services HouseValue;
   datalines;
5700     12.8     2500     270     25000
1000     10.9     600      10      10000
3400     8.8      1000     10      9000
3800     13.6     1700     140     25000
4000     12.8     1600     140     25000
8200     8.3      2600     60      12000
1200     11.4     400      10      16000
9100     11.5     3300     60      14000
9900     12.5     3400     180     18000
9600     13.7     3600     390     25000
9600     9.6      3300     80      12000
9400     11.4     4000     100     13000
;

title3 'Principal Component Analysis';
proc factor data=SocioEconomics simple corr;
run;
```

Correlations

	Population	School	Employment	Services	HouseValue
Population	1.00000	0.00975	0.97245	0.43887	0.02241
School	0.00975	1.00000	0.15428	0.69141	0.86307
Employment	0.97245	0.15428	1.00000	0.51472	0.12193
Services	0.43887	0.69141	0.51472	1.00000	0.77765
HouseValue	0.02241	0.86307	0.12193	0.77765	1.00000

Five Socioeconomic Variables
See Page 14 of Harman: Modern Factor Analysis, 3rd Ed
Principal Component Analysis

The FACTOR Procedure
Initial Factor Method: Principal Components

→ Prior Communality Estimates: ONE

Eigenvalues of the Correlation Matrix: Total = 5 Average = 1

	Eigenvalue	Difference	Proportion	Cumulative
1	2.87331359	1.07665350	0.5747	0.5747
2	1.79666009	1.58182321	0.3593	0.9340
3	0.21483689	0.11490283	0.0430	0.9770
4	0.09993405	0.08467868	0.0200	0.9969
5	0.01525537		0.0031	1.0000

```
proc factor data=SocioEconomics
   priors=smc msa residual
   rotate=promax reorder
   outstat=fact_all
   plots=(scree initloadings preloadings loadings);
run;
```

Partial Correlations Controlling all other Variables

	Population	School	Employment	Services	HouseValue
Population	1.00000	-0.54465	0.97083	0.09612	0.15871
School	-0.54465	1.00000	0.54373	0.04996	0.64717
Employment	0.97083	0.54373	1.00000	0.06689	-0.25572
Services	0.09612	0.04996	0.06689	1.00000	0.59415
HouseValue	0.15871	0.64717	-0.25572	0.59415	1.00000

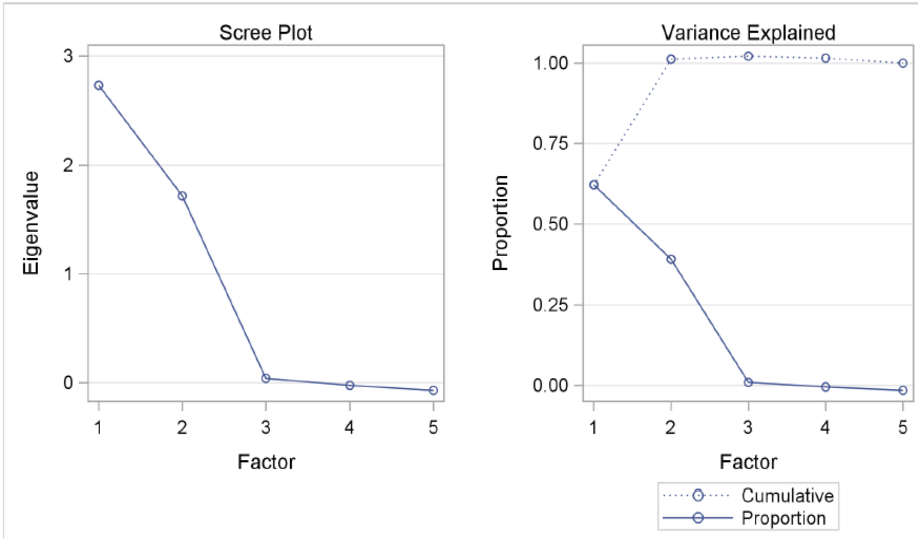
Kaiser's Measure of Sampling Adequacy: Overall MSA = 0.57536759

Population	School	Employment	Services	HouseValue
0.47207897	0.55158839	0.48851137	0.80664365	0.61281377

Prior Communality Estimates: SMC

Population	School	Employment	Services	HouseValue
0.96859160	0.82228514	0.96918082	0.78572440	0.84701921

Output 33.2.2 Scree and Variance Explained Plots



Output 33.2.6 Varimax Rotation: Transform Matrix and Rotated Pattern

```
Five Socioeconomic Variables
See Page 14 of Harman: Modern Factor Analysis, 3rd Ed
Principal Factor Analysis with Promax Rotation

The FACTOR Procedure
Prerotation Method: Varimax

Orthogonal Transformation Matrix

                1             2

1         0.78895       0.61446
2        -0.61446       0.78895
```

Orthogonal Matrix: $\mathbf{PP' = P'P = I}$

Rotated Factor Pattern

	Factor1	Factor2
HouseValue	0.94072	-0.00004
School	0.90419	0.00055
Services	0.79085	0.41509
Population	0.02255	0.98874
Employment	0.14625	0.97499

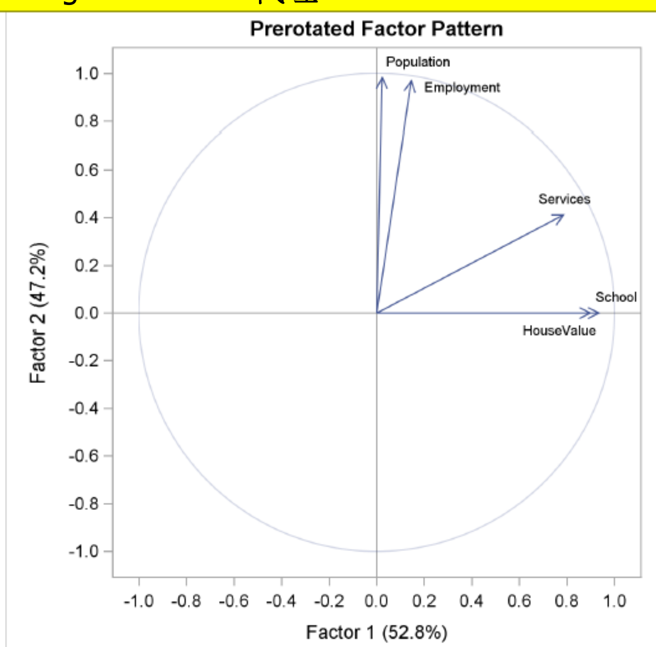
Variance Explained by Each Factor

Factor1	Factor2
2.3498567	2.1005128

Final Communality Estimates: Total = 4.450370

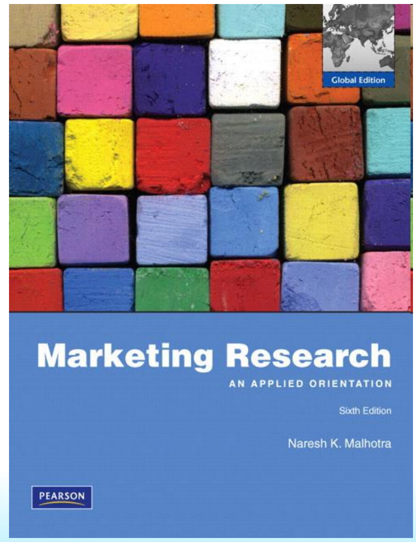
Population	School	Employment	Services	HouseValue
0.97811334	0.81756387	0.97199928	0.79774304	0.88494998

surrogate variable 代理 for the associated factor



Chapter Twenty-Two

Structural Equation Modeling and Path Analysis



LISREL, an acronym for linear structural relations, is a statistical software package used in structural equation modeling. (ASIMPLIS, AMOS, EQA, Mplus, Mx, RAMONA)

Structural equation modeling (SEM), a procedure for estimating a series of dependence relationships among a set of concepts or constructs represented by multiple measured variables and incorporated into an integrated model.

Path analysis A special case of SEM with only single indicators for each of the variables in the causal model. In other words, path analysis is SEM with a structural model, but no measurement model.

Review - SAS manual

The equation for the common factor model is

$$y_{ij} = x_{i1}b_{1j} + x_{i2}b_{2j} + \cdots + x_{iq}b_{qj} + e_{ij}$$

where

- y_{ij} is the value of the ith observation on the jth variable
- x_{ik} is the value of the ith observation on the kth common factor
- b_{kj} is the regression coefficient of the kth common factor for predicting the jth variable
- e_{ij} is the value of the ith observation on the jth unique factor
- q is the number of common factors

It is assumed, for convenience, that all variables have a mean of 0. In matrix terms, these equations reduce to

$$\mathbf{Y} = \mathbf{XB} + \mathbf{E}$$

In the preceding equation, $\mathbf{X}$ is the matrix of factor scores, and $\mathbf{B}'$ is the factor pattern.

When the factors are initially extracted, it is also assumed, for convenience, that the common factors are uncorrelated with each other and have unit variance. In this case, the common factor model implies that the covariance s_{jk} between the jth and kth variables, $j \neq k$, is given by

$$s_{jk} = b_{1j}b_{1k} + b_{2j}b_{2k} + \cdots + b_{qj}b_{qk}$$

or

$$\mathbf{S} = \mathbf{B}'\mathbf{B} + \mathbf{U}^2$$

where $\mathbf{S}$ is the covariance matrix of the observed variables, and $\mathbf{U}^2$ is the diagonal covariance matrix of the unique factors.

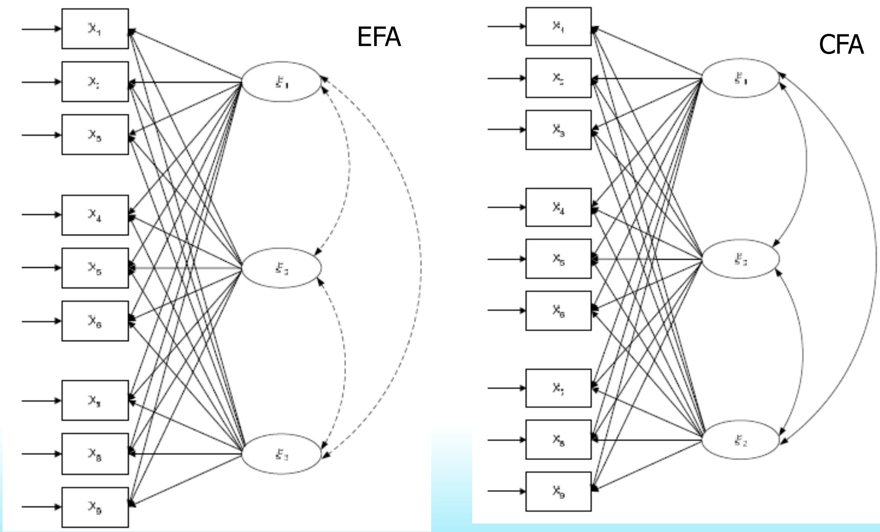
If the original variables are standardized to unit variance, the preceding formula yields correlations instead of covariances. It is in this sense that common factors explain the correlations among the observed variables. When considering the diagonal elements of standardized $\mathbf{S}$, the variance of the jth variable is expressed as

$$s_{jj} = 1 = b_{1j}^2 + b_{2j}^2 + \cdots + b_{qj}^2 + [\mathbf{U}^2]_{jj}$$

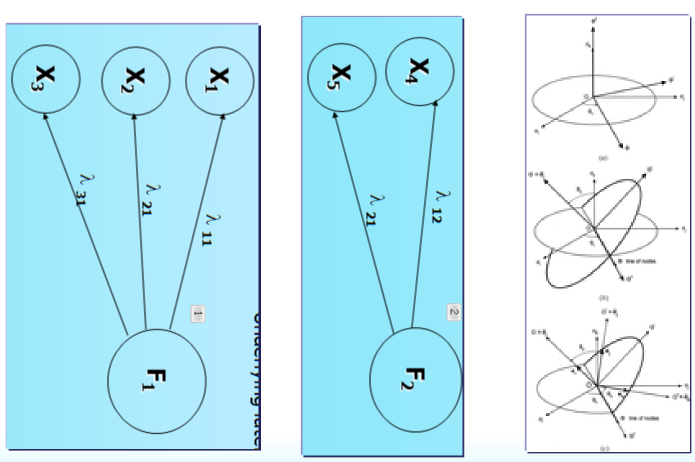
$$\text{BAC} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$$

where $b_{1j}^2 + b_{2j}^2 + \cdots + b_{qj}^2$ and $[\mathbf{U}^2]_{jj}$ are the communality and uniqueness, respectively, of the jth variable. The communality represents the extent of the overlap with the common factors. In other words, it is the proportion of variance accounted for by the common factors.

Exploratory vs. Confirmatory FA



Oblique Rotation in CFA



EFA

- is a variable reduction technique which identifies the number of latent constructs and the underlying factor structure of a set of variables
- hypothesizes an underlying construct, a variable not measured directly
- estimates factors which influence responses on observed variables
- allows you to describe and identify the number of latent constructs (factors)
- includes unique factors, error due to unreliability in measurement
- traditionally has been used to explore the possible underlying factor structure of a set of measured variables without imposing any preconceived structure on the outcome (Child, 1990).

Goals of factor analysis are

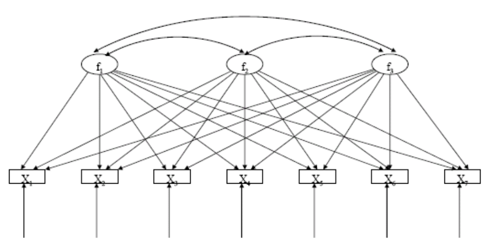
1) to help an investigator determine the number of latent constructs underlying a set of items (variables)
2) to provide a means of explaining variation among variables (items) using a few newly created variables (factors), e.g., condensing information
3) to define the content or meaning of factors, e.g., latent constructs

Assumptions underlying EFA are

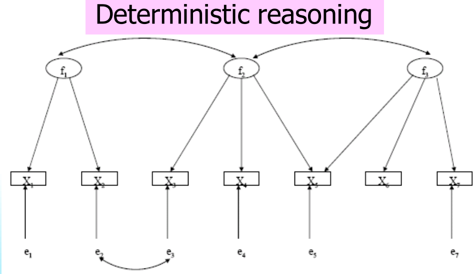
- Interval or ratio level of measurement
- Random sampling
- Relationship between observed variables is linear
- A normal distribution (each observed variable)
- A bivariate normal distribution (each pair of observed variables)
- Multivariate normality

EFA vs. CFA

CFA assumption: knows exactly which item loads on what factor



Deterministic reasoning



Confirmatory VS. Exploratory

Differences between CFA and EFA

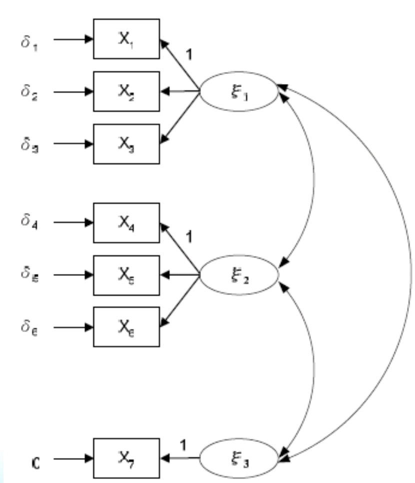
CFA requires specification of

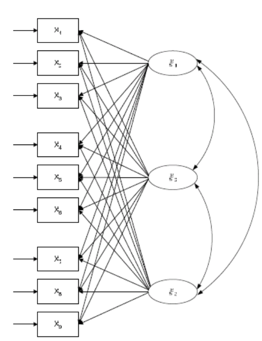
- a model a priori
- the number of factors
- which items load on each factor
- a model supported by theory or previous research
- error explicitly

Inductive reasoning 歸納推理: Use the observed data to confirm or define the hypothetical construct as general rule to aid prediction and expectation.

X^2, the discrepancy measure, compared the sample (observed) covariance matrix with the implies model covariance matrix computed from the hypothetical structure and all the identified model parameters

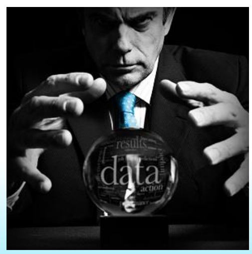
Confirmatory Factor Analysis



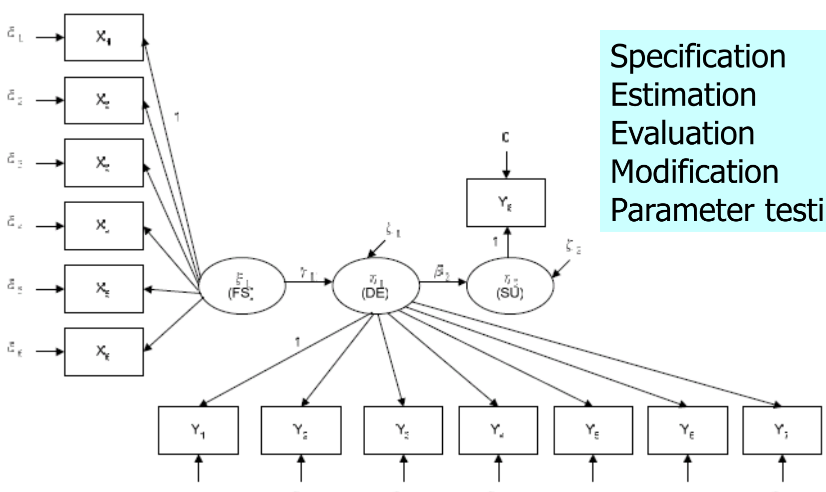


What might be wrong about data mining (a CFA without theory)?

PROC CALIS offers an analysis of linear dependencies in the information matrix (approximate Hessian matrix) that might be helpful in detecting unidentified models. You also can save the information matrix and the approximate covariance matrix of the parameter estimates (inverse of the information matrix), together with parameter estimates, gradient, and approximate standard errors, in an output data set for further analysis.



Structure Equation Model
SEM=CFA + PA



Specification
Estimation
Evaluation
Modification
Parameter testing

SAS **TCALIS**: FACTOR, LINEQS, LISMOD, PATH, and RAM

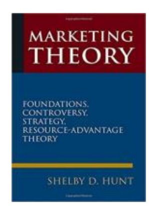
4.2 CAUSAL EXPLANATIONS

Other marketing researchers are vigorously pursuing the structural equation approach to causal modeling. Originally conceptualized by Bock and Borgman (1966) and later developed by Joreskog (1968, 1973), structural equation modeling (SEM) uses, among others, the maximum likelihood method for estimating parameters. Bagozzi (1980) introduced the approach in marketing and used it to explore for causal relationships between performance and satisfaction among industrial salespeople. Bentler (1990), Bollen (1989), Fornell (1983), Rigdon (1995), and Rigdon and Ferguson (1991), among others, develop the approach. Major advantages of SEM include the ability to control for measurement error, an enhanced ability to test the effects of experimental manipulations, the ability to test complex theoretical structures, the ability to link micro and macro perspectives, and more powerful ways to assess measure reliability and validity (MacKenzie 2001).

In conclusion, the use of the concepts *cause* and *causation* remain and should remain in marketing. Indeed, the search for true causal relationships is central to the mission of marketing science. However, we must never delude ourselves into believing that we can ever know any causal relationship with certainty. Purportedly causal relationships are always only more or less probable, and we should always diligently explore the possibility that the relationships are actually spurious. The very essence of science is that all statements are tentative; all are subject to change and revision on the basis of future evidence.

4

EXPLANATION: ISSUES AND ASPECTS

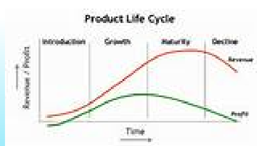


Whenever we propose a solution to a problem we ought to try as hard as we can to overthrow our solution rather than defend it. Few of us, unfortunately, practice this precept; but other people, fortunately, will supply the criticism for us if we fail to supply it ourselves.

—*Karl R. Popper*

Inductive Reasoning 歸納推理

- Locke, Berkeley, and Hume (1711-1776)
- *Logical* reasoning: one makes a series of observations and infers a new claim based on them
- 少年們得到了許多超速罰單 → 所以所有少年都超速。
- *Psychological* reasoning (I-S explanation): one draws inferences from a limited number of observations to a general rule which will aid us in prediction and expectation (observations → statistical inferences → decision rule→ prediction and expectation)
- 冰是冷的 →所有冰都是冷的 (future resemble the past)
- Physics: Newton's Theory of Gravitation



統計歸納法的推論"原罪"

Keesling-Wiley-Jöreskog LISREL (Linear Structural Relationship) Model (Keesling 1972; Wiley 1973; Jöreskog 1973)

Str $\eta = \beta\eta + \gamma\xi$ nd measurement models:

$$\eta = \mathbf{B}\eta + \mathbf{\Gamma}\xi + \zeta \quad , \quad \mathbf{y} = \mathbf{\Lambda}_y\eta + \varepsilon \quad , \quad \mathbf{x} = \mathbf{\Lambda}_x\xi + \delta$$

where η and ξ are vectors of latent variables (factors), and $\mathbf{x}$ and $\mathbf{y}$ are vectors of manifest variables. The components of η correspond to endogenous latent variables; the components of ξ correspond to exogenous latent variables. The endogenous and exogenous latent variables are connected by a system of linear equations (the structural model) with coefficient matrices $\mathbf{B}$ and $\mathbf{\Gamma}$ and an error vector ζ. It is assumed that matrix $\mathbf{I} - \mathbf{B}$ is nonsingular. The random vectors $\mathbf{y}$ and $\mathbf{x}$ correspond to manifest variables that are related to the latent variables η and ξ by two systems of linear equations (the measurement model) with coefficients $\mathbf{\Lambda}_y$ and $\mathbf{\Lambda}_x$ and with measurement errors ε and δ.

$$\mathbf{C} = \mathbf{J}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{P}((\mathbf{I}-\mathbf{A})^{-1})'\mathbf{J}'$$

$$A = \begin{pmatrix} 0 & 0 & \mathbf{\Lambda}_y & 0 \\ 0 & 0 & 0 & \mathbf{\Lambda}_x \\ 0 & 0 & \mathbf{B} & \mathbf{\Gamma} \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad P = \begin{pmatrix} \mathbf{\Theta}_\varepsilon & & & \\ & \mathbf{\Theta}_\delta & & \\ & & \mathbf{\Psi} & \\ & & & \mathbf{\Phi} \end{pmatrix}$$

with selection matrix $\mathbf{J}$, $\mathbf{\Phi} = \mathcal{E}\{\xi\xi'\}$, $\mathbf{\Psi} = \mathcal{E}\{\zeta\zeta'\}$, $\mathbf{\Theta}_\delta = \mathcal{E}\{\delta\delta'\}$, and $\mathbf{\Theta}_\varepsilon = \mathcal{E}\{\varepsilon\varepsilon'\}$.

恆真句(Truism)的套套邏輯

How is knowledge acquired or developed?

- 恆真句(的同義反覆論述): C(A + B) = CA + CB
- 套套邏輯是指同義重覆。同義重覆的句子不可能被事實否証或推翻，因此恆真句沒有解釋能力。
- Survival of the fittest (適者生存)
- widow of the late Mr. Smith (已故史密斯先生的遺孀)
- 資源能力學派(RBV)：*VRIN principle (valuable, rare, inimitable, and non-substitutable)*
- *CFA: two constructs are correlated, therefore the path coefficient (loading) is significant.*

Determinism and Tautology

- Immanuel Kant (康德): to a great extent we impose our structures on the world, in particular the world is Euclidean because this is the way we organize spatial positions.
- *Determinism* in Newton's theory (Einstein's Theory of Relativity)
- *Irrational* problem in scientific thinking
- *Tautology* in strategic management: Porter's generic strategy and resource-based view (a(b+c) = ab + ac)
- Tautology in marketing: confirmation study in PLC, TAM, AIDA, and hierarchical models (did not generate any new knowledge)

「如果這不是關說，
那什麼才是關說」