

# **SAS/STAT® 9.2 User's Guide**

## **The FACTOR Procedure**

### **(Book Excerpt)**



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## Chapter 33

# The FACTOR Procedure

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## Overview: FACTOR Procedure

The FACTOR procedure performs a variety of common factor and component analyses and rotations. Input can be multivariate data, a correlation matrix, a covariance matrix, a factor pattern, or a matrix of scoring coefficients. The procedure can factor either the correlation or covariance matrix, and you can save most results in an output data set.

PROC FACTOR can process output from other procedures. For example, it can rotate the canonical coefficients from multivariate analyses in the GLM procedure.

The methods for factor extraction are principal component analysis, principal factor analysis, iterated principal factor analysis, unweighted least squares factor analysis, maximum likelihood (canonical) factor analysis, alpha factor analysis, image component analysis, and Harris component analysis. A variety of methods for prior communality estimation is also available.

Specific methods for orthogonal rotation are varimax, quartimax, biquartimax, equamax, parsimax, and factor parsimax. Oblique versions of these methods are also available. In addition, quartimin, biquartimin, and covarimin methods for (direct) oblique rotation are available. General methods for orthogonal rotation are orthomax with user-specified gamma, Crawford-Ferguson family with user-specified weights on variable parsimony and factor parsimony, and generalized Crawford-Ferguson family with user-specified weights. General methods for oblique rotation are direct oblimin with user-specified tau, Crawford-Ferguson family with user-specified weights on variable parsimony and factor parsimony, generalized Crawford-Ferguson family with user-specified weights, promax with user-specified exponent, Harris-Kaiser case II with user-specified exponent, and Procrustes with a user-specified target pattern.

Output includes means, standard deviations, correlations, Kaiser's measure of sampling adequacy, eigenvalues, a scree plot, eigenvectors, prior and final communality estimates, the unrotated factor pattern, residual and partial correlations, the rotated primary factor pattern, the primary factor structure, interfactor correlations, the reference structure, reference axis correlations, the variance explained by each factor both ignoring and eliminating other factors, plots of both rotated and unrotated factors, squared multiple correlation of each factor with the variables, standard error estimates, confidence limits, coverage displays, and scoring coefficients.

The FACTOR procedure now uses ODS Graphics to create graphs as part of its output. For general information about ODS Graphics, see Chapter 21, "[Statistical Graphics Using ODS](#)."

Any topics that are not given explicit references are discussed in Mulaik (1972) or Harman (1976).

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## Background

See Chapter 69, "[The PRINCOMP Procedure](#)," for a discussion of principal component analysis. See Chapter 25, "[The CALIS Procedure](#)," for a discussion of confirmatory factor analysis.

Common factor analysis was invented by Spearman (1904). Kim and Mueller (1978a, 1978b) provide a very elementary discussion of the common factor model. Gorsuch (1974) presents a broad

survey of factor analysis, and Gorsuch (1974) and Cattell (1978) are useful as guides to practical research methodology. Harman (1976) gives a lucid discussion of many of the more technical aspects of factor analysis, especially oblique rotation. Morrison (1976) and Mardia, Kent, and Bibby (1979) provide excellent statistical treatments of common factor analysis. Mulaik (1972) provides the most thorough and authoritative general reference on factor analysis and is highly recommended to anyone familiar with matrix algebra. Stewart (1981) gives a nontechnical presentation of some issues to consider when deciding whether or not a factor analysis might be appropriate.

A frequent source of confusion in the field of factor analysis is the term *factor*. It sometimes refers to a hypothetical, unobservable variable, as in the phrase *common factor*. In this sense, *factor analysis* must be distinguished from component analysis since a component is an observable linear combination. *Factor* is also used in the sense of *matrix factor*, in that one matrix is a factor of a second matrix if the first matrix multiplied by its transpose equals the second matrix. In this sense, *factor analysis* refers to all methods of data analysis that use matrix factors, including component analysis and common factor analysis.

A *common factor* is an unobservable, hypothetical variable that contributes to the variance of at least two of the observed variables. The unqualified term “factor” often refers to a common factor. A *unique factor* is an unobservable, hypothetical variable that contributes to the variance of only one of the observed variables. The model for common factor analysis posits one unique factor for each observed variable.

The equation for the common factor model is

$$y_{ij} = x_{i1}b_{1j} + x_{i2}b_{2j} + \cdots + x_{iq}b_{qj} + e_{ij}$$

where

$y_{ij}$	is the value of the $i$ th observation on the $j$ th variable
$x_{ik}$	is the value of the $i$ th observation on the $k$ th common factor
$b_{kj}$	is the regression coefficient of the $k$ th common factor for predicting the $j$ th variable
$e_{ij}$	is the value of the $i$ th observation on the $j$ th unique factor
$q$	is the number of common factors

It is assumed, for convenience, that all variables have a mean of 0. In matrix terms, these equations reduce to

$$\mathbf{Y} = \mathbf{XB} + \mathbf{E}$$

In the preceding equation,  $\mathbf{X}$  is the matrix of factor scores, and  $\mathbf{B}'$  is the factor pattern.

There are two critical assumptions:

- The unique factors are uncorrelated with each other.
- The unique factors are uncorrelated with the common factors.

In principal component analysis, the residuals are generally correlated with each other. In common factor analysis, the unique factors play the role of residuals and are defined to be uncorrelated both

with each other and with the common factors. Each common factor is assumed to contribute to at least two variables; otherwise, it would be a unique factor.

When the factors are initially extracted, it is also assumed, for convenience, that the common factors are uncorrelated with each other and have unit variance. In this case, the common factor model implies that the covariance  $s_{jk}$  between the  $j$ th and  $k$ th variables,  $j \neq k$ , is given by

$$s_{jk} = b_{1j}b_{1k} + b_{2j}b_{2k} + \cdots + b_{qj}b_{qk}$$

or

$$\mathbf{S} = \mathbf{B}'\mathbf{B} + \mathbf{U}^2$$

where  $\mathbf{S}$  is the covariance matrix of the observed variables, and  $\mathbf{U}^2$  is the diagonal covariance matrix of the unique factors.

If the original variables are standardized to unit variance, the preceding formula yields correlations instead of covariances. It is in this sense that common factors explain the correlations among the observed variables. When considering the diagonal elements of standardized  $\mathbf{S}$ , the variance of the  $j$ th variable is expressed as

$$s_{jj} = 1 = b_{1j}^2 + b_{2j}^2 + \cdots + b_{qj}^2 + [\mathbf{U}^2]_{jj}$$

where  $b_{1j}^2 + b_{2j}^2 + \cdots + b_{qj}^2$  and  $[\mathbf{U}^2]_{jj}$  are the communality and uniqueness, respectively, of the  $j$ th variable. The communality represents the extent of the overlap with the common factors. In other words, it is the proportion of variance accounted for by the common factors.

The difference between the correlation predicted by the common factor model and the actual correlation is the *residual correlation*. A good way to assess the goodness of fit of the common factor model is to examine the residual correlations.

The common factor model implies that the partial correlations among the variables, removing the effects of the common factors, must all be zero. When the common factors are removed, only unique factors, which are by definition uncorrelated, remain.

The assumptions of common factor analysis imply that the common factors are, in general, not linear combinations of the observed variables. In fact, even if the data contain measurements on the entire population of observations, you cannot compute the scores of the observations on the common factors. Although the common factor scores cannot be computed directly, they can be estimated in a variety of ways.

The problem of factor score indeterminacy has led several factor analysts to propose methods yielding components that can be considered approximations to common factors. Since these components are defined as linear combinations, they are computable. The methods include Harris component analysis and image component analysis. The advantage of producing determinate component scores is offset by the fact that, even if the data fit the common factor model perfectly, component methods do not generally recover the correct factor solution. You should not use any type of component analysis if you really want a common factor analysis (Dziuban and Harris 1973; Lee and Comrey 1979).

After the factors are estimated, it is necessary to interpret them. Interpretation usually means assigning to each common factor a name that reflects the *salience* of the factor in predicting each of the observed variables—that is, the coefficients in the pattern matrix corresponding to the factor. Factor interpretation is a subjective process. It can sometimes be made less subjective by *rotating* the common factors—that is, by applying a nonsingular linear transformation. A rotated pattern matrix in which all the coefficients are close to 0 or  $\pm 1$  is easier to interpret than a pattern with many intermediate elements. Therefore, most rotation methods attempt to optimize a simplicity function of the rotated pattern matrix that measures, in some sense, how close the elements are to 0 or  $\pm 1$ . Because the loading estimates are subject to sampling variability, it is useful to obtain the standard error estimates for the loadings for assessing the uncertainty due to random sampling. Notice that the *salience* of a factor loading refers to the magnitude of the loading, while statistical *significance* refers to the statistical evidence against a particular hypothetical value. A loading significantly different from 0 does not automatically mean it must be salient. For example, if salience is defined as a magnitude larger than 0.4 while the entire 95% confidence interval for a loading lies between 0.1 and 0.3, the loading is statistically significant larger than 0 but it is not salient. Under the maximum likelihood method, you can obtain standard errors and confidence intervals for judging the salience of factor loadings.

After the initial factor extraction, the common factors are uncorrelated with each other. If the factors are rotated by an *orthogonal transformation*, the rotated factors are also uncorrelated. If the factors are rotated by an *oblique transformation*, the rotated factors become correlated. Oblique rotations often produce more useful patterns than do orthogonal rotations. However, a consequence of correlated factors is that there is no single unambiguous measure of the importance of a factor in explaining a variable. Thus, for oblique rotations, the pattern matrix does not provide all the necessary information for interpreting the factors; you must also examine the *factor structure* and the *reference structure*.

Rotating a set of factors does not change the statistical explanatory power of the factors. You cannot say that any rotation is better than any other rotation from a statistical point of view; all rotations, orthogonal or oblique, are equally good statistically. Therefore, the choice among different rotations must be based on nonstatistical grounds. For most applications, the preferred rotation is that which is most easily interpretable, or most compatible with substantive theories.

If two rotations give rise to different interpretations, those two interpretations must not be regarded as conflicting. Rather, they are two different ways of looking at the same thing, two different points of view in the common-factor space. Any conclusion that depends on one and only one rotation being correct is invalid.

---

## Outline of Use

### Principal Component Analysis

One important type of analysis performed by the FACTOR procedure is principal component analysis. The following statements result in a principal component analysis:

```
proc factor;
run;
```

The output includes all the eigenvalues and the pattern matrix for eigenvalues greater than one.

Most applications require additional output. For example, you might want to compute principal component scores for use in subsequent analyses or obtain a graphical aid to help decide how many components to keep. You can save the results of the analysis in a permanent SAS data library by using the **OUTSTAT=** option. (Refer to the *SAS Language Reference: Dictionary* for more information about permanent SAS data libraries and librefs.) Assuming that your SAS data library has the libref `save` and that the data are in a SAS data set called `raw`, you could do a principal component analysis as follows:

```
proc factor data=raw method=principal scree mineigen=0 score
  outstat=save.fact_all;
run;
```

The **SCREE** option produces a plot of the eigenvalues that is helpful in deciding how many components to use. Alternative, you can use the **PLOTS=SCREE** option to produce high-quality scree plots. The **MINEIGEN=0** option causes all components with variance greater than zero to be retained. The **SCORE** option requests that scoring coefficients be computed. The **OUTSTAT=** option saves the results in a specially structured SAS data set. The name of the data set, in this case `fact_all`, is arbitrary. To compute principal component scores, use the **SCORE** procedure:

```
proc score data=raw score=save.fact_all out=save.scores;
run;
```

The **SCORE** procedure uses the data and the scoring coefficients that are saved in `save.fact_all` to compute principal component scores. The component scores are placed in variables named `Factor1`, `Factor2`, ..., `Factorm` and are saved in the data set `save.scores`. If you know ahead of time how many principal components you want to use, you can obtain the scores directly from **PROC FACTOR** by specifying the **NFACTORS=** and **OUT=** options. To get scores from three principal components, specify the following:

```
proc factor data=raw method=principal
  nfactors=3 out=save.scores;
run;
```

To plot the scores for the first three components, use the **PLOT** procedure:

```
proc plot;
  plot factor2*factor1 factor3*factor1 factor3*factor2;
run;
```

## Principal Factor Analysis

The simplest and computationally most efficient method of common factor analysis is principal factor analysis, which is obtained in the same way as principal component analysis except for the use of the **PRIORS=** option. The usual form of the initial analysis is as follows:

```
proc factor data=raw method=principal scree
  mineigen=0 priors=smc outstat=save.fact_all;
run;
```

The squared multiple correlations (SMC) of each variable with all the other variables are used as the prior communality estimates. If your correlation matrix is singular, you should specify **PRIORS=MAX** instead of **PRIORS=SMC**. The **SCREE** and **MINEIGEN=** options serve the same



purpose as in the preceding principal component analysis. Saving the results with the **OUTSTAT=** option enables you to examine the eigenvalues and scree plot before deciding how many factors to rotate and to try several different rotations without reextracting the factors. The **OUTSTAT=** data set is automatically marked **TYPE=FACTOR**, so the **FACTOR** procedure realizes that it contains statistics from a previous analysis instead of raw data.

After looking at the eigenvalues to estimate the number of factors, you can try some rotations. Two and three factors can be rotated with the following statements:

```
proc factor data=save.fact_all method=principal n=2
    rotate=promax reorder score outstat=save.fact_2;
proc factor data=save.fact_all method=principal n=3
    rotate=promax reorder score outstat=save.fact_3;
run;
```

The output data set from the previous run is used as input for these analyses. The options **N=2** and **N=3** specify the number of factors to be rotated. The specification **ROTATE=PROMAX** requests a promax rotation, which has the advantage of providing both orthogonal and oblique rotations with only one invocation of **PROC FACTOR**. The **REORDER** option causes the variables to be reordered in the output so that variables associated with the same factor appear next to each other.

You can now compute and plot factor scores for the two-factor promax-rotated solution as follows:

```
proc score data=raw score=save.fact_2 out=save.scores;
proc plot;
    plot factor2*factor1;
run;
```

## Maximum Likelihood Factor Analysis

Although principal factor analysis is perhaps the most commonly used method of common factor analysis, most statisticians prefer maximum likelihood (ML) factor analysis (Lawley and Maxwell 1971). The ML method of estimation has desirable asymptotic properties (Bickel and Doksum 1977) and produces better estimates than principal factor analysis in large samples. You can test hypotheses about the number of common factors by using the ML method. You can also obtain standard error and confidence interval estimates for many classes of rotated or unrotated factor loadings, factor correlations, and structure loadings under the ML theory.

The unrotated ML solution is equivalent to Rao's (1955) canonical factor solution and Howe's solution maximizing the determinant of the partial correlation matrix (Morrison 1976). Thus, as a descriptive method, ML factor analysis does not require a multivariate normal distribution. The validity of Bartlett's  $\chi^2$  test for the number of factors does require approximate normality plus additional regularity conditions that are usually satisfied in practice (Geweke and Singleton 1980).

Lawley and Maxwell (1971) derive the standard error formulas for unrotated loadings, while Archer and Jennrich (1973) and Jennrich (1973, 1974) derive the standard error formulas for several classes of rotated solutions. Extended results appear in Browne et al. (2004), Hayashi and Yung (1999), and Yung and Hayashi (2001). A combination of these methods is used in **PROC FACTOR** to compute standard errors in an efficient manner. Confidence intervals are computed by using the asymptotic normality of the estimates. To ensure that the confidence intervals fall within the admissible parameter range, transformation methods due to Browne (1982) are used. The validity of the standard

error estimates and confidence limits requires the assumptions of multivariate normality and a fixed number of factors.

The ML method is more computationally demanding than principal factor analysis for two reasons. First, the communalities are estimated iteratively, and each iteration takes about as much computer time as principal factor analysis. The number of iterations typically ranges from about five to twenty. Second, if you want to extract different numbers of factors, as is often the case, you must run the FACTOR procedure once for each number of factors. Therefore, an ML analysis can take 100 times as long as a principal factor analysis. This does not include the time for computing standard error estimates, which is even more computationally demanding. For analyses with fewer than 35 variables, the computing time for the ML method, including the computation of standard errors, usually ranges from a few seconds to well under a minute. This seems to be a reasonable performance.

You can use principal factor analysis to get a rough idea of the number of factors before doing an ML analysis. If you think that there are between one and three factors, you can use the following statements for the ML analysis:

```
proc factor data=raw method=ml n=1
    outstat=save.fact1;
run;
proc factor data=raw method=ml n=2 rotate=promax
    outstat=save.fact2;
run;
proc factor data=raw method=ml n=3 rotate=promax
    outstat=save.fact3;
run;
```

The output data sets can be used for trying different rotations, computing scoring coefficients, or restarting the procedure in case it does not converge within the allotted number of iterations.

If you can determine how many factors should be retained before an analysis, as in the following statements, you can get the standard errors and confidence limits to aid interpretations for the ML analysis:

```
proc factor data=raw method=ml n=3 rotate=quartimin se
    cover=.4;
run;
```

In this analysis, you specify the quartimin rotation in the **ROTATE=** option. The **SE** option requests the computation of standard error estimates. In the **COVER=** option, you require absolute values of 0.4 or greater in order for loadings to be salient. In the output of coverage display, loadings that are salient would have their entire confidence intervals spanning beyond the 0.4 mark (or the -0.4 mark in the opposite direction). Only those salient loadings should be used for interpreting the factors. See the section “[Confidence Intervals and the Salience of Factor Loadings](#)” on page 1571 for more details.

The ML method cannot be used with a singular correlation matrix, and it is especially prone to Heywood cases. See the section “[Heywood Cases and Other Anomalies about Communality Estimates](#)” on page 1576 for a discussion of Heywood cases. If you have problems with ML, the best alternative is to use the **METHOD=ULS** option for unweighted least squares factor analysis.

## Factor Rotation

After the initial factor extraction, the factors are uncorrelated with each other. If the factors are rotated by an *orthogonal transformation*, the rotated factors are also uncorrelated. If the factors are rotated by an *oblique transformation*, the rotated factors become correlated. Oblique rotations often produce more useful patterns than orthogonal rotations do. However, a consequence of correlated factors is that there is no single unambiguous measure of the importance of a factor in explaining a variable. Thus, for oblique rotations, the pattern matrix does not provide all the necessary information for interpreting the factors; you must also examine the *factor structure* and the *reference structure*.

Nowadays, most rotations are done analytically. There are many choices for orthogonal and oblique rotations. An excellent summary of a wide class of analytic rotations is in Crawford and Ferguson (1970). The Crawford-Ferguson family of orthogonal rotations includes the orthomax rotation as a subclass and the popular varimax rotation as a special case. To illustrate these relationships, the following four specifications for orthogonal rotations with different **ROTATE=** options will give the same results for a data set with nine observed variables:

```
/* Orthogonal Crawford-Ferguson Family with
   variable parsimony weight = nvar - 1 = 8, and
   factor parsimony weight = 1 */
proc factor data=raw n=3 rotate=orthcf(8,1);
run;

/* Orthomax without the GAMMA= option */
proc factor data=raw n=3 rotate=orthomax(1);
run;

/* Orthomax with the GAMMA= option */
proc factor data=raw n=3 rotate=orthomax gamma=1;
run;

/* Varimax */
proc factor data=raw n=3 rotate=varimax;
run;
```

You can also get the oblique versions of the varimax in two equivalent ways:

```
/* Oblique Crawford-Ferguson Family with
   variable parsimony weight = nvar - 1 = 8, and
   factor parsimony weight = 1; */
proc factor data=raw n=3 rotate=oblicf(8,1);
run;

/* Oblique Varimax */
proc factor data=raw n=3 rotate=obvarimax;
run;
```

Jennrich (1973) proposes a generalized Crawford-Ferguson family that includes the Crawford-Ferguson family and the (direct) oblimin family (refer to Harman 1976) as subclasses. The better-known quartimin rotation is a special case of the oblimin class, and hence a special case of the generalized Crawford-Ferguson family. For example, the following four specifications of oblique rotations are equivalent:

```

/* Oblique generalized Crawford-Ferguson Family
   with weights 0, 1, 0, -1 */
proc factor data=raw n=3 rotate=oblimin(0,1,0,-1);
run;

/* Oblimin family without the TAU= option */
proc factor data=raw n=3 rotate=oblimin(0);
run;

/* Oblimin family with the TAU= option */
proc factor data=raw n=3 rotate=oblimin tau=0;
run;

/* Quartimin */
proc factor data=raw n=3 rotate=quartimin;
run;

```

In addition to the generalized Crawford-Ferguson family, the available oblique rotation methods in PROC FACTOR include Harris-Kaiser, promax, and Procrustes. See the section “[Simplicity Functions for Rotations](#)” on page 1572 for details about the definitions of various rotations. Refer to Harman (1976) and Mulaik (1972) for further information.

---

## Getting Started: FACTOR Procedure

The following example demonstrates how you can use the FACTOR procedure to perform common factor analysis and factor rotation.

In this example, 103 police officers were rated by their supervisors on 14 scales (variables). You conduct a common factor analysis on these variables to see what latent factors are operating behind these ratings. The overall rating variable is excluded from the factor analysis.

The following DATA step creates the SAS data set jobratings:

```
options validvarname=any;
data jobratings;
  input ('Communication Skills'n
        'Problem Solving'n
        'Learning Ability'n
        'Judgment Under Pressure'n
        'Observational Skills'n
        'Willingness to Confront Problems'n
        'Interest in People'n
        'Interpersonal Sensitivity'n
        'Desire for Self-Improvement'n
        'Appearance'n
        'Dependability'n
        'Physical Ability'n
        'Integrity'n
        'Overall Rating'n) (1.);
  datalines;
26838853879867
74758876857667
56757863775875
67869777988997
99997798878888
89897899888799
8999889899798

... more lines ...

99899899899899
76656399567486
;
```

The following statements invoke the FACTOR procedure:

```
proc factor data=jobratings(drop='Overall Rating'n) priors=smc
  rotate=varimax;
run;
```

The **DATA=** option in PROC FACTOR specifies the SAS data set jobratings as the input data set. The **DROP=** option drops the Overall Rating variable from the analysis. To conduct a common factor analysis, you need to set the prior communality estimate to less than one for each variable. Otherwise, the factor solution would simply be a recast of the principal components solution, in which “factors” are linear combinations of observed variables. However, in the common factor model you always assume that observed variables are functions of underlying factors. In this example, the **PRIORS=** option specifies that the squared multiple correlations (SMC) of each variable with all the other variables are used as the prior communality estimates. Note that squared multiple correlations are usually less than one. By default, the principal factor extraction is used if the **METHOD=** option is not specified. To facilitate interpretations, the **ROTATE=** option specifies the VARIMAX orthogonal factor rotation to be used.

The output from the factor analysis is displayed in [Figure 33.1](#) through [Figure 33.5](#).

As displayed in Figure 33.1, the prior communality estimates are set to the squared multiple correlations. Figure 33.1 also displays the table of eigenvalues (the variances of the principal factors) of the reduced correlation matrix. Each row of the table pertains to a single eigenvalue. Following the column of eigenvalues are three measures of each eigenvalue's relative size and importance. The first of these displays the difference between the eigenvalue and its successor. The last two columns display the individual and cumulative proportions that the corresponding factor contributes to the total variation. The last line displayed in Figure 33.1 states that three factors are retained, as determined by the PROPORTION criterion.

**Figure 33.1** Table of Eigenvalues from PROC FACTOR

The FACTOR Procedure				
Initial Factor Method: Principal Factors				
Prior Communality Estimates: SMC				
Communication Skills	Problem Solving	Learning Ability	Judgment Under Pressure	Observational Skills
0.62981394	0.58657431	0.61009871	0.63766021	0.67187583
Willingness to Confront Problems	Interest in People	Interpersonal Sensitivity	Desire for Self-Improvement	
0.64779805	0.75641519	0.75584891	0.57460176	
Appearance	Dependability	Physical Ability	Integrity	
0.45505304	0.63449045	0.42245324	0.68195454	
Eigenvalues of the Reduced Correlation Matrix:				
Total = 8.06463816 Average = 0.62035678				
	Eigenvalue	Difference	Proportion	Cumulative
1	6.17760549	4.71531946	0.7660	0.7660
2	1.46228602	0.90183348	0.1813	0.9473
3	0.56045254	0.28093933	0.0695	1.0168
4	0.27951322	0.04766016	0.0347	1.0515
5	0.23185305	0.16113428	0.0287	1.0802
6	0.07071877	0.07489624	0.0088	1.0890
7	-.00417747	0.03387533	-0.0005	1.0885
8	-.03805279	0.04776534	-0.0047	1.0838
9	-.08581814	0.02438060	-0.0106	1.0731
10	-.11019874	0.01452741	-0.0137	1.0595
11	-.12472615	0.02356465	-0.0155	1.0440
12	-.14829080	0.05823605	-0.0184	1.0256
13	-.20652684		-0.0256	1.0000
3 factors will be retained by the PROPORTION criterion.				

Figure 33.2 displays the initial factor pattern matrix. The factor pattern matrix represents standardized regression coefficients for predicting the variables by using the extracted factors. Because the initial factors are uncorrelated, the pattern matrix is also equal to the correlations between variables and the common factors.

**Figure 33.2** Factor Pattern Matrix from PROC FACTOR

Factor Pattern			
	Factor1	Factor2	Factor3
Communication Skills	0.75441	0.07707	-0.25551
Problem Solving	0.68590	0.08026	-0.34788
Learning Ability	0.65904	0.34808	-0.25249
Judgment Under Pressure	0.73391	-0.21405	-0.23513
Observational Skills	0.69039	0.45292	0.10298
Willingness to Confront Problems	0.66458	0.47460	0.09210
Interest in People	0.70770	-0.53427	0.10979
Interpersonal Sensitivity	0.64668	-0.61284	-0.07582
Desire for Self-Improvement	0.73820	0.12506	0.09062
Appearance	0.57188	0.20052	0.16367
Dependability	0.79475	-0.04516	0.16400
Physical Ability	0.51285	0.10251	0.34860
Integrity	0.74906	-0.35091	0.18656

The pattern matrix suggests that Factor1 represents general ability. All loadings for Factor1 in the Factor Pattern are at least 0.5. Factor2 consists of high positive loadings on certain task-related skills (Willingness to Confront Problems, Observational Skills, and Learning Ability) and high negative loadings on some interpersonal skills (Interpersonal Sensitivity, Interest in People, and Integrity). This factor measures individuals' relative strength in these skills. Theoretically, individuals with high positive scores on this factor would exhibit better task-related skills than interpersonal skills. Individuals with high negative scores would exhibit better interpersonal skills than task-related skills. Individuals with scores near zero would have those skills balanced. Factor3 does not have a cluster of very high or very low factor loadings. Therefore, interpreting this factor is difficult.

Figure 33.3 displays the proportion of variance explained by each factor and the final communality estimates, including the total communality. The final communality estimates are the proportion of variance of the variables accounted for by the common factors. When the factors are orthogonal, the final communalities are calculated by taking the sum of squares of each row of the factor pattern matrix.

**Figure 33.3** Variance Explained and Final Communality Estimates

Variance Explained by Each Factor		
Factor1	Factor2	Factor3
6.1776055	1.4622860	0.5604525

Figure 33.3 continued

Final Communality Estimates: Total = 8.200344				
Communication Skills	Problem Solving	Learning Ability	Judgment Under Pressure	Observational Skills
0.64036292	0.59791844	0.61924167	0.63972863	0.69237485
Willingness to Confront Problems	Interest in People	Interpersonal Sensitivity	Desire for Self-Improvement	
0.67538695	0.79833968	0.79951357	0.56879171	
Appearance	Dependability	Physical Ability	Integrity	
0.39403630	0.66056907	0.39504805	0.71903222	

Figure 33.4 displays the results of the VARIMAX rotation of the three extracted factors and the corresponding orthogonal transformation matrix. The rotated factor pattern matrix is calculated by postmultiplying the original factor pattern matrix (Figure 33.4) by the transformation matrix.

Figure 33.4 Transformation Matrix and Rotated Factor Pattern

The FACTOR Procedure			
Rotation Method: Varimax			
Orthogonal Transformation Matrix			
	1	2	3
1	0.59125	0.59249	0.54715
2	-0.80080	0.51170	0.31125
3	0.09557	0.62219	-0.77701
Rotated Factor Pattern			
	Factor1	Factor2	Factor3
Communication Skills	0.35991	0.32744	0.63530
Problem Solving	0.30802	0.23102	0.67058
Learning Ability	0.08679	0.41149	0.66512
Judgment Under Pressure	0.58287	0.17901	0.51764
Observational Skills	0.05533	0.70488	0.43870
Willingness to Confront Problems	0.02168	0.69391	0.43978
Interest in People	0.85677	0.21422	0.13562
Interpersonal Sensitivity	0.86587	0.02239	0.22200
Desire for Self-Improvement	0.34498	0.55775	0.37242
Appearance	0.19319	0.54327	0.24814
Dependability	0.52174	0.54981	0.29337
Physical Ability	0.25445	0.57321	0.04165
Integrity	0.74172	0.38033	0.15567



The rotated factor pattern matrix is somewhat simpler to interpret. If a magnitude of at least 0.5 is required to indicate a salient variable-factor relationship, Factor1 now represents interpersonal skills (Interpersonal Sensitivity, Interest in People, Integrity, Judgment Under Pressure, and Dependability). Factor2 measures physical skills and job enthusiasm (Observational Skills, Willingness to Confront Problems, Physical Ability, Desire for Self-Improvement, Dependability, and Appearance). Factor3 measures cognitive skills (Communication Skills, Problem Solving, Learning Ability, and Judgment Under Pressure).

However, using 0.5 for determining a salient variable-factor relationship does not take sampling variability into account. If the underlying assumptions for the maximum likelihood estimation are approximately satisfied, you can output standard error estimates and the confidence intervals with **METHOD=ML**. You can then determine the salience of the variable-factor relationship by using the coverage displays. See the section “[Confidence Intervals and the Salience of Factor Loadings](#)” on page 1571 for more details.

Figure 33.5 displays the variance explained by each factor and the final communality estimates after the orthogonal rotation. Even though the variances explained by the rotated factors are different from that of the unrotated factor (compare with Figure 33.3), the cumulative variance explained by the common factors remains the same. Note also that the final communalities for variables, as well as the total communality, remain unchanged after rotation. Although rotating a factor solution will not increase or decrease the statistical quality of the factor model, it can simplify the interpretations of the factors and redistribute the variance explained by the factors.

**Figure 33.5** Variance Explained and Final Communality Estimates after Rotation

Variance Explained by Each Factor				
	Factor1	Factor2	Factor3	
	3.1024330	2.7684489	2.3294622	
Final Communality Estimates: Total = 8.200344				
Communication Skills	Problem Solving	Learning Ability	Judgment Under Pressure	Observational Skills
0.64036292	0.59791844	0.61924167	0.63972863	0.69237485
Willingness to Confront Problems	Interest in People	Interpersonal Sensitivity	Desire for Self-Improvement	
0.67538695	0.79833968	0.79951357	0.56879171	
Appearance	Dependability	Physical Ability	Integrity	
0.39403630	0.66056907	0.39504805	0.71903222	

## Syntax: FACTOR Procedure

You can specify the following statements with the FACTOR procedure:

```
PROC FACTOR < options > ;
  VAR variables ;
  PRIORS communalities ;
  PARTIAL variables ;
  FREQ variable ;
  WEIGHT variable ;
  BY variables ;
```

Usually only the VAR statement is needed in addition to the PROC FACTOR statement. The descriptions of the BY, FREQ, PARTIAL, PRIORS, VAR, and WEIGHT statements follow the description of the PROC FACTOR statement in alphabetical order.

## PROC FACTOR Statement

```
PROC FACTOR < options > ;
```

The options available with the PROC FACTOR statement are listed in the following table and then described in alphabetical order.

**Table 33.1** Options Available in the PROC FACTOR Statement

Option	Description
<b>Data Set Options</b>	
<b>DATA=</b>	specifies input SAS data set
<b>OUT=</b>	specifies output SAS data set
<b>OUTSTAT=</b>	specifies output data set containing statistical results
<b>TARGET=</b>	specifies input data set containing the target pattern for rotation
<b>Factor Extraction and Communalities</b>	
<b>HEYWOOD</b>	sets to 1 any communality greater than 1
<b>METHOD=</b>	specifies the estimation method
<b>PRIORS=</b>	specifies the method for computing prior communality estimates
<b>RANDOM=</b>	specifies the seed for pseudo-random number generation
<b>ULTRAHEYWOOD</b>	allows communalities to exceed 1
<b>Number of Factors</b>	
<b>MINEIGEN=</b>	specifies the smallest eigenvalue for retaining a factor
<b>NAFACTORS=</b>	specifies the number of factors to retain

Table 33.1 *continued*

Option	Description
PROPORTION=	specifies the proportion of common variance in extracted factors
<b>Data Analysis Options</b>	
ALPHA=	specifies the confidence level for interval construction
COVARIANCE	requests factoring of the covariance matrix
COVER=	computes the confidence interval and specifies the coverage reference point
NOINT	omits the intercept from computing covariances or correlations
SE	requests the standard error estimates in ML estimation
VARDEF=	specifies the divisor used in calculating covariances or correlations
WEIGHT	factors a weighted correlation or covariance matrix
<b>Rotation Method and Properties</b>	
GAMMA=	specifies the orthomax weight
HKPOWER=	specifies the power in Harris-Kaiser rotation
NORM=	specifies the method for row normalization in rotation
NOPROMAXNORM	turns off row normalization in promax rotation
POWER=	specifies the power to be used in promax rotation
PREROTATE=	specifies the prerotation method in promax rotation
RCONVERGE=	specifies the convergence criterion for rotation cycles
RITER=	specifies the maximum number of cycles for rotation
ROTATE=	specifies the rotation method
TAU=	specifies the oblimin weight
<b>ODS Graphics</b>	
PLOTS=	specifies ODS Graphics selection
<b>Control Display Output</b>	
ALL	displays all optional output except plots
CORR	displays the (partial) correlation matrix
EIGENVECTORS	displays the eigenvectors of the reduced correlation matrix
FLAG=	specifies the minimum absolute value to be flagged in the correlation and loading matrices
FUZZ=	specifies the maximum absolute value to be displayed as missing in the correlation and loading matrices
MSA	computes Kaiser's measure of sampling adequacy and the related partial correlations
NOPRINT	suppresses the display of all output
NPLOT=	specifies the number of factors to be plotted
PLOT	plots the rotated factor pattern
PLOTREF	plots the reference structure
PREPLOT	plots the factor pattern before rotation

**Table 33.1** *continued*

Option	Description
PRINT	displays the input factor pattern or scoring coefficients and related statistics
REORDER	reorders the rows (variables) of various factor matrices
RESIDUALS	displays the residual correlation matrix and the associated partial correlation matrix
ROUND	prints correlation and loading matrices with rounded values
SCORE	displays the factor scoring coefficients
SCREE	displays the scree plot of the eigenvalues
SIMPLE	displays means, standard deviations, and number of observations
<b>Numerical Properties</b>	
CONVERGE=	specifies the convergence criterion
MAXITER=	specifies the maximum number of iterations
SINGULAR=	specifies the singularity criterion
<b>Miscellaneous</b>	
NOCORR	excludes the correlation matrix from the OUTSTAT= data set
NOBS=	specifies the number of observations
PARPREFIX=	specifies the prefix for the residual variables in the output data sets
PREFIX=	specifies the prefix for naming factors

**ALL**

displays all optional output except plots. When the input data set is TYPE=CORR, TYPE=UCORR, TYPE=COV, TYPE=UCOV, or TYPE=FACTOR, simple statistics, correlations, and MSA are not displayed.

**ALPHA= $p$** 

specifies the level of confidence  $1-p$  for interval construction. By default,  $p = 0.05$ , corresponding to  $1-p = 95\%$  confidence intervals. If  $p$  is greater than one, it is interpreted as a percentage and divided by 100. With multiple confidence intervals to be constructed, the ALPHA= value is applied to each interval construction one at a time. This will not control the coverage probability of the intervals simultaneously. To control familywise coverage probability, you might consider supplying a nonconventional  $p$  by using methods such as Bonferroni adjustment.

**CONVERGE= $p$** **CONV= $p$** 

specifies the convergence criterion for the METHOD=PRINIT, METHOD=ULS, METHOD=ALPHA, or METHOD=ML option. Iteration stops when the maximum change in the communalities is less than the value of the CONVERGE= option. The default value is 0.001. Negative values are not allowed.

**CORR****C**

displays the correlation matrix or partial correlation matrix.

**COVARIANCE****COV**

requests factoring of the covariance matrix instead of the correlation matrix. The COV option is effective only with the [METHOD=PRINCIPAL](#), [METHOD=PRINIT](#), [METHOD=ULS](#), or [METHOD=IMAGE](#) option. For other methods, PROC FACTOR produces the same results with or without the COV option.

**COVER <=p>****CI <=p>**

computes the confidence intervals and optionally specifies the value of factor loading for coverage detection. By default,  $p = 0$ . The specified value is represented by an asterisk (\*) in the coverage display. This is useful for determining the salience of loadings. For example, if COVER=0.4, a display '0\*[ ]' indicates that the entire confidence interval is above 0.4, implying strong evidence for the salience of the loading. See the section "[Confidence Intervals and the Salience of Factor Loadings](#)" on page 1571 for more details.

**DATA=SAS-data-set**

specifies the input data set, which can be an ordinary SAS data set or a specially structured SAS data set as described in the section "[Input Data Set](#)" on page 1566. If the DATA= option is omitted, the most recently created SAS data set is used.

**EIGENVECTORS****EV**

displays the eigenvectors of the reduced correlation matrix, of which the diagonal elements are replaced with the communality estimates. When [METHOD=ML](#), the eigenvectors are for the weighted reduced correlation matrix. PROC FACTOR chooses the solution that makes the sum of the elements of each eigenvector nonnegative. If the sum of the elements is equal to zero, then the sign depends on how the number is rounded off.

**FLAG=p**

flags absolute values larger than  $p$  with an asterisk in the correlation and loading matrices. Negative values are not allowed for  $p$ . Values printed in the matrices are multiplied by 100 and rounded to the nearest integer (see the [ROUND](#) option). The FLAG= option has no effect when standard errors or confidence intervals are also printed.

**FUZZ=p**

prints correlations and factor loadings with absolute values less than  $p$  printed as missing. For partial correlations, the FUZZ= value is divided by 2. For residual correlations, the FUZZ= value is divided by 4. The exact values in any matrix can be obtained from the [OUTSTAT=](#) and ODS output data sets. Negative values are not allowed. The FUZZ= option has no effect when standard errors or confidence intervals are also printed.

**GAMMA=*p***

specifies the orthomax weight used with the option [ROTATE=ORTHOMAX](#) or [PREROTATE=ORTHOMAX](#). Alternatively, you can use [ROTATE=ORTHOMAX\(\*p\*\)](#) with *p* representing the orthomax weight. There is no restriction on valid values for the orthomax weight, although the most common values are between zero and the number of variables. The default GAMMA= value is one, resulting in the varimax rotation. See the section “[Simplicity Functions for Rotations](#)” on page 1572 for more details.

**HEYWOOD****HEY**

sets to 1 any communality greater than 1, allowing iterations to proceed. See the section “[Heywood Cases and Other Anomalies about Communality Estimates](#)” on page 1576 for a discussion of Heywood cases.

**HKPOWER=*p*****HKP=*p***

specifies the power of the square roots of the eigenvalues used to rescale the eigenvectors for Harris-Kaiser ([ROTATE=HK](#)) rotation, assuming that the factors are extracted by the principal factor method. If the principal factor method is not used for factor extraction, the eigenvectors are replaced by the normalized columns of the unrotated factor matrix, and the eigenvalues are replaced by the column normalizing constants. HKPOWER= values between 0.0 and 1.0 are reasonable. The default value is 0.0, yielding the independent cluster solution, in which each variable tends to have a large loading on only one factor. An HKPOWER= value of 1.0 is equivalent to an orthogonal rotation, with the varimax rotation as the default. You can also specify the HKPOWER= option with [ROTATE=QUARTIMAX](#), [ROTATE=BIQUARTIMAX](#), [ROTATE=EQUAMAX](#), or [ROTATE=ORTHOMAX](#), and so on. The only restriction is that the Harris-Kaiser rotation must be associated with an orthogonal rotation.

**MAXITER=*n***

specifies the maximum number of iterations for factor extraction. You can use the MAXITER= option with the PRINIT, ULS, ALPHA, or ML method. The default is 30.

**METHOD=*name*****M=*name***

specifies the method for extracting factors. The default is METHOD=PRINCIPAL unless the [DATA=](#) data set is TYPE=FACTOR, in which case the default is METHOD=PATTERN. Valid values for *name* are as follows:

ALPHA   A	produces alpha factor analysis.
HARRIS   H	yields Harris component analysis of $\mathbf{S}^{-1}\mathbf{RS}^{-1}$ (Harris 1962), a noniterative approximation to canonical component analysis.
IMAGE   I	yields principal component analysis of the image covariance matrix, not Kaiser’s (1963, 1970) or Kaiser and Rice’s (1974) image analysis. A nonsingular correlation matrix is required.
ML   M	performs maximum likelihood factor analysis with an algorithm due, except for minor details, to Fuller (1987). The option METHOD=ML requires a nonsingular correlation matrix.

PATTERN	reads a factor pattern from a TYPE=FACTOR, TYPE=CORR, TYPE=UCORR, TYPE=COV, or TYPE=UCOV data set. If you create a TYPE=FACTOR data set in a DATA step, only observations containing the factor pattern (_TYPE_='PATTERN') and, if the factors are correlated, the interfactor correlations (_TYPE_='FCORR') are required.
PRINCIPAL   PRIN   P	yields principal component analysis if no PRIORS option or statement is used or if you specify PRIORS=ONE; if you specify a PRIORS statement or a PRIORS= value other than PRIORS=ONE, a principal factor analysis is performed.
PRINIT	yields iterated principal factor analysis.
SCORE	reads scoring coefficients (_TYPE_='SCORE') from a TYPE=FACTOR, TYPE=CORR, TYPE=UCORR, TYPE=COV, or TYPE=UCOV data set. The data set must also contain either a correlation or a covariance matrix. Scoring coefficients are also displayed if you specify the OUT= option.
ULS   U	produces unweighted least squares factor analysis.

**MINEIGEN=*p*****MIN=*p***

specifies the smallest eigenvalue for which a factor is retained. If you specify two or more of the MINEIGEN=, NFACTORS=, and PROPORTION= options, the number of factors retained is the minimum number satisfying any of the criteria. The MINEIGEN= option cannot be used with either the METHOD=PATTERN or the METHOD=SCORE option. Negative values are not allowed. The default is 0 unless you omit both the NFACTORS= and the PROPORTION= options and one of the following conditions holds:

- If you specify the METHOD=ALPHA or METHOD=HARRIS option, then MINEIGEN=1.
- If you specify the METHOD=IMAGE option, then

$$\text{MINEIGEN} = \frac{\text{total image variance}}{\text{number of variables}}$$

- For any other METHOD= specification, if prior communality estimates of 1.0 are used, then

$$\text{MINEIGEN} = \frac{\text{total weighted variance}}{\text{number of variables}}$$

When an unweighted correlation matrix is factored, this value is 1.

**MSA**

produces the partial correlations between each pair of variables controlling for all other variables (the negative anti-image correlations) and Kaiser's measure of sampling adequacy (Kaiser 1970; Kaiser and Rice 1974; Cerny and Kaiser 1977).

**NFACTORS=*n*****NFACT=*n*****N=*n***

specifies the maximum number of factors to be extracted and determines the amount of memory to be allocated for factor matrices. The default is the number of variables. Specifying a number that is small relative to the number of variables can substantially decrease the amount of memory required to run PROC FACTOR, especially with oblique rotations. If you specify two or more of the NFACTORS=, [MINEIGEN=](#), and [PROPORTION=](#) options, the number of factors retained is the minimum number satisfying any of the criteria. If you specify the option NFACTORS=0, eigenvalues are computed, but no factors are extracted. If you specify the option NFACTORS=-1, neither eigenvalues nor factors are computed. You can use the [NFACTORS=](#) option with the [METHOD=PATTERN](#) or [METHOD=SCORE](#) option to specify a smaller number of factors than are present in the data set.

**NOBS=*n***

specifies the number of observations. If the [DATA=](#) input data set is a raw data set, *nobs* is defined by default to be the number of observations in the raw data set. The NOBS= option overrides this default definition. If the [DATA=](#) input data set contains a covariance, correlation, or scalar product matrix, the number of observations can be specified either by using the NOBS= option in the PROC FACTOR statement or by including a `_TYPE_='N'` observation in the [DATA=](#) input data set.

**NOCORR**

prevents the correlation matrix from being transferred to the [OUTSTAT=](#) data set when you specify the [METHOD=PATTERN](#) option. The NOCORR option greatly reduces memory requirements when there are many variables but few factors. The NOCORR option is not effective if the correlation matrix is required for other requested output; for example, if the scores or the residual correlations are displayed (for example, by using the SCORE, [RESIDUALS](#), or [ALL](#) option).

**NOINT**

omits the intercept from the analysis; covariances or correlations are not corrected for the mean.

**NOPRINT**

suppresses the display of all output. Note that this option temporarily disables the Output Delivery System (ODS). For more information, see Chapter 20, “[Using the Output Delivery System](#).”

**NOPROMAXNORM | NOPMAXNORM**

turns off the default row normalization of the prerotated factor pattern, which is used in computing the promax target matrix.

**NORM=COV | KAISER | NONE | RAW | WEIGHT**

specifies the method for normalizing the rows of the factor pattern for rotation. If you specify the option NORM=KAISER, Kaiser’s normalization is used ( $\sum_j p_{ij}^2 = 1$ ). If you specify the option NORM=WEIGHT, the rows are weighted by the Cureton-Mulaik technique (Cureton and Mulaik 1975). If you specify the option NORM=COV, the rows of the pattern matrix are rescaled to represent covariances instead of correlations. If you specify



the option `NORM=NONE` or `NORM=RAW`, normalization is not performed. The default is `NORM=KAISER`.

#### **NPLOTS | NPLOT=*n***

specifies the number of factors to be plotted. The default is to plot all factors. The smallest allowable value is 2. If you specify the option `NPLOTS=n`, all pairs of the first *n* factors are plotted, producing a total of  $n(n - 1)/2$  plots.

#### **OUT=SAS-data-set**

creates a data set containing all the data from the `DATA=` data set plus variables called Factor1, Factor2, and so on, containing estimated factor scores. The `DATA=` data set must contain multivariate data, not correlations or covariances. You must also specify the `NFACTORS=` option to determine the number of factor score variables. If you specify partial variables in the `PARTIAL` statement, the `OUT=` data set will also contain the residual variables that are used for factor analysis. The output data set is described in detail in the section “[Output Data Sets](#)” on page 1568. If you want to create a permanent SAS data set, you must specify a two-level name. Refer to “SAS Files” in *SAS Language Reference: Concepts* for more information about permanent data sets.

#### **OUTSTAT=SAS-data-set**

specifies an output data set containing most of the results of the analysis. The output data set is described in detail in the section “[Output Data Sets](#)” on page 1568. If you want to create a permanent SAS data set, you must specify a two-level name. Refer to “SAS Files” in *SAS Language Reference: Concepts* for more information about permanent data sets.

#### **PARPREFIX=name**

specifies the prefix for the residual variables in the `OUT=` and the `OUTSTAT=` data sets when partial variables are specified in the `PARTIAL` statement.

#### **PLOT**

plots the factor pattern after rotation. This option produces printer plots. High-quality ODS graphical plots for factor patterns can be requested with the `PLOTS=LOADINGS` or `PLOTS=INITLOADINGS` option.

#### **PLOTREF**

plots the reference structure instead of the default factor pattern after oblique rotation.

#### **PLOTS <(global-plot-options)> = plot-request <(options)>**

#### **PLOTS <(global-plot-options)> = (plot-request <(options)> <...plot-request <(options)> > )**

specifies one or more ODS graphical plots in PROC FACTOR. When you specify only one *plot-request*, you can omit the parentheses around the *plot-request*. Here are some examples:

```
plots=all
plots(flip)=loadings
plots=(loadings(flip) scree(unpack))
```

You must enable ODS graphics before requesting plots. For example:

```
ods graphics on;
proc factor plots=all;
run;
ods graphics off;
```

For more information about the ODS GRAPHICS statement, see Chapter 21, “[Statistical Graphics Using ODS](#).” For an example containing graphical displays of factor analysis results, see [Example 33.2](#).

The following table shows the available *plot-requests* and their available suboptions:

<i>Plot-Request</i>	<i>Plot Description</i>	<i>Suboptions</i>
ALL	all available plots	all
INITLOADINGS	unrotated factor loadings	CIRCLE=, FLIP, NPLOTS=, PLOTREF, and VECTOR
LOADINGS	rotated factor loadings	CIRCLE=, FLIP, NPLOTS=, PLOTREF, and VECTOR
NONE	no ODS graphical plots	
PRELOADINGS	prerotated factor loadings	CIRCLE=, FLIP, NPLOTS=, PLOTREF, and VECTOR
SCREE	scree and variance explained	UNPACK

The following are the available *global-plot-options* or *options* for plots:

**CIRCLE | CIRCLES** *<= numbers >* specifies circles with prescribed areas to be drawn in scatter plots or vector plots, where the optional *numbers* represent proportions or percentages of areas enclosed by the circles in the plots. These *numbers* must lie between 0 and 100, inclusively. When a number is between 0 and 1 (inclusively), it is interpreted as a proportion; otherwise, it is interpreted as a percentage. A maximum of 5 numbers for the circles will be used. The CIRCLE option applies to the scatter or vector plots requested by the INITLOADINGS, LOADINGS, and PRELOADINGS options. By default, a unit-circle, which represents 100% of the total area, is drawn for the vector plots. However, no circle will be drawn for scatter plots unless the CIRCLE option is specified. Two special cases for this option are: (1) With no *numbers* following the CIRCLE option, a 100% circle will be drawn. (2) With CIRCLE=0, no circle will be drawn. This special case is primarily used to turn off the default unit-circle in vector plots.

**FLIP** switches the X and Y axes. It applies to the INITLOADINGS, LOADINGS, and PRELOADINGS *plot-requests*.

**NPLOT | NPLOTS**=*n* specifies the number of factors *n* ( $n \geq 2$ ) to be plotted. It applies to the INITLOADINGS, LOADINGS, and PRELOADINGS *plot-requests*. Since this option can also be specified in the PROC FACTOR statement, the final value of *n* is determined by the following steps. The NPLOTS= value of the PROC FACTOR is read first. If the NPLOTS= option is specified as a *global-plot-option*, the value of *n* will be updated. Then, if the NPLOTS= option is again specified in an individual *plot-request*, the value will be updated again for that individual *plot-request*. For example, in the following statement, four factors are extracted with the N=4 option:

```
proc factor n=4 npLOTS=3 plots(npLOTS=4)=
              (loadings preloadings(npLOTS=2));
```

Initially, plots of the first three factors are specified with the `NPLOTS=3` option. When you are producing ODS graphical plots, the *global-plot-option* `NPLOTS=4` is used. As a result, the `LOADINGS plot-request` will produce plots for all pairs of the first 4 factors. However, because the `NPLOTS=2` is specified locally for the `PRELOADINGS plot-request`, it will produce a prerotated factor loading plot for the first two factors only. The default `NPLOTS=` value is 5 or the total number of factors ( $m$ ), whichever is smaller. If you specify an `NPLOTS=` value that is greater than  $m$ , `NPLOTS=m` will be used.

PLOTREF	plots the reference structures rather than the factor pattern loadings. It applies to the <code>INITLOADINGS</code> , <code>LOADINGS</code> , and <code>PRELOADINGS plot-requests</code> when the factor solution is oblique. This option can also be set globally as an option in the PROC FACTOR statement.
UNPACK	plots component graphs separately. It applies to the <code>SCREE plot-request</code> only.
VECTOR	plots loadings in vector form. It applies to the <code>INITLOADINGS</code> , <code>LOADINGS</code> , and <code>PRELOADINGS plot-requests</code> when the factor solution is orthogonal. For oblique solutions, the <code>VECTOR</code> option is ignored and the default scatter plots for factor loadings or reference structures are displayed.

Be aware that the `PLOT` option in the PROC FACTOR statement requests only the printer plots of factor loadings. The current option `PLOTS=` or `PLOT=`, however, is for ODS graphical plots.

You can specify options for the requested ODS graphical plots as *global-plot-options* or as *local options*. *Global-plot-options* apply to all appropriate individual *plot-requests* specified. For example, because the `SCREE` plot is not subject to axes flipping, the following two specifications are equivalent:

```
plots(flip)=(loadings preloadings scree)
plots=(loadings(flip) preloadings(flip) scree)
```

*Options* specified locally after each *plot-request* apply to that plot-request only. For example, consider the following specification:

```
plots=(scree(unpack) loadings(plotref) preloadings(flip))
```

The `FLIP` option applies to the `PRELOADINGS plot-request` but not the `LOADINGS plot-request`; the `PLOTREF` option applies to the `LOADINGS plot-request` but not the `PRELOADINGS plot-request`; and the `UNPACK` option applies to the `SCREE plot-request` only.

#### POWER= $n$

specifies the power to be used in computing the target pattern for the option `ROTATE=PROMAX`. Valid values must be integers  $\geq 1$ . The default value is 3. You can also specify the `POWER=` value in the `ROTATE=` option—for example, `ROTATE=PROMAX(4)`.

**PREFIX=***name*

specifies a prefix for naming the factors. By default, the names are Factor1, Factor2, ..., Factor*n*. If you specify PREFIX=ABC, the factors are named ABC1, ABC2, ABC3, and so on. The number of characters in the prefix plus the number of digits required to designate the variables should not exceed the current name length defined by the VALIDVARNAME= system option.

**PREPLOT**

plots the factor pattern before rotation. This option produces printer plots. High-quality ODS graphical plots for factor patterns can be requested with the [PLOTS=PRELOADINGS](#) option.

**PREROTATE=***name***PRE=***name*

specifies the prerotation method for the option [ROTATE=PROMAX](#). Any rotation method other than PROMAX or PROCUSTES can be used. The default is PREROTATE=VARIMAX. If a previously rotated pattern is read using the option [METHOD=PATTERN](#), you should specify the PREROTATE=NONE option.

**PRINT**

displays the input factor pattern or scoring coefficients and related statistics. In oblique cases, the reference and factor structures are computed and displayed. The PRINT option is effective only with the option [METHOD=PATTERN](#) or [METHOD=SCORE](#).

**PRIORS=***name*

specifies a method for computing prior communality estimates. You can specify numeric values for the prior communality estimates by using the PRIORS statement. Valid values for *name* are as follows:

ASMC   A	sets the prior communality estimates proportional to the squared multiple correlations but adjusted so that their sum is equal to that of the maximum absolute correlations (Cureton 1968).
INPUT   I	reads the prior communality estimates from the first observation with either <code>_TYPE_='PRIORS'</code> or <code>_TYPE_='COMMUNAL'</code> in the <a href="#">DATA=</a> data set (which cannot be <code>TYPE=DATA</code> ).
MAX   M	sets the prior communality estimate for each variable to its maximum absolute correlation with any other variable.
ONE   O	sets all prior communalities to 1.0.
RANDOM   R	sets the prior communality estimates to pseudo-random numbers uniformly distributed between 0 and 1.
SMC   S	sets the prior communality estimate for each variable to its squared multiple correlation with all other variables.

The default prior communality estimates are as follows:

METHOD=	PRIORS=
PRINCIPAL	ONE
PRINIT	ONE
ALPHA	SMC
ULS	SMC
ML	SMC
HARRIS	(not applicable)
IMAGE	(not applicable)
PATTERN	(not applicable)
SCORE	(not applicable)

By default, the options **METHOD=PRINIT**, **METHOD=ULS**, **METHOD=ALPHA**, and **METHOD=ML** stop iterating and set the number of factors to 0 if an estimated communality exceeds 1. The options **HEYWOOD** and **ULTRAHEYWOOD** allow processing to continue.

**PROPORTION=*p***

**PERCENT=*p***

**P=*p***

specifies the proportion of common variance to be accounted for by the retained factors. The proportion of common variance is computed using the total prior communality estimates as the basis. If the value is greater than one, it is interpreted as a percentage and divided by 100. The options **PROPORTION=0.75** and **PERCENT=75** are equivalent. The default value is 1.0 or 100%. You cannot specify the **PROPORTION=** option with the **METHOD=PATTERN** or **METHOD=SCORE** option. If you specify two or more of the **PROPORTION=**, **NFACTORS=**, and **MINEIGEN=** options, the number of factors retained is the minimum number satisfying any of the criteria.

**RANDOM=*n***

specifies a positive integer as a starting value for the pseudo-random number generator for use with the option **PRIORS=RANDOM**. If you do not specify the **RANDOM=** option, the time of day is used to initialize the pseudo-random number sequence. Valid values must be integers  $\geq 1$ .

**RCONVERGE=*p***

**RCONV=*p***

specifies the convergence criterion for rotation cycles. Rotation stops when the scaled change of the simplicity function value is less than the **RCONVERGE=** value. The default convergence criterion is

$$|f_{new} - f_{old}|/K < \epsilon$$

where  $f_{new}$  and  $f_{old}$  are simplicity function values of the current cycle and the previous cycle, respectively,  $K = \max(1, |f_{old}|)$  is a scaling factor, and  $\epsilon$  is 1E-9 by default and is modified by the **RCONVERGE=** value.

**REORDER****RE**

causes the rows (variables) of various factor matrices to be reordered on the output. Variables with their highest absolute loading (reference structure loading for oblique rotations) on the first factor are displayed first, from largest to smallest loading, followed by variables with their highest absolute loading on the second factor, and so on. The order of the variables in the output data set is not affected. The factors are not reordered.

**RESIDUALS****RES**

displays the residual correlation matrix and the associated partial correlation matrix. The diagonal elements of the residual correlation matrix are the unique variances.

**RITER=*n***

specifies the maximum number of cycles *n* for factor rotation. Except for promax and Procrustes, you can use the RITER= option with all rotation methods. The default *n* is the maximum between 10 times the number of variables and 100.

**ROTATE=*name*****R=*name***

specifies the rotation method. The default is ROTATE=NONE.

Valid *names* for orthogonal rotations are as follows:

BIQUARTIMAX | BIQMAX specifies orthogonal biquartimax rotation. This corresponds to the specification ROTATE=ORTHOMAX(.5).

EQUAMAX | E specifies orthogonal equamax rotation. This corresponds to the specification ROTATE=ORTHOMAX with [GAMMA=number of factors/2](#).

FACTORPARSIMAX | FPA specifies orthogonal factor parsimax rotation. This corresponds to the specification ROTATE=ORTHOMAX with [GAMMA=number of variables](#).

NONE | N specifies that no rotation be performed, leaving the original orthogonal solution.

ORTHCF(*p1,p2*) | ORCF(*p1,p2*) specifies the orthogonal Crawford-Ferguson rotation with the weights *p1* and *p2* for variable parsimony and factor parsimony, respectively. See the definitions of weights in the section “[Simplicity Functions for Rotations](#)” on page 1572.

ORTHGENCF(*p1,p2,p3,p4*) | ORGENCF(*p1,p2,p3,p4*) specifies the orthogonal generalized Crawford-Ferguson rotation with the four weights *p1*, *p2*, *p3*, and *p4*. See the definitions of weights in the section “[Simplicity Functions for Rotations](#)” on page 1572.

ORTHOMAX<*p*> | ORMAX<*p*> specifies the orthomax rotation with orthomax weight *p*. If ROTATE=ORTHOMAX is used, the default *p* value is 1 unless specified otherwise in the [GAMMA=](#) option. Alternatively, ROTATE=ORTHOMAX(*p*) specifies *p* as the orthomax weight or the [GAMMA=](#) value. See the definition of the orthomax weight in the section “[Simplicity Functions for Rotations](#)” on page 1572.

PARSIMAX | PA specifies orthogonal parsimax rotation. This corresponds to the specification ROTATE=ORTHOMAX with

$$\text{GAMMA} = \frac{nvar \times (nfact - 1)}{nvar + nfact - 2}$$

where *nvar* is the number of variables and *nfact* is the number of factors.

QUARTIMAX | QMAX | Q specifies orthogonal quartimax rotation. This corresponds to the specification ROTATE=ORTHOMAX(0).

VARIMAX | V specifies orthogonal varimax rotation. This corresponds to the specification ROTATE=ORTHOMAX with [GAMMA=1](#).

Valid *names* for oblique rotations are as follows:

BIQUARTIMIN | BIQMIN specifies biquartimin rotation. It corresponds to the specification ROTATE=OBLIMIN(.5) or ROTATE=OBLIMIN with [TAU=0.5](#).

COVARIMIN | CVMIN specifies covarimin rotation. It corresponds to the specification ROTATE=OBLIMIN(1) or ROTATE=OBLIMIN with [TAU=1](#).

HK<(p)> | H<(p)> specifies Harris-Kaiser case II orthoblique rotation. When specifying this option, you can use the [HKPOWER=](#) option to set the power of the square roots of the eigenvalues by which the eigenvectors are scaled, assuming that the factors are extracted by the principal factor method. For other extraction methods, the unrotated factor pattern is column normalized. The power is then applied to the column normalizing constants, instead of the eigenvalues. You can also use ROTATE=HK(*p*), with *p* representing the [HKPOWER=](#) value. The default associated orthogonal rotation with ROTATE=HK is the varimax rotation without Kaiser normalization. You might associate the Harris-Kaiser with other orthogonal rotations by using the ROTATE= option together with the [HKPOWER=](#) option.

OBBIQUARTIMAX | OBIQMAX specifies oblique biquartimax rotation.

OBEQUAMAX | OE specifies oblique equamax rotation.

OBFACORPARSIMAX | OFPA specifies oblique factor parsimax rotation.

OBLICF(*p1,p2*) | OBCF(*p1,p2*) specifies the oblique Crawford-Ferguson rotation with the weights *p1* and *p2* for variable parsimony and factor parsimony, respectively. See the definitions of weights in the section “[Simplicity Functions for Rotations](#)” on page 1572.

OBLIGENCF(*p1,p2,p3,p4*) | OBGENCF(*p1,p2,p3,p4*) specifies the oblique generalized Crawford-Ferguson rotation with the four weights *p1*, *p2*, *p3*, and *p4*. See the definitions of weights in the section “[Simplicity Functions for Rotations](#)” on page 1572.

OBLIMIN<(p)> | OBMIN<(p)> specifies the oblimin rotation with oblimin weight *p*. If ROTATE=OBLIMIN is used, the default *p* value is zero unless specified otherwise in the [TAU=](#) option. Alternatively, ROTATE=OBLIMIN(*p*) specifies *p* as the oblimin weight or the [TAU=](#) value. See the definition of



the oblimin weight in the section “[Simplicity Functions for Rotations](#)” on page 1572.

**OBPARSIMAX** | **OPA** specifies oblique parsimax rotation.

**OBQUARTIMAX** | **OQMAX** specifies oblique quartimax rotation. This is the same as the **QUARTIMIN** method.

**OBVARIMAX** | **OV** specifies oblique varimax rotation.

**PROCRUSTES** specifies oblique Procrustes rotation with the target pattern provided by the **TARGET=** data set. The unrestricted least squares method is used with factors scaled to unit variance after rotation.

**PROMAX**<(p)> | **P**<(p)> specifies oblique promax rotation. You can use the **PREROTATE=** option to set the desirable prerotation method, orthogonal or oblique. When used with **ROTATE=PROMAX**, the **POWER=** option lets you specify the power for forming the target. You can also use **ROTATE=PROMAX(p)**, where *p* represents the **POWER=** value.

**QUARTIMIN** | **QMIN** specifies quartimin rotation. It is the same as the oblique quartimax method. It also corresponds to the specification **ROTATE=OBLIMIN(0)** or **ROTATE=OBLIMIN** with **TAU=0**.

## ROUND

prints correlation and loading matrices with entries multiplied by 100 and rounded to the nearest integer. The exact values can be obtained from the **OUTSTAT=** and ODS output data sets. The **ROUND** option also flags absolute values larger than the **FLAG=** value with an asterisk in correlation and loading matrices (see the **FLAG=** option). If the **FLAG=** option is not specified, the root mean square of all the values in the matrix printed is used as the default **FLAG=** value. The **ROUND** option has no effect when standard errors or confidence intervals are also printed.

## SCORE

displays the factor scoring coefficients. The squared multiple correlation of each factor with the variables is also displayed except in the case of unrotated principal components. The **SCORE** option also outputs the factor scoring coefficients in the **\_TYPE\_=SCORE** or **\_TYPE\_=USCORE** observations in the **OUTSTAT=** data set. Unless you specify the **NOINT** option in **PROC FACTOR**, the scoring coefficients should be applied to standardized variables—variables that are centered by subtracting the original variable means and then divided by the original variable standard deviations. With the **NOINT** option, the scoring coefficients should be applied to data without centering.

## SCREE

displays a scree plot of the eigenvalues (Cattell 1966, 1978; Cattell and Vogelman 1977; Horn and Engstrom 1979). This option produces printer plots. High-quality scree plots can be requested with the **PLOTS=SCREE** option.

## SE

## STDERR

computes standard errors for various classes of unrotated and rotated solutions under the maximum likelihood estimation.



**SIMPLE****S**

displays means, standard deviations, and the number of observations.

**SINGULAR= $p$** **SING= $p$** 

specifies the singularity criterion, where  $0 < p < 1$ . The default value is  $1E-8$ .

**TARGET=SAS-data-set**

specifies an input data set containing the target pattern for Procrustes rotation (see the description of the **ROTATE=** option). The **TARGET=** data set must contain variables with the same names as those being factored. Each observation in the **TARGET=** data set becomes one column of the target factor pattern. Missing values are treated as zeros. The **\_NAME\_** and **\_TYPE\_** variables are not required and are ignored if present.

**TAU= $p$** 

specifies the oblimin weight used with the option **ROTATE=OBLIMIN** or **PREROTATE=OBLIMIN**. Alternatively, you can use **ROTATE=OBLIMIN( $p$ )** with  $p$  representing the oblimin weight. There is no restriction on valid values for the oblimin weight, although for practical purposes a negative or zero value is recommended. The default **TAU=** value is 0, resulting in the quartimin rotation. See the section “[Simplicity Functions for Rotations](#)” on page 1572 for more details.

**ULTRAHEYWOOD****ULTRA**

allows communalities to exceed 1. The **ULTRAHEYWOOD** option can cause convergence problems because communalities can become extremely large, and ill-conditioned Hessians might occur. See the section “[Heywood Cases and Other Anomalies about Communality Estimates](#)” on page 1576 for a discussion of Heywood cases.

**VARDEF=DF | N | WDF | WEIGHT | WGT**

specifies the divisor used in the calculation of variances and covariances. The default value is **VARDEF=DF**. The values and associated divisors are displayed in the following table where  $i = 0$  if the **NOINT** option is used and  $i = 1$  otherwise, and where  $k$  is the number of partial variables specified in the **PARTIAL** statement.

Value	Description	Divisor
DF	degrees of freedom	$n - k - i$
N	number of observations	$n - k$
WDF	sum of weights DF	$\sum_i w_i - k - i$
WEIGHT   WGT	sum of weights	$\sum_i w_i - k$

**WEIGHT**

factors a weighted correlation or covariance matrix. The **WEIGHT** option can be used only with the **METHOD=PRINCIPAL**, **METHOD=PRINIT**, **METHOD=ULS**, or **METHOD=IMAGE** option. The input data set must be of type **CORR**, **UCORR**, **COV**, **UCOV**, or **FACTOR**, and the variable weights are obtained from an observation with **\_TYPE\_='WEIGHT'**.

---

## BY Statement

**BY variables ;**

You can specify a BY statement with PROC FACTOR to obtain separate analyses on observations in groups defined by the BY variables. When a BY statement appears, the procedure expects the input data set to be sorted in order of the BY variables.

If your input data set is not sorted in ascending order, use one of the following alternatives:

- Sort the data by using the SORT procedure with a similar BY statement.
- Specify the BY statement option NOTSORTED or DESCENDING in the BY statement for PROC FACTOR. The NOTSORTED option does not mean that the data are unsorted but rather that the data are arranged in groups (according to values of the BY variables) and that these groups are not necessarily in alphabetical or increasing numeric order.
- Create an index on the BY variables by using the DATASETS procedure.

If you specify the **TARGET=** option and the **TARGET=** data set does not contain any of the BY variables, then the entire **TARGET=** data set is used as a Procrustean target for each BY group in the **DATA=** data set.

If the **TARGET=** data set contains some but not all of the BY variables, or if some BY variables do not have the same type or length in the **TARGET=** data set as in the **DATA=** data set, then PROC FACTOR displays an error message and stops.

If all the BY variables appear in the **TARGET=** data set with the same type and length as in the **DATA=** data set, then each BY group in the **TARGET=** data set is used as a Procrustean target for the corresponding BY group in the **DATA=** data set. The BY groups in the **TARGET=** data set must be in the same order as in the **DATA=** data set. If you specify the NOTSORTED option in the BY statement, there must be identical BY groups in the same order in both data sets. If you do not specify the NOTSORTED option, some BY groups can appear in one data set but not in the other.

For more information about the BY statement, see *SAS Language Reference: Concepts*. For more information about the DATASETS procedure, see the *Base SAS Procedures Guide*.

---

## FREQ Statement

**FREQ variable ;**

If a variable in the data set represents the frequency of occurrence for the other values in the observation, include the variable's name in a FREQ statement. The procedure then treats the data set as if each observation appears  $n$  times, where  $n$  is the value of the FREQ variable for the observation. The total number of observations is considered to be equal to the sum of the FREQ variable when the procedure determines degrees of freedom for significance probabilities.

If the value of the FREQ variable is missing or is less than one, the observation is not used in the analysis. If the value is not an integer, the value is truncated to an integer.

The WEIGHT and FREQ statements have a similar effect, except in determining the number of observations for significance tests.

---

## PARTIAL Statement

**PARTIAL** *variables* ;

If you want the analysis to be based on a partial correlation or covariance matrix, use the PARTIAL statement to list the variables that are used to partial out the variables in the analysis.

---

## PRIORS Statement

**PRIORS** *communalities* ;

The PRIORS statement specifies numeric values between 0.0 and 1.0 for the prior communality estimates for each variable. The first numeric value corresponds to the first variable in the VAR statement, the second value to the second variable, and so on. The number of numeric values must equal the number of variables. For example:

```
proc factor;
  var      x  y  z;
  priors .7 .8 .9;
run;
```

You can specify various methods for computing prior communality estimates with the **PRIORS=** option in the PROC FACTOR statement. Refer to the description of that option for more information about the default prior communality estimates.

---

## VAR Statement

**VAR** *variables* ;

The VAR statement specifies the numeric variables to be analyzed. If the VAR statement is omitted, all numeric variables not specified in other statements are analyzed.

---

## WEIGHT Statement

**WEIGHT** *variable* ;

If you want to use relative weights for each observation in the input data set, specify a variable containing weights in a WEIGHT statement. This is often done when the variance associated with each observation is different and the values of the weight variable are proportional to the reciprocals of the variances. If a variable value is negative or is missing, it is excluded from the analysis.

---

## Details: FACTOR Procedure

---

### Input Data Set

The FACTOR procedure can read an ordinary SAS data set containing raw data or a special data set specified as a TYPE=CORR, TYPE=UCORR, TYPE=SSCP, TYPE=COV, TYPE=UCOV, or TYPE=FACTOR data set containing previously computed statistics. A TYPE=CORR data set can be created by the CORR procedure or various other procedures such as the PRINCOMP procedure. It contains means, standard deviations, the sample size, the correlation matrix, and possibly other statistics if it is created by some procedure other than PROC CORR. A TYPE=COV data set is similar to a TYPE=CORR data set but contains a covariance matrix. A TYPE=UCORR or TYPE=UCOV data set contains a correlation or covariance matrix that is not corrected for the mean. The default VAR variable list does not include Intercept if the DATA= data set is TYPE=SSCP. If the Intercept variable is explicitly specified in the VAR statement with a TYPE=SSCP data set, the NOINT option is activated. A TYPE=FACTOR data set can be created by the FACTOR procedure and is described in the section “Output Data Sets” on page 1568.

If your data set has many observations and you plan to run FACTOR several times, you can save computer time by first creating a TYPE=CORR data set and using it as input to PROC FACTOR, as in the following statements:

```
proc corr data=raw out=correl;      /* create TYPE=CORR data set */
proc factor data=correl method=ml; /* maximum likelihood      */
proc factor data=correl;           /* principal components  */
```

The data set created by the CORR procedure is automatically given the TYPE=CORR data set option, so you do not have to specify TYPE=CORR. However, if you use a DATA step with a SET statement to modify the correlation data set, you must use the TYPE=CORR attribute in the new data set. You can use a VAR statement with PROC FACTOR when reading a TYPE=CORR data set to select a subset of the variables or change the order of the variables.

Problems can arise from using the CORR procedure when there are missing data. By default, PROC CORR computes each correlation from all observations that have values present for the pair of variables involved (pairwise deletion). The resulting correlation matrix might have negative

eigenvalues. If you specify the NOMISS option with the CORR procedure, observations with any missing values are completely omitted from the calculations (listwise deletion), and there is no danger of negative eigenvalues.

PROC FACTOR can also create a TYPE=FACTOR data set, which includes all the information in a TYPE=CORR data set, and use it for repeated analyses. For a TYPE=FACTOR data set, the default value of the [METHOD=](#) option is PATTERN. The following PROC FACTOR statements produce the same results as the previous example:

```
proc factor data=raw method=ml outstat=fact; /* max. likelihood */
proc factor data=fact method=prin;          /* principal components */
```

You can use a TYPE=FACTOR data set to try several different rotation methods on the same data without repeatedly extracting the factors. In the following example, the second and third PROC FACTOR statements use the data set fact created by the first PROC FACTOR statement:

```
proc factor data=raw outstat=fact; /* principal components */
proc factor rotate=varimax;        /* varimax rotation      */
proc factor rotate=quartimax;      /* quartimax rotation  */
```

You can create a TYPE=CORR, TYPE=UCORR, or TYPE=FACTOR data set in a DATA step for PROC FACTOR to read as input. For example, in the following a TYPE=CORR data set is created and is read as input data set by the subsequent PROC FACTOR statement:

```
data correl(type=corr);
  _TYPE_='CORR';
  input _NAME_ $ x y z;
  datalines;
x  1.0  .  .
y   .7 1.0  .
z   .5 .4 1.0
;
proc factor;
run;
```

Be sure to specify the TYPE= option in parentheses after the data set name in the DATA statement and include the \_TYPE\_ and \_NAME\_ variables. In a TYPE=CORR data set, only the correlation matrix (\_TYPE\_='CORR') is necessary. It can contain missing values as long as every pair of variables has at least one nonmissing value.

You can also create a TYPE=FACTOR data set containing only a factor pattern (\_TYPE\_='PATTERN') and use the FACTOR procedure to rotate it, as these statements show:

```
data pat(type=factor);
  _TYPE_='PATTERN';
  input _NAME_ $ x y z;
  datalines;
factor1  .5  .7  .3
factor2  .8  .2  .8
;
proc factor rotate=promax prerotate=none;
run;
```

If the input factors are oblique, you must also include the interfactor correlation matrix with `_TYPE_='FCORR'`, as shown here:

```
data pat (type=factor);
  input _TYPE_ $ _NAME_ $ x y z;
  datalines;
pattern factor1 .5 .7 .3
pattern factor2 .8 .2 .8
fcorr factor1 1.0 .2 .
fcorr factor2 .2 1.0 .
;
proc factor rotate=promax prerotate=none;
run;
```

Some procedures, such as the PRINCOMP and CANDISC procedures, produce `TYPE=CORR` or `TYPE=UCORR` data sets containing scoring coefficients (`_TYPE_='SCORE'` or `_TYPE_='USCORE'`). These coefficients can be input to PROC FACTOR and rotated by using the `METHOD=SCORE` option, as in the following statements:

```
proc princomp data=raw n=2 outstat=prin;
run;
proc factor data=prin method=score rotate=varimax;
run;
```

Notice that the input data set prin must contain the correlation matrix as well as the scoring coefficients.

---

## Output Data Sets

### The OUT= Data Set

The `OUT=` data set contains all the data in the `DATA=` data set plus new variables called Factor1, Factor2, and so on, containing estimated factor scores. Each estimated factor score is computed as a linear combination of the standardized values of the variables that are factored. The coefficients are always displayed if the `OUT=` option is specified, and they are labeled “Standardized Scoring Coefficients.”

If partial variables are specified in the PARTIAL statement, the factor analysis is on the residuals of the variables, which are regressed on the partial variables. In this case, the `OUT=` data set also contains the (unstandardized) residuals, which are prefixed by `R_` by default. For example, the residual of variable X is named `R_X` in the `OUT=` data set. You might also assign the prefix by the `PARPREFIX=` option. Because the residuals are factor-analyzed, the estimated factor scores are computed as linear combinations of the standardized values of the residuals, but not the original variables.

## The OUTSTAT= Data Set

The **OUTSTAT=** data set is similar to the **TYPE=CORR** or **TYPE=UCORR** data set produced by the **CORR** procedure, but it is a **TYPE=FACTOR** data set and it contains many results in addition to those produced by **PROC CORR**. The **OUTSTAT=** data set contains observations with **\_TYPE\_='UCORR'** and **\_TYPE\_='USTD'** if you specify the **NOINT** option.

The output data set contains the following variables:

- the **BY** variables, if any
- two new character variables, **\_TYPE\_** and **\_NAME\_**
- the variables analyzed—those in the **VAR** statement, or, if there is no **VAR** statement, all numeric variables not listed in any other statement. If partial variables are specified in the **PARTIAL** statement, the residuals are included instead. By default, the residual variable names are prefixed by **R\_**, unless you specify something different in the **PARPREFIX=** option.

Each observation in the output data set contains some type of statistic as indicated by the **\_TYPE\_** variable. The **\_NAME\_** variable is blank except where otherwise indicated. The values of the **\_TYPE\_** variable are as follows:

MEAN	means
STD	standard deviations
USTD	uncorrected standard deviations
N	sample size
CORR	correlations. The <b>_NAME_</b> variable contains the name of the variable corresponding to each row of the correlation matrix.
UCORR	uncorrected correlations. The <b>_NAME_</b> variable contains the name of the variable corresponding to each row of the uncorrected correlation matrix.
IMAGE	image coefficients. The <b>_NAME_</b> variable contains the name of the variable corresponding to each row of the image coefficient matrix.
IMAGECOV	image covariance matrix. The <b>_NAME_</b> variable contains the name of the variable corresponding to each row of the image covariance matrix.
COMMUNAL	final communality estimates
PRIORS	prior communality estimates, or estimates from the last iteration for iterative methods
WEIGHT	variable weights
SUMWGT	sum of the variable weights
EIGENVAL	eigenvalues
UNROTATE	unrotated factor pattern. The <b>_NAME_</b> variable contains the name of the factor.
SE_UNROT	standard error estimates for the unrotated loadings. The <b>_NAME_</b> variable contains the name of the factor.

RESIDUAL	residual correlations. The <code>_NAME_</code> variable contains the name of the variable corresponding to each row of the residual correlation matrix.
PRETRANS	transformation matrix from prerotation. The <code>_NAME_</code> variable contains the name of the factor.
PREFCORR	prerotated interfactor correlations. The <code>_NAME_</code> variable contains the name of the factor.
SE_PREFC	standard error estimates for prerotated interfactor correlations. The <code>_NAME_</code> variable contains the name of the factor.
PREROTAT	prerotated factor pattern. The <code>_NAME_</code> variable contains the name of the factor.
SE_PREPA	standard error estimates for the prerotated loadings. The <code>_NAME_</code> variable contains the name of the factor.
PRERCORR	prerotated reference axis correlations. The <code>_NAME_</code> variable contains the name of the factor.
PREREFER	prerotated reference structure. The <code>_NAME_</code> variable contains the name of the factor.
PRESTRUC	prerotated factor structure. The <code>_NAME_</code> variable contains the name of the factor.
SE_PREST	standard error estimates for prerotated structure loadings. The <code>_NAME_</code> variable contains the name of the factor.
PRESCORE	prerotated scoring coefficients. The <code>_NAME_</code> variable contains the name of the factor.
TRANSFOR	transformation matrix from rotation. The <code>_NAME_</code> variable contains the name of the factor.
FCORR	interfactor correlations. The <code>_NAME_</code> variable contains the name of the factor.
SE_FCORR	standard error estimates for interfactor correlations. The <code>_NAME_</code> variable contains the name of the factor.
PATTERN	factor pattern. The <code>_NAME_</code> variable contains the name of the factor.
SE_PAT	standard error estimates for the rotated loadings. The <code>_NAME_</code> variable contains the name of the factor.
RCORR	reference axis correlations. The <code>_NAME_</code> variable contains the name of the factor.
REFERENC	reference structure. The <code>_NAME_</code> variable contains the name of the factor.
STRUCTUR	factor structure. The <code>_NAME_</code> variable contains the name of the factor.
SE_STRUC	standard error estimates for structure loadings. The <code>_NAME_</code> variable contains the name of the factor.
SCORE	scoring coefficients to be applied to standardized variables if the <code>SCORE</code> option is specified on the <code>PROC FACTOR</code> statement. The <code>_NAME_</code> variable contains the name of the factor.
USCORE	scoring coefficients to be applied without subtracting the mean from the raw variables if the <code>SCORE</code> option is specified on the <code>PROC FACTOR</code> statement. The <code>_NAME_</code> variable contains the name of the factor.



## Confidence Intervals and the Saliency of Factor Loadings

The traditional approach to determining salient loadings (loadings that are considered large in absolute values) employs rules of thumb such as 0.3 or 0.4. However, this does not use the statistical evidence efficiently. The asymptotic normality of the distribution of factor loadings enables you to construct confidence intervals to gauge the saliency of factor loadings. To guarantee the range-respecting properties of confidence intervals, a transformation procedure such as in CEFA (Browne et al. 2004) is used. For example, because the orthogonal rotated factor loading  $\theta$  must be bounded between  $-1$  and  $+1$ , the Fisher transformation

$$\varphi = \frac{1}{2} \log\left(\frac{1 + \theta}{1 - \theta}\right)$$

is employed so that  $\varphi$  is an unbounded parameter. Assuming the asymptotic normality of  $\hat{\varphi}$ , a symmetric confidence interval for  $\varphi$  is constructed. Then, a back-transformation on the confidence limits yields an asymmetric confidence interval for  $\theta$ . Applying the results of Browne (1982), a  $(1-\alpha)100\%$  confidence interval for the orthogonal factor loading  $\theta$  is

$$(\hat{\theta}_l = \frac{a/b - 1}{a/b + 1}, \hat{\theta}_u = \frac{a \times b - 1}{a \times b + 1})$$

where

$$a = \frac{1 + \hat{\theta}}{1 - \hat{\theta}}, \quad b = \exp(z_{\alpha/2} \times \frac{2\hat{\sigma}}{1 - \hat{\theta}^2})$$

and  $\hat{\theta}$  is the estimated factor loading,  $\hat{\sigma}$  is the standard error estimate of the factor loading, and  $z_{\alpha/2}$  is the  $(1 - \alpha/2)100$  percentile point of a standard normal distribution.

Once the confidence limits are constructed, you can use the corresponding coverage displays for determining the saliency of the variable-factor relationship. In a coverage display, the **COVER=** value is represented by an asterisk (\*). The following table summarizes various displays and their interpretations.

**Table 33.2** Interpretations of the Coverage Displays

Positive Estimate	Negative Estimate	COVER=0 Specified	Interpretation
[0]*	*[0]		The estimate is not significantly different from zero, and the CI covers a region of values that are smaller in magnitude than the COVER= value. This is strong statistical evidence for the nonsalience of the variable-factor relationship.
0[ ]*	*[ ]0		The estimate is significantly different from zero, but the CI covers a region of values that are smaller in magnitude than the COVER= value. This is strong statistical evidence for the nonsalience of the variable-factor relationship.
[0*]	[*0]	[0]	The estimate is not significantly different from zero or the COVER= value. The population value might have been larger or smaller in magnitude than the COVER= value. There is no statistical evidence for the salience of the variable-factor relationship.
0[*]	[*]0		The estimate is significantly different from zero but not from the COVER= value. This is marginal statistical evidence for the salience of the variable-factor relationship.
0*[ ]	[ ]*0	0[ ] or [ ]0	The estimate is significantly different from zero, and the CI covers a region of values that are larger in magnitude than the COVER= value. This is strong statistical evidence for the salience of the variable-factor relationship.

See [Example 33.4](#) for an illustration of the use of confidence intervals for interpreting factors.

## Simplicity Functions for Rotations

To rotate a factor pattern is to apply a nonsingular linear transformation to the unrotated factor pattern matrix. An optimal transformation is usually defined as a minimum or maximum point of a simplicity function. Different rotation methods are based on different simplicity functions employed.

For the promax or the Procrustes rotation, the simplicity function used is the sum of squared differences between the rotated factor pattern and the target matrix. The optimal transformation is obtained by minimizing this simplicity function with respect to the choices of all possible transformation.

For the class of the generalized Crawford-Ferguson family (Jennrich 1973), the simplicity function being optimized is

$$f = k_1 Z + k_2 H + k_3 V + k_4 Q$$

where

$$Z = \left( \sum_j \sum_i b_{ij}^2 \right)^2, \quad H = \sum_i \left( \sum_j b_{ij}^2 \right)^2$$

$$V = \sum_j \left( \sum_i b_{ij}^2 \right)^2, \quad Q = \sum_j \sum_i b_{ij}^4$$

$k_1, k_2, k_3$ , and  $k_4$  are constants, and  $b_{ij}$  represents an element of the rotated pattern matrix. Except for specialized research purposes, it is uncommon in practice to use this simplicity function directly for rotation. However, this simplicity function reduces to many well-known classes of rotations. One of these is the Crawford-Ferguson family (Crawford and Ferguson 1970), which minimizes

$$f_{cf} = c_1(H - Q) + c_2(V - Q)$$

where  $c_1$  and  $c_2$  are constants,  $(H - Q)$  represents variable (row) parsimony, and  $(V - Q)$  represents factor (column) parsimony. Therefore, the relative importance of both the variable parsimony and of the factor parsimony is adjusted using the constants  $c_1$  and  $c_2$ . The orthomax class (see Harman 1976) maximizes the function

$$f_{or} = pQ - \gamma V$$

where  $\gamma$  is the orthomax weight and is usually between 0 and the number of variables  $p$ . The oblimin class minimizes the function

$$f_{ob} = p(H - Q) - \tau(Z - V)$$

where  $\tau$  is the oblimin weight. For practical purposes, a negative or zero value for  $\tau$  is recommended.

All of the preceding definitions are for rotations without row normalization. For rotations with Kaiser normalization, the definition of  $b_{ij}$  is replaced by  $b_{ij}/h_i$ , where  $h_i$  is the communality estimate of variable  $i$ .

---

## Missing Values

If the **DATA=** data set contains data (rather than a matrix or factor pattern), then observations with missing values for any variables in the analysis are omitted from the computations. If a correlation or covariance matrix is read, it can contain missing values as long as every pair of variables has at least one nonmissing entry. Missing values in a pattern or scoring coefficient matrix are treated as zeros.

---

## Cautions

- The amount of time that FACTOR takes is roughly proportional to the cube of the number of variables. Factoring 100 variables, therefore, takes about 1,000 times as long as factoring 10 variables. Iterative methods (PRINIT, ALPHA, ULS, ML) can also take 100 times as long as noniterative methods (PRINCIPAL, IMAGE, HARRIS).
- No computer program is capable of reliably determining the optimal number of factors, since the decision is ultimately subjective. You should not blindly accept the number of factors obtained by default; instead, use your own judgment to make a decision.
- Singular correlation matrices cause problems with the options **PRIORS=SMC** and **METHOD=ML**. Singularities can result from using a variable that is the sum of other variables, coding too many dummy variables from a classification variable, or having more variables than observations.
- If you use the CORR procedure to compute the correlation matrix and there are missing data and the NOMISS option is not specified, then the correlation matrix might have negative eigenvalues.
- If a TYPE=CORR, TYPE=UCORR, or TYPE=FACTOR data set is copied or modified using a DATA step, the new data set does not automatically have the same TYPE as the old data set. You must specify the TYPE= data set option in the DATA statement. If you try to analyze a data set that has lost its TYPE=CORR attribute, PROC FACTOR displays a warning message saying that the data set contains \_NAME\_ and \_TYPE\_ variables but analyzes the data set as an ordinary SAS data set.
- For a TYPE=FACTOR data set, the default is **METHOD=PATTERN**, not **METHOD=PRIN**.

---

## Factor Scores

The FACTOR procedure can compute estimated factor scores directly if you specify the **NFACTORS=** and **OUT=** options, or indirectly using the SCORE procedure. The latter method is preferable if you use the FACTOR procedure interactively to determine the number of factors, the rotation method, or various other aspects of the analysis. To compute factor scores for each observation by using the SCORE procedure, do the following:

- Use the **SCORE** option in the PROC FACTOR statement.
- Create a TYPE=FACTOR output data set with the **OUTSTAT=** option.
- Use the SCORE procedure with both the raw data and the TYPE=FACTOR data set.
- Do not use the TYPE= option in the PROC SCORE statement.

For example, the following statements could be used:

```
proc factor data=raw score outstat=fact;
run;
proc score data=raw score=fact out=scores;
run;
```

or

```
proc corr data=raw out=correl;
run;
proc factor data=correl score outstat=fact;
run;
proc score data=raw score=fact out=scores;
run;
```

For a more detailed example, see [Example 76.1](#) in Chapter 76, “[The SCORE Procedure](#).”

A component analysis (principal, image, or Harris) produces scores with mean zero and variance one. If you have done a common factor analysis, the true factor scores have mean zero and variance one, but the computed factor scores are only estimates of the true factor scores. These estimates have mean zero but variance equal to the squared multiple correlation of the factor with the variables. The estimated factor scores might have small nonzero correlations even if the true factors are uncorrelated.

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## Variable Weights and Variance Explained

A principal component analysis of a correlation matrix treats all variables as equally important. A principal component analysis of a covariance matrix gives more weight to variables with larger variances. A principal component analysis of a covariance matrix is equivalent to an analysis of a weighted correlation matrix, where the weight of each variable is equal to its variance. Variables with large weights tend to have larger loadings on the first component and smaller residual correlations than variables with small weights.

You might want to give weights to variables by using values other than their variances. Mulaik (1972) explains how to obtain a maximally reliable component by means of a weighted principal component analysis. With the FACTOR procedure, you can indirectly give arbitrary weights to the variables by using the COV option and rescaling the variables to have variance equal to the desired weight, or you can give arbitrary weights directly by using the [WEIGHT](#) option and including the weights in a TYPE=CORR data set.

Arbitrary variable weights can be used with the [METHOD=PRINCIPAL](#), [METHOD=PRINIT](#), [METHOD=ULS](#), or [METHOD=IMAGE](#) option. Alpha and ML factor analyses compute variable weights based on the communalities (Harman 1976, pp. 217–218). For alpha factor analysis, the weight of a variable is the reciprocal of its communality. In ML factor analysis, the weight is the reciprocal of the uniqueness. Harris component analysis uses weights equal to the reciprocal of one minus the squared multiple correlation of each variable with the other variables.

For uncorrelated factors, the variance explained by a factor can be computed with or without taking the weights into account. The usual method for computing variance accounted for by a factor is to take the sum of squares of the corresponding column of the factor pattern, yielding an unweighted result. If the square of each loading is multiplied by the weight of the variable before the sum is taken, the result is the weighted variance explained, which is equal to the corresponding eigenvalue except in image analysis. Whether the weighted or unweighted result is more important depends on the purpose of the analysis.

In the case of correlated factors, the variance explained by a factor can be computed with or without taking the other factors into account. If you want to ignore the other factors, the variance explained is given by the weighted or unweighted sum of squares of the appropriate column of the factor structure since the factor structure contains simple correlations. If you want to subtract the variance explained by the other factors from the amount explained by the factor in question (the Type II variance explained), you can take the weighted or unweighted sum of squares of the appropriate column of the reference structure because the reference structure contains semipartial correlations. There are other ways of measuring the variance explained. For example, given a prior ordering of the factors, you can eliminate from each factor the variance explained by previous factors and compute a Type I variance explained. Harman (1976, pp. 268–270) provides another method, which is based on direct and joint contributions.

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## Heywood Cases and Other Anomalies about Communality Estimates

Since communalities are squared correlations, you would expect them always to lie between 0 and 1. It is a mathematical peculiarity of the common factor model, however, that final communality estimates might exceed 1. If a communality equals 1, the situation is referred to as a Heywood case, and if a communality exceeds 1, it is an ultra-Heywood case. An ultra-Heywood case implies that some unique factor has negative variance, a clear indication that something is wrong. Possible causes include the following:

- bad prior communality estimates
- too many common factors
- too few common factors
- not enough data to provide stable estimates
- the common factor model is not an appropriate model for the data

An ultra-Heywood case renders a factor solution invalid. Factor analysts disagree about whether or not a factor solution with a Heywood case can be considered legitimate.

With **METHOD=PRINIT**, **METHOD=ULS**, **METHOD=ALPHA**, or **METHOD=ML**, the FACTOR procedure, by default, stops iterating and sets the number of factors to 0 if an estimated communality exceeds 1. To enable processing to continue with a Heywood or ultra-Heywood case, you can use the **HEYWOOD** or **ULTRAHEYWOOD** option in the PROC FACTOR statement. The **HEYWOOD**

option sets the upper bound of any communality to 1, while the **ULTRAHEYWOOD** option allows communalities to exceed 1.

Theoretically, the communality of a variable should not exceed its reliability. Violation of this condition is called a quasi-Heywood case and should be regarded with the same suspicion as an ultra-Heywood case.

Elements of the factor structure and reference structure matrices can exceed 1 only in the presence of an ultra-Heywood case. On the other hand, an element of the factor pattern might exceed 1 in an oblique rotation.

The maximum likelihood method is especially susceptible to quasi- or ultra-Heywood cases. During the iteration process, a variable with high communality is given a high weight; this tends to increase its communality, which increases its weight, and so on.

It is often stated that the squared multiple correlation of a variable with the other variables is a lower bound to its communality. This is true if the common factor model fits the data perfectly, but it is not generally the case with real data. A final communality estimate that is less than the squared multiple correlation can, therefore, indicate poor fit, possibly due to not enough factors. It is by no means as serious a problem as an ultra-Heywood case. Factor methods that use the Newton-Raphson method can actually produce communalities less than 0, a result even more disastrous than an ultra-Heywood case.

The squared multiple correlation of a factor with the variables might exceed 1, even in the absence of ultra-Heywood cases. This situation is also cause for alarm. Alpha factor analysis seems to be especially prone to this problem, but it does not occur with maximum likelihood. If a squared multiple correlation is negative, there are too many factors retained.

With data that do not fit the common factor model perfectly, you can expect some of the eigenvalues to be negative. If an iterative factor method converges properly, the sum of the eigenvalues corresponding to rejected factors should be 0; hence, some eigenvalues are positive and some negative. If a principal factor analysis fails to yield any negative eigenvalues, the prior communality estimates are probably too large. Negative eigenvalues cause the cumulative proportion of variance explained to exceed 1 for a sufficiently large number of factors. The cumulative proportion of variance explained by the retained factors should be approximately 1 for principal factor analysis and should converge to 1 for iterative methods. Occasionally, a single factor can explain more than 100 percent of the common variance in a principal factor analysis, indicating that the prior communality estimates are too low.

If a squared canonical correlation or a coefficient alpha is negative, there are too many factors retained.

Principal component analysis, unlike common factor analysis, has none of these problems if the covariance or correlation matrix is computed correctly from a data set with no missing values. Various methods for missing value correlation or severe rounding of the correlations can produce negative eigenvalues in principal components.

## Time Requirements

- $n$  = number of observations  
 $v$  = number of variables  
 $f$  = number of factors  
 $i$  = number of iterations during factor extraction  
 $r$  = length of iterations during factor rotation

The time required to compute. . .	is roughly proportional to
an overall factor analysis	$iv^3$
the correlation matrix	$nv^2$
PRIORS=SMC or ASMC	$v^3$
PRIORS=MAX	$v^2$
eigenvalues	$v^3$
final eigenvectors	$fv^2$
generalized Crawford-Ferguson family of rotations, PROMAX, or HK	$rvf^2$
ROTATE=PROCRUSTES	$vf^2$

Each iteration in the PRINIT or ALPHA method requires computation of eigenvalues and  $f$  eigenvectors.

Each iteration in the ML or ULS method requires computation of eigenvalues and  $v - f$  eigenvectors.

The amount of time that PROC FACTOR takes is roughly proportional to the cube of the number of variables. Factoring 100 variables, therefore, takes about 1000 times as long as factoring 10 variables. Iterative methods (PRINIT, ALPHA, ULS, ML) can also take 100 times as long as noniterative methods (PRINCIPAL, IMAGE, HARRIS).



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## Displayed Output

PROC FACTOR output includes the following.

- Mean and Std Dev (standard deviation) of each variable and the number of observations, if you specify the **SIMPLE** option
- Correlations, if you specify the **CORR** option
- Inverse Correlation Matrix, if you specify the **ALL** option
- Partial Correlations Controlling all other Variables (negative anti-image correlations), if you specify the **MSA** option. If the data are appropriate for the common factor model, the partial correlations should be small.
- Kaiser's Measure of Sampling Adequacy (Kaiser 1970; Kaiser and Rice 1974; Cerny and Kaiser 1977) both overall and for each variable, if you specify the **MSA** option. The **MSA** is a summary of how small the partial correlations are relative to the ordinary correlations. Values greater than 0.8 can be considered good. Values less than 0.5 require remedial action, either by deleting the offending variables or by including other variables related to the offenders.
- Prior Communality Estimates, unless 1.0s are used or unless you specify the **METHOD=IMAGE**, **METHOD=HARRIS**, **METHOD=PATTERN**, or **METHOD=SCORE** option
- Squared Multiple Correlations of each variable with all the other variables, if you specify the **METHOD=IMAGE** or **METHOD=HARRIS** option
- Image Coefficients, if you specify the **METHOD=IMAGE** option
- Image Covariance Matrix, if you specify the **METHOD=IMAGE** option
- Preliminary Eigenvalues based on the prior communalities, if you specify the **METHOD=PRINIT**, **METHOD=ALPHA**, **METHOD=ML**, or **METHOD=ULS** option. The table produced includes the Total and the Average of the eigenvalues, the Difference between successive eigenvalues, the Proportion of variation represented, and the Cumulative proportion of variation.
- the number of factors that are retained, unless you specify the **METHOD=PATTERN** or **METHOD=SCORE** option
- the Scree Plot of Eigenvalues, if you specify the **SCREE** option. The preliminary eigenvalues are used if you specify the **METHOD=PRINIT**, **METHOD=ALPHA**, **METHOD=ML**, or **METHOD=ULS** option.
- the iteration history, if you specify the **METHOD=PRINIT**, **METHOD=ALPHA**, **METHOD=ML**, or **METHOD=ULS** option. The table produced contains the iteration number (Iter); the Criterion being optimized (Jöreskog 1977); the Ridge value for the iteration if you specify the **METHOD=ML** or **METHOD=ULS** option; the maximum Change in any communality estimate; and the Communalities.

- Significance tests, if you specify the option **METHOD=ML**, including Bartlett's chi-square, df, and  $\text{Prob} > \chi^2$  for  $H_0$ : No common factors and  $H_0$ : factors retained are sufficient to explain the correlations. The variables should have an approximate multivariate normal distribution for the probability levels to be valid. Lawley and Maxwell (1971) suggest that the number of observations should exceed the number of variables by 50 or more, although Geweke and Singleton (1980) claim that as few as 10 observations are adequate with five variables and one common factor. Certain regularity conditions must also be satisfied for Bartlett's  $\chi^2$  test to be valid (Geweke and Singleton 1980), but in practice these conditions usually are satisfied. The notation  $\text{Prob}>\chi^2$  means "the probability under the null hypothesis of obtaining a greater  $\chi^2$  statistic than that observed." The chi-square value is displayed with and without Bartlett's correction.
- Akaike's Information Criterion, if you specify the **METHOD=ML** option. Akaike's information criterion (AIC) (Akaike 1973, 1974, 1987) is a general criterion for estimating the best number of parameters to include in a model when maximum likelihood estimation is used. The number of factors that yields the smallest value of AIC is considered best. Like the chi-square test, AIC tends to include factors that are statistically significant but inconsequential for practical purposes.
- Schwarz's Bayesian Criterion, if you specify the **METHOD=ML** option. Schwarz's Bayesian Criterion (SBC) (Schwarz 1978) is another criterion, similar to AIC, for determining the best number of parameters. The number of factors that yields the smallest value of SBC is considered best; SBC seems to be less inclined to include trivial factors than either AIC or the chi-square test.
- Tucker and Lewis's Reliability Coefficient, if you specify the **METHOD=ML** option (Tucker and Lewis 1973)
- Squared Canonical Correlations, if you specify the **METHOD=ML** option. These are the same as the squared multiple correlations for predicting each factor from the variables.
- Coefficient Alpha for Each Factor, if you specify the **METHOD=ALPHA** option
- Eigenvectors, if you specify the **EIGENVECTORS** or **ALL** option, unless you also specify the **METHOD=PATTERN** or **METHOD=SCORE** option
- Eigenvalues of the (Weighted) (Reduced) (Image) Correlation or Covariance Matrix, unless you specify the **METHOD=PATTERN** or **METHOD=SCORE** option. Included are the Total and the Average of the eigenvalues, the Difference between successive eigenvalues, the Proportion of variation represented, and the Cumulative proportion of variation.
- the Factor Pattern, which is equal to both the matrix of standardized regression coefficients for predicting variables from common factors and the matrix of correlations between variables and common factors since the extracted factors are uncorrelated. Standard error estimates are included if the **SE** option is specified with **METHOD=ML**. Confidence limits and coverage displays are included if **COVER=** option is specified with **METHOD=ML**.
- Variance explained by each factor, both Weighted and Unweighted, if variable weights are used

- Final Communality Estimates, including the Total communality; or Final Communality Estimates and Variable Weights, including the Total communality, both Weighted and Unweighted, if variable weights are used. Final communality estimates are the squared multiple correlations for predicting the variables from the estimated factors, and they can be obtained by taking the sum of squares of each row of the factor pattern, or a weighted sum of squares if variable weights are used.
- Residual Correlations with Uniqueness on the Diagonal, if you specify the RESIDUAL or ALL option
- Root Mean Square Off-diagonal Residuals, both Over-all and for each variable, if you specify the RESIDUAL or ALL option
- Partial Correlations Controlling Factors, if you specify the RESIDUAL or ALL option
- Root Mean Square Off-diagonal Partial, both Over-all and for each variable, if you specify the RESIDUAL or ALL option
- Plots of Factor Pattern for unrotated factors, if you specify the PREPLOT option. The number of plots is determined by the NPLOT= option.
- Variable Weights for Rotation, if you specify the NORM=WEIGHT option
- Factor Weights for Rotation, if you specify the HKPOWER= option
- Orthogonal Transformation Matrix, if you request an orthogonal rotation
- Rotated Factor Pattern, if you request an orthogonal rotation. Standard error estimates are included if the SE option is specified with METHOD=ML. Confidence limits and coverage displays are included if COVER= option is specified with METHOD=ML.
- Variance explained by each factor after rotation. If you request an orthogonal rotation and if variable weights are used, both weighted and unweighted values are produced.
- Target Matrix for Procrustean Transformation, if you specify the ROTATE=PROMAX or ROTATE=PROCRUSTES option
- the Procrustean Transformation Matrix, if you specify the ROTATE=PROMAX or ROTATE=PROCRUSTES option
- the Normalized Oblique Transformation Matrix, if you request an oblique rotation, which, for the option ROTATE=PROMAX, is the product of the prerotation and the Procrustes rotation
- Inter-factor Correlations, if you specify an oblique rotation. Standard error estimates are included if the SE option is specified with METHOD=ML. Confidence limits and coverage displays are included if COVER= option is specified with METHOD=ML.
- Rotated Factor Pattern (Std Reg Coefs), if you specify an oblique rotation, giving standardized regression coefficients for predicting the variables from the factors. Standard error estimates are included if the SE option is specified with METHOD=ML. Confidence limits and coverage displays are included if COVER= option is specified with METHOD=ML.

- Reference Axis Correlations if you specify an oblique rotation. These are the partial correlations between the primary factors when all factors other than the two being correlated are partialled out.
- Reference Structure (Semipartial Correlations), if you request an oblique rotation. The reference structure is the matrix of semipartial correlations (Kerlinger and Pedhazur 1973) between variables and common factors, removing from each common factor the effects of other common factors. If the common factors are uncorrelated, the reference structure is equal to the factor pattern.
- Variance explained by each factor eliminating the effects of all other factors, if you specify an oblique rotation. Both Weighted and Unweighted values are produced if variable weights are used. These variances are equal to the (weighted) sum of the squared elements of the reference structure corresponding to each factor.
- Factor Structure (Correlations), if you request an oblique rotation. The (primary) factor structure is the matrix of correlations between variables and common factors. If the common factors are uncorrelated, the factor structure is equal to the factor pattern. Standard error estimates are included if the **SE** option is specified with **METHOD=ML**. Confidence limits and coverage displays are included if **COVER=** option is specified with **METHOD=ML**.
- Variance explained by each factor ignoring the effects of all other factors, if you request an oblique rotation. Both Weighted and Unweighted values are produced if variable weights are used. These variances are equal to the (weighted) sum of the squared elements of the factor structure corresponding to each factor.
- Final Communality Estimates for the rotated factors if you specify the **ROTATE=** option. The estimates should equal the unrotated communalities.
- Squared Multiple Correlations of the Variables with Each Factor, if you specify the **SCORE** or **ALL** option, except for unrotated principal components
- Standardized Scoring Coefficients, if you specify the **SCORE** or **ALL** option
- Plots of the Factor Pattern for rotated factors, if you specify the **PLOT** option and you request an orthogonal rotation. The number of plots is determined by the **NPLOT=** option.
- Plots of the Reference Structure for rotated factors, if you specify the **PLOT** option and you request an oblique rotation. The number of plots is determined by the **NPLOT=** option. Included are the Reference Axis Correlation and the Angle between the Reference Axes for each pair of factors plotted.

If you specify the **ROTATE=PROMAX** option, the output includes results for both the prerotation and the Procrustes rotation.

## ODS Table Names

PROC FACTOR assigns a name to each table it creates. You can use these names to reference the table when using the Output Delivery System (ODS) to select tables and create output data sets. These names are listed in the [Table 33.3](#). For more information about ODS, see Chapter 20, “[Using the Output Delivery System](#).”

**Table 33.3** ODS Tables Produced by PROC FACTOR

ODS Table Name	Description	Option
AlphaCoef	Coefficient alpha for each factor	METHOD=ALPHA
CanCorr	Squared canonical correlations	METHOD=ML
CondStdDev	Conditional standard deviations	SIMPLE with PARTIAL
ConvergenceStatus	Convergence status	METHOD=PRINIT, ALPHA, ML, or ULS
Corr	Correlations	CORR
Eigenvalues	Eigenvalues	default, SCREE
Eigenvectors	Eigenvectors	EIGENVECTORS
FactorWeightRotate	Factor weights for rotation	HKPOWER=
FactorPattern	Factor pattern	default
FactorStructure	Factor structure	ROTATE= any oblique rotation
FinalCommun	Final communalities	default
FinalCommunWgt	Final communalities with weights	METHOD=ML or ALPHA
FitMeasures	Measures of fit	METHOD=ML
ImageCoef	Image coefficients	METHOD=IMAGE
ImageCov	Image covariance matrix	METHOD=IMAGE
ImageFactors	Image factor matrix	METHOD=IMAGE
InputFactorPattern	Input factor pattern	METHOD=PATTERN with PRINT or ALL
InputScoreCoef	Standardized input scoring coefficients	METHOD=SCORE with PRINT or ALL
InterFactorCorr	Interfactor correlations	ROTATE= any oblique rotation
InvCorr	Inverse correlation matrix	ALL
IterHistory	Iteration history	METHOD=PRINIT, ALPHA, ML, or ULS
MultipleCorr	Squared multiple correlations	METHOD=IMAGE or METHOD=HARRIS
NormObliqueTrans	Normalized oblique transformation matrix	ROTATE= any oblique rotation
ObliqueRotFactPat	Rotated factor pattern	ROTATE= any oblique rotation
ObliqueTrans	Oblique transformation matrix	HKPOWER=
OrthRotFactPat	Rotated factor pattern	ROTATE= any orthogonal rotation
OrthTrans	Orthogonal transformation matrix	ROTATE= any orthogonal rotation
ParCorrControlFactor	Partial correlations controlling factors	RESIDUAL

**Table 33.3** *continued*

ODS Table Name	Description	Option
ParCorrControlVar	Partial correlations controlling other variables	MSA
PartialCorr	Partial correlations	MSA, CORR with PARTIAL
PriorCommunalEst	Prior communality estimates	PRIORS=, METHOD=ML or ALPHA
ProcrustesTarget	Target matrix for Procrustean transformation	ROTATE=PROCRUSTES, ROTATE=PROMAX
ProcrustesTrans	Procrustean transformation matrix	ROTATE=PROCRUSTES, ROTATE=PROMAX
RMSOffDiagPartials	Root mean square off-diagonal partials	RESIDUAL
RMSOffDiagResids	Root mean square off-diagonal residuals	RESIDUAL
ReferenceAxisCorr	Reference axis correlations	ROTATE= any oblique rotation
ReferenceStructure	Reference structure	ROTATE= any oblique rotation
ResCorrUniqueDiag	Residual correlations with uniqueness on the diagonal	RESIDUAL
SamplingAdequacy	Kaiser's measure of sampling adequacy	MSA
SignifTests	Significance tests	METHOD=ML
SimpleStatistics	Simple statistics	SIMPLE
StdScoreCoef	Standardized scoring coefficients	SCORE
VarExplain	Variance explained	default
VarExplainWgt	Variance explained with weights	METHOD=ML, or ALPHA
VarFactorCorr	Squared multiple correlations of the variables with each factor	SCORE
VarWeightRotate	Variable weights for rotation	NORM=WEIGHT, ROTATE=

## ODS Graphics

To request graphics with PROC FACTOR, you must first enable ODS Graphics by specifying the **ods graphics on** statement. See Chapter 21, “[Statistical Graphics Using ODS](#),” for more information. The names of the graphs that PROC FACTOR generates are listed in [Table 33.4](#), along with the required statements and options.

**Table 33.4** ODS Graphics Produced by PROC FACTOR

ODS Graph Name	Plot Description	Option
PatternPlot	Rotated factor pattern	PLOTS=LOADINGS
InitPatternPlot	Initial factor pattern	PLOTS=INITLOADINGS
InitRefStructurePlot	Initial reference structures	PLOTS=INITLOADINGS and PLOTREF
PrePatternPlot	Prerotated factor pattern	PLOTS=PRELOADINGS
PreRefStructurePlot	Prerotated reference structures	PLOTS=PRELOADINGS and PLOTREF
RefStructurePlot	Rotated reference structures	PLOTS=LOADINGS and PLOTREF
ScreePlot	Scree and variance explained plots	PLOTS=SCREE
VariancePlot	Plot of explained variance	PLOTS=SCREE(UNPACK)

## Examples: FACTOR Procedure

### Example 33.1: Principal Component Analysis

This example analyzes socioeconomic data provided by Harman (1976). The five variables represent total population, median school years, total employment, miscellaneous professional services, and median house value. Each observation represents one of twelve census tracts in the Los Angeles Standard Metropolitan Statistical Area.

The first analysis is a principal component analysis. Simple descriptive statistics and correlations are also displayed. The following statements produce [Output 33.1.1](#):

```
data SocioEconomics;
  title 'Five Socioeconomic Variables';
  title2 'See Page 14 of Harman: Modern Factor Analysis, 3rd Ed';
  input Population School Employment Services HouseValue;
  datalines;
5700      12.8      2500      270      25000
1000      10.9      600      10      10000
3400      8.8      1000      10      9000
3800      13.6      1700      140      25000
4000      12.8      1600      140      25000
8200      8.3      2600      60      12000
1200      11.4      400      10      16000
9100      11.5      3300      60      14000
9900      12.5      3400      180      18000
9600      13.7      3600      390      25000
9600      9.6      3300      80      12000
9400      11.4      4000      100      13000
;

title3 'Principal Component Analysis';
proc factor data=SocioEconomics simple corr;
run;
```

There are two large eigenvalues, 2.8733 and 1.7967, which together account for 93.4% of the standardized variance. Thus, the first two principal components provide an adequate summary of the data for most purposes. Three components, explaining 97.7% of the variation, should be sufficient for almost any application. PROC FACTOR retains two components on the basis of the eigenvalues-greater-than-one rule since the third eigenvalue is only 0.2148.

The first component has large positive loadings for all five variables. The correlation with Services (0.9324) is especially high. The second component is a contrast of Population (0.8064) and Employment (0.7261) against School (−0.5448) and HouseValue (−0.5582), with a very small loading on Services (−0.1043).



**Output 33.1.1** Principal Component Analysis

Five Socioeconomic Variables  
 See Page 14 of Harman: Modern Factor Analysis, 3rd Ed  
 Principal Component Analysis

## The FACTOR Procedure

## Means and Standard Deviations from 12 Observations

Variable	Mean	Std Dev
Population	6241.667	3439.9943
School	11.442	1.7865
Employment	2333.333	1241.2115
Services	120.833	114.9275
HouseValue	17000.000	6367.5313

## Correlations

	Population	School	Employment	Services	HouseValue
Population	1.00000	0.00975	0.97245	0.43887	0.02241
School	0.00975	1.00000	0.15428	0.69141	0.86307
Employment	0.97245	0.15428	1.00000	0.51472	0.12193
Services	0.43887	0.69141	0.51472	1.00000	0.77765
HouseValue	0.02241	0.86307	0.12193	0.77765	1.00000

Five Socioeconomic Variables  
 See Page 14 of Harman: Modern Factor Analysis, 3rd Ed  
 Principal Component Analysis

## The FACTOR Procedure

## Initial Factor Method: Principal Components

Prior Communality Estimates: ONE

Eigenvalues of the Correlation Matrix: Total = 5 Average = 1

	Eigenvalue	Difference	Proportion	Cumulative
1	2.87331359	1.07665350	0.5747	0.5747
2	1.79666009	1.58182321	0.3593	0.9340
3	0.21483689	0.11490283	0.0430	0.9770
4	0.09993405	0.08467868	0.0200	0.9969
5	0.01525537		0.0031	1.0000

**Output 33.1.1** *continued*

2 factors will be retained by the MINEIGEN criterion.				
Factor Pattern				
	Factor1	Factor2		
Population	0.58096	0.80642		
School	0.76704	-0.54476		
Employment	0.67243	0.72605		
Services	0.93239	-0.10431		
HouseValue	0.79116	-0.55818		
Variance Explained by Each Factor				
	Factor1	Factor2		
	2.8733136	1.7966601		
Final Communality Estimates: Total = 4.669974				
Population	School	Employment	Services	HouseValue
0.98782629	0.88510555	0.97930583	0.88023562	0.93750041

The final communality estimates show that all the variables are well accounted for by two components, with final communality estimates ranging from 0.8802 for Services to 0.9878 for Population.

## Example 33.2: Principal Factor Analysis

This example uses the data presented in [Example 33.1](#), and performs a principal factor analysis with squared multiple correlations for the prior communality estimates (**PRIORS=SMC**). Unlike [Example 33.1](#), which analyzes the principal components, the current analysis is based on a common factor model.

To help determine whether the common factor model is appropriate, Kaiser's measure of sampling adequacy (**MSA**) is requested, and the residual correlations and partial correlations are computed (**RESIDUAL**).

The **ROTATE=** and **REORDER** options are specified to enhance factor interpretability. The **ROTATE=PROMAX** option produces an orthogonal varimax prerotation (default) followed by an oblique Procrustes rotation, and the **REORDER** option reorders the variables according to their largest factor loadings. An **OUTSTAT=** data set is created by PROC FACTOR and displayed in [Output 33.2.12](#).

PROC FACTOR can produce high-quality graphs that are very useful to interpret the factor solutions. To request these graphs, you must first enable ODS Graphics by specifying the **ods**

**graphics on** statement, as shown in the following code. All ODS graphs in PROC FACTOR are requested with the **PLOTS=** option. In this example, a scree plot is requested to help you determine the number of factors. Loading plots for the initial unrotated solution, prerotated (varimax) solution, and promax-rotated solution are also requested to help you visualize the patterns of factor loadings in various stages.

```
ods graphics on;

title3 'Principal Factor Analysis with Promax Rotation';
proc factor data=SocioEconomics
  priors=smc msa residual
  rotate=promax reorder
  outstat=fact_all
  plots=(scree initloadings preloadings loadings);
run;

ods graphics off;
```

Output 33.2.1 displays the results of the principal factor extraction.

#### Output 33.2.1 Principal Factor Analysis

Five Socioeconomic Variables					
See Page 14 of Harman: Modern Factor Analysis, 3rd Ed					
Principal Factor Analysis with Promax Rotation					
The FACTOR Procedure					
Initial Factor Method: Principal Factors					
Partial Correlations Controlling all other Variables					
	Population	School	Employment	Services	HouseValue
Population	1.00000	-0.54465	0.97083	0.09612	0.15871
School	-0.54465	1.00000	0.54373	0.04996	0.64717
Employment	0.97083	0.54373	1.00000	0.06689	-0.25572
Services	0.09612	0.04996	0.06689	1.00000	0.59415
HouseValue	0.15871	0.64717	-0.25572	0.59415	1.00000
Kaiser's Measure of Sampling Adequacy: Overall MSA = 0.57536759					
Population	School	Employment	Services	HouseValue	
0.47207897	0.55158839	0.48851137	0.80664365	0.61281377	
Prior Communality Estimates: SMC					
Population	School	Employment	Services	HouseValue	
0.96859160	0.82228514	0.96918082	0.78572440	0.84701921	

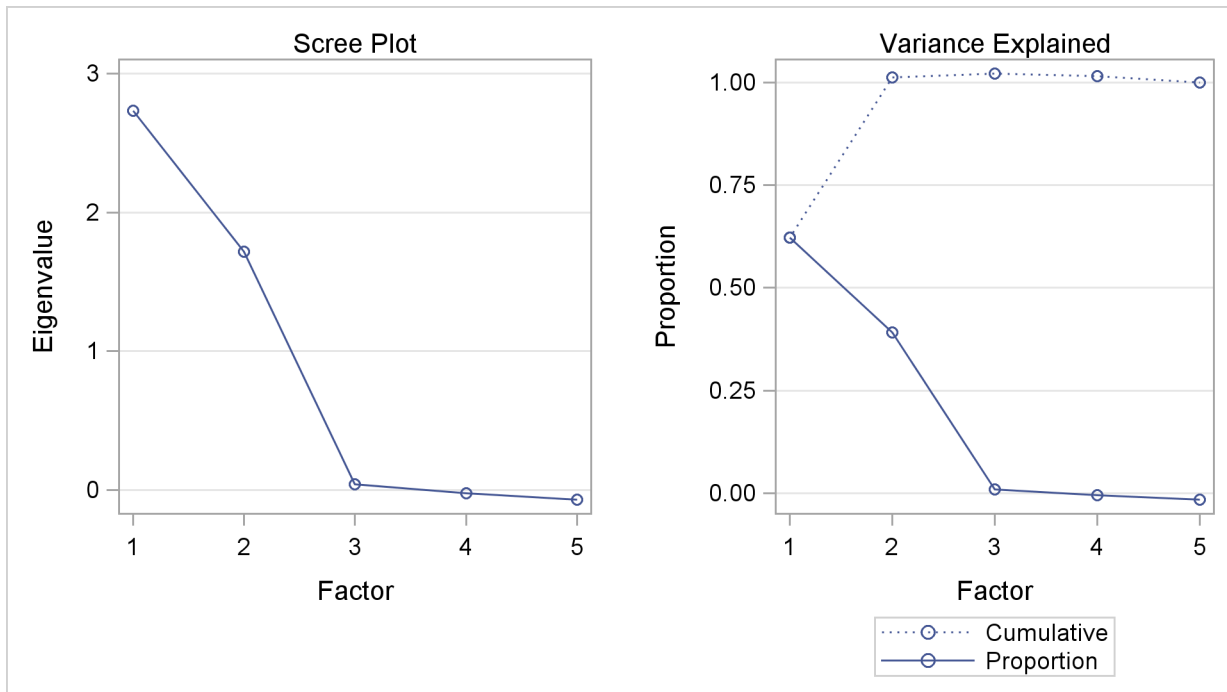
**Output 33.2.1** *continued*

Eigenvalues of the Reduced Correlation Matrix:				
Total = 4.39280116    Average = 0.87856023				
	Eigenvalue	Difference	Proportion	Cumulative
1	2.73430084	1.01823217	0.6225	0.6225
2	1.71606867	1.67650586	0.3907	1.0131
3	0.03956281	0.06408626	0.0090	1.0221
4	-.02452345	0.04808427	-0.0056	1.0165
5	-.07260772		-0.0165	1.0000

If the data are appropriate for the common factor model, the partial correlations controlling the other variables should be small compared to the original correlations. The partial correlation between the variables *School* and *HouseValue*, for example, is 0.65, slightly less than the original correlation of 0.86. The partial correlation between *Population* and *School* is  $-0.54$ , which is much larger in absolute value than the original correlation; this is an indication of trouble. Kaiser's *MSA* is a summary, for each variable and for all variables together, of how much smaller the partial correlations are than the original correlations. Values of 0.8 or 0.9 are considered good, while *MSAs* below 0.5 are unacceptable. The variables *Population*, *School*, and *Employment* have very poor *MSAs*. Only the *Services* variable has a good *MSA*. The overall *MSA* of 0.58 is sufficiently poor that additional variables should be included in the analysis to better define the common factors. A commonly used rule is that there should be at least three variables per factor. In the following analysis, there seems to be two common factors in these data, so more variables are needed for a reliable analysis.

The *SMCs* are all fairly large; hence, the factor loadings do not differ greatly from those in the principal component analysis.

The eigenvalues in [Output 33.2.1](#) show clearly that two common factors are present. The first two largest positive eigenvalues account for 101.31% of the common variance. This is possible because the reduced correlation matrix, in general, is not necessarily positive definite, and negative eigenvalues for the matrix are possible. These cumulative proportions of common variance explained by factors are plotted in the right panel of [Output 33.2.2](#), which shows that the curve flattens out essentially after the second factor. Showing in the left panel of [Output 33.2.2](#) is the scree plot, which displays a sharp bend at the third eigenvalue, reinforcing the conclusion that two common factors are present.

**Output 33.2.2** Scree and Variance Explained Plots

As displayed in [Output 33.2.3](#), the principal factor pattern is similar to the principal component pattern seen in [Example 33.1](#). For example, the variable Services has the largest loading on the first factor, and the Population variable has the smallest. The variables Population and Employment have large positive loadings on the second factor, and the HouseValue and School variables have large negative loadings.

**Output 33.2.3** Initial Factor Pattern Matrix and Communalities

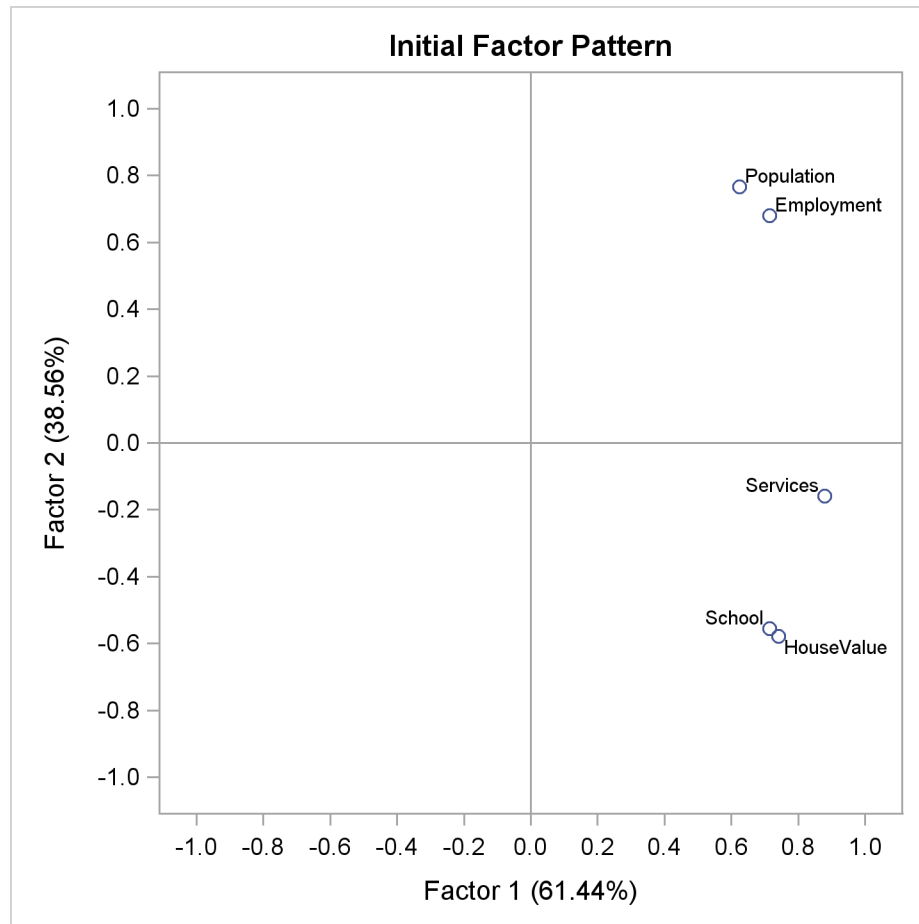
Factor Pattern				
	Factor1	Factor2		
Services	0.87899	-0.15847		
HouseValue	0.74215	-0.57806		
Employment	0.71447	0.67936		
School	0.71370	-0.55515		
Population	0.62533	0.76621		
Variance Explained by Each Factor				
	Factor1	Factor2		
	2.7343008	1.7160687		
Final Communality Estimates: Total = 4.450370				
Population	School	Employment	Services	HouseValue
0.97811334	0.81756387	0.97199928	0.79774304	0.88494998

The final communality estimates are all fairly close to the priors. Only the communality for the variable `HouseValue` increased appreciably, from 0.847 to 0.885. Nearly 100% of the common variance is accounted for. The residual correlations (off-diagonal elements) are low, the largest being 0.03 ([Output 33.2.4](#)). The partial correlations are not quite as impressive, since the uniqueness values are also rather small. These results indicate that the SMCs are good but not quite optimal communality estimates.

#### Output 33.2.4 Residual and Partial Correlations

Residual Correlations With Uniqueness on the Diagonal					
	Population	School	Employment	Services	HouseValue
Population	0.02189	-0.01118	0.00514	0.01063	0.00124
School	-0.01118	0.18244	0.02151	-0.02390	0.01248
Employment	0.00514	0.02151	0.02800	-0.00565	-0.01561
Services	0.01063	-0.02390	-0.00565	0.20226	0.03370
HouseValue	0.00124	0.01248	-0.01561	0.03370	0.11505
Root Mean Square Off-Diagonal Residuals: Overall = 0.01693282					
Population	School	Employment	Services	HouseValue	
0.00815307	0.01813027	0.01382764	0.02151737	0.01960158	
Partial Correlations Controlling Factors					
	Population	School	Employment	Services	HouseValue
Population	1.00000	-0.17693	0.20752	0.15975	0.02471
School	-0.17693	1.00000	0.30097	-0.12443	0.08614
Employment	0.20752	0.30097	1.00000	-0.07504	-0.27509
Services	0.15975	-0.12443	-0.07504	1.00000	0.22093
HouseValue	0.02471	0.08614	-0.27509	0.22093	1.00000
Root Mean Square Off-Diagonal Partial Correlations: Overall = 0.18550132					
Population	School	Employment	Services	HouseValue	
0.15850824	0.19025867	0.23181838	0.15447043	0.18201538	

As displayed in [Output 33.2.5](#), the unrotated factor pattern reveals two tight clusters of variables, with the variables `HouseValue` and `School` at the negative end of `Factor2` axis and the variables `Employment` and `Population` at the positive end. The `Services` variable is in between but closer to the `HouseValue` and `School` variables. A good rotation would put the reference axes through the two clusters.

**Output 33.2.5** Unrotated Factor Pattern Plot

[Output 33.2.6](#) and [Output 33.2.7](#) display the results of the varimax rotation. This rotation puts one axis through the variables HouseValue and School but misses the Population and Employment variables slightly.

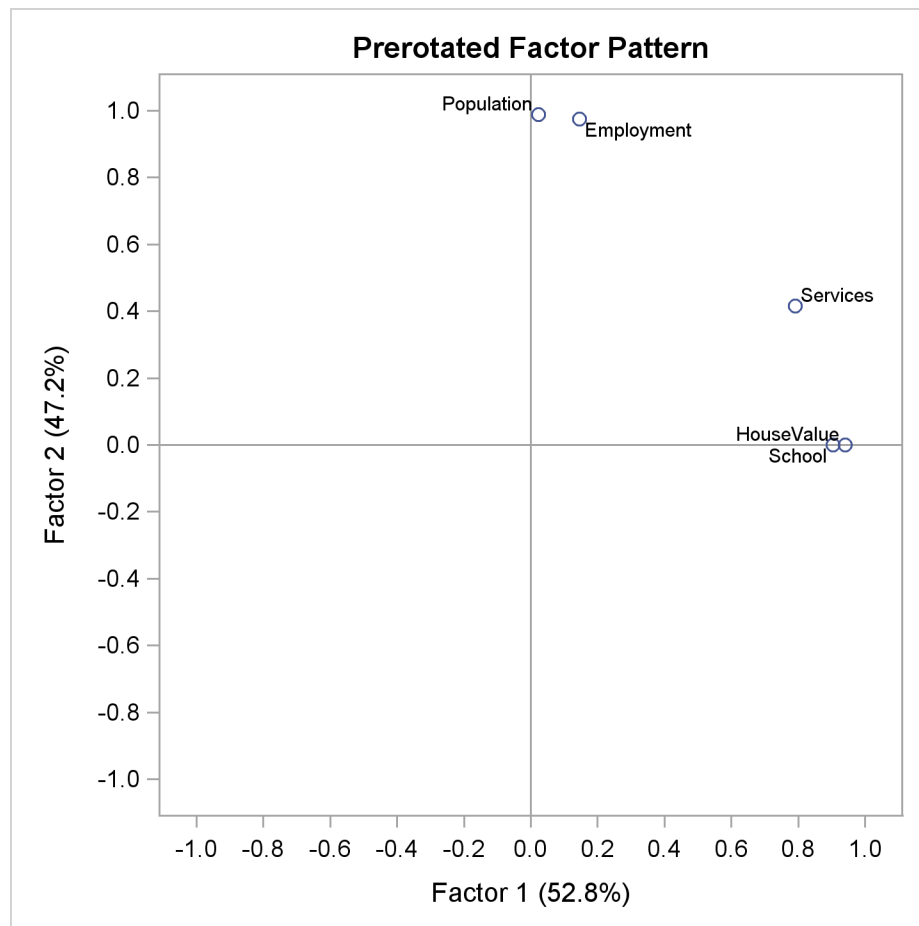
**Output 33.2.6** Varimax Rotation: Transform Matrix and Rotated Pattern

Five Socioeconomic Variables		
See Page 14 of Harman: Modern Factor Analysis, 3rd Ed		
Principal Factor Analysis with Promax Rotation		
The FACTOR Procedure		
Prerotation Method: Varimax		
Orthogonal Transformation Matrix		
	1	2
1	0.78895	0.61446
2	-0.61446	0.78895

**Output 33.2.6** *continued*

Rotated Factor Pattern				
	Factor1	Factor2		
HouseValue	0.94072	-0.00004		
School	0.90419	0.00055		
Services	0.79085	0.41509		
Population	0.02255	0.98874		
Employment	0.14625	0.97499		
Variance Explained by Each Factor				
	Factor1	Factor2		
	2.3498567	2.1005128		
Final Communality Estimates: Total = 4.450370				
Population	School	Employment	Services	HouseValue
0.97811334	0.81756387	0.97199928	0.79774304	0.88494998

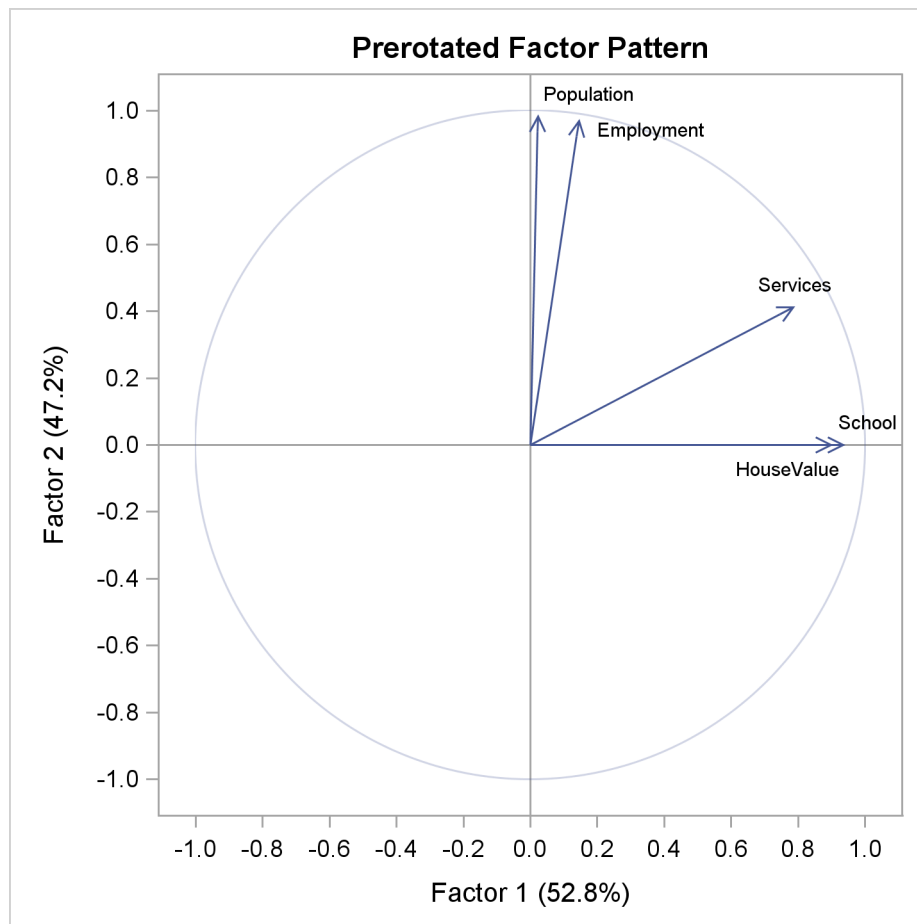


**Output 33.2.7** Varimax-Rotated Factor Loadings

An alternative to the scatter plot of factor loadings as shown in [Output 33.2.7](#) is the so-called vector plot of loadings. The vector plot is requested with the suboption VECTOR in the **PLOTS=** option. For example:

```
plots=preloadings(vector)
```

This will generate the vector plot of loadings as shown in [Output 33.2.8](#).

**Output 33.2.8** Varimax-Rotated Factor Loadings: Vector Plot

The results of oblique promax rotation are shown in [Output 33.2.9](#) and [Output 33.2.10](#). The corresponding plot of factor loadings is shown in [Output 33.2.11](#).

**Output 33.2.9** Promax Rotation: Procrustean Target and Transformation

Five Socioeconomic Variables		
See Page 14 of Harman: Modern Factor Analysis, 3rd Ed		
Principal Factor Analysis with Promax Rotation		
The FACTOR Procedure		
Rotation Method: Promax (power = 3)		
Target Matrix for Procrustean Transformation		
	Factor1	Factor2
HouseValue	1.00000	-0.00000
School	1.00000	0.00000
Services	0.69421	0.10045
Population	0.00001	1.00000
Employment	0.00326	0.96793

**Output 33.2.9** *continued*

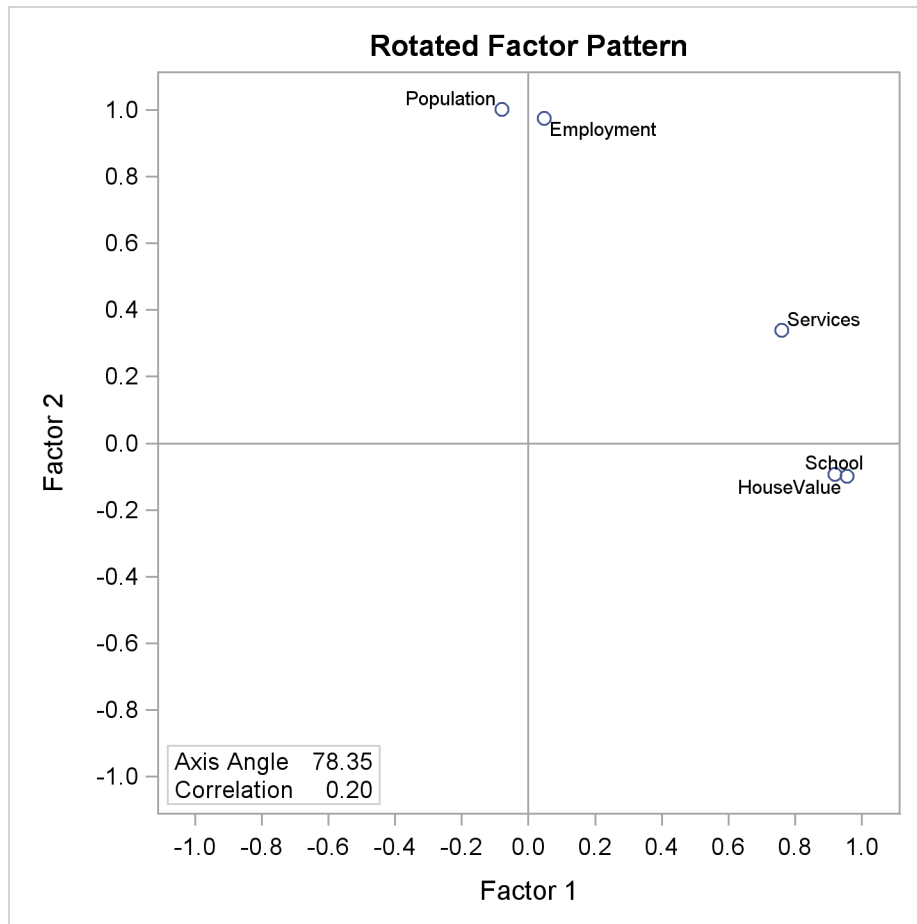
Procrustean Transformation Matrix			
	1	2	
1	1.04116598	-0.0986534	
2	-0.1057226	0.96303019	
Normalized Oblique Transformation Matrix			
	1	2	
1	0.73803	0.54202	
2	-0.70555	0.86528	

**Output 33.2.10** Promax Rotation: Promax-Rotated Factor Solution

Inter-Factor Correlations		
	Factor1	Factor2
Factor1	1.00000	0.20188
Factor2	0.20188	1.00000
Rotated Factor Pattern (Standardized Regression Coefficients)		
	Factor1	Factor2
HouseValue	0.95558485	-0.0979201
School	0.91842142	-0.0935214
Services	0.76053238	0.33931804
Population	-0.0790832	1.00192402
Employment	0.04799	0.97509085
Reference Axis Correlations		
	Factor1	Factor2
Factor1	1.00000	-0.20188
Factor2	-0.20188	1.00000
Reference Structure (Semipartial Correlations)		
	Factor1	Factor2
HouseValue	0.93591	-0.09590
School	0.89951	-0.09160
Services	0.74487	0.33233
Population	-0.07745	0.98129
Employment	0.04700	0.95501

Output 33.2.10 *continued*

Variance Explained by Each Factor Eliminating Other Factors				
	Factor1	Factor2		
	2.2480892	2.0030200		
Factor Structure (Correlations)				
	Factor1	Factor2		
HouseValue	0.93582	0.09500		
School	0.89954	0.09189		
Services	0.82903	0.49286		
Population	0.12319	0.98596		
Employment	0.24484	0.98478		
Variance Explained by Each Factor Ignoring Other Factors				
	Factor1	Factor2		
	2.4473495	2.2022803		
Final Communality Estimates: Total = 4.450370				
Population	School	Employment	Services	HouseValue
0.97811334	0.81756387	0.97199928	0.79774304	0.88494998

**Output 33.2.11** Promax-Rotated Factor Loadings

As shown in [Output 33.2.11](#), the promax solution places an axis through the variables Population and Employment but misses the HouseValue and School variables. Since an independent-cluster solution would be possible if it were not for the variable Services, a Harris-Kaiser rotation weighted by the Cureton-Mulaik technique could be used. Rather than reanalyze the entire problem with the Harris-Kaiser rotation, you can simply use the preceding results stored in the [OUTSTAT=](#) data set.

First, the [OUTSTAT=](#) data set is printed using this code:

```
title3 'Factor Output Data Set';
proc print data=fact_all;
run;
```

The output data set is displayed in [Output 33.2.12](#).

**Output 33.2.12** Output Data Set

Five Socioeconomic Variables							
See Page 14 of Harman: Modern Factor Analysis, 3rd Ed							
Factor Output Data Set							
Obs	_TYPE_	_NAME_	Population	School	Employment	Services	House Value
1	MEAN		6241.67	11.4417	2333.33	120.833	17000.00
2	STD		3439.99	1.7865	1241.21	114.928	6367.53
3	N		12.00	12.0000	12.00	12.000	12.00
4	CORR	Population	1.00	0.0098	0.97	0.439	0.02
5	CORR	School	0.01	1.0000	0.15	0.691	0.86
6	CORR	Employment	0.97	0.1543	1.00	0.515	0.12
7	CORR	Services	0.44	0.6914	0.51	1.000	0.78
8	CORR	HouseValue	0.02	0.8631	0.12	0.778	1.00
9	COMMUNAL		0.98	0.8176	0.97	0.798	0.88
10	PRIORS		0.97	0.8223	0.97	0.786	0.85
11	EIGENVAL		2.73	1.7161	0.04	-0.025	-0.07
12	UNROTATE	Factor1	0.63	0.7137	0.71	0.879	0.74
13	UNROTATE	Factor2	0.77	-0.5552	0.68	-0.158	-0.58
14	RESIDUAL	Population	0.02	-0.0112	0.01	0.011	0.00
15	RESIDUAL	School	-0.01	0.1824	0.02	-0.024	0.01
16	RESIDUAL	Employment	0.01	0.0215	0.03	-0.006	-0.02
17	RESIDUAL	Services	0.01	-0.0239	-0.01	0.202	0.03
18	RESIDUAL	HouseValue	0.00	0.0125	-0.02	0.034	0.12
19	PRETRANS	Factor1	0.79	-0.6145	.	.	.
20	PRETRANS	Factor2	0.61	0.7889	.	.	.
21	PREROTAT	Factor1	0.02	0.9042	0.15	0.791	0.94
22	PREROTAT	Factor2	0.99	0.0006	0.97	0.415	-0.00
23	TRANSFOR	Factor1	0.74	-0.7055	.	.	.
24	TRANSFOR	Factor2	0.54	0.8653	.	.	.
25	FCORR	Factor1	1.00	0.2019	.	.	.
26	FCORR	Factor2	0.20	1.0000	.	.	.
27	PATTERN	Factor1	-0.08	0.9184	0.05	0.761	0.96
28	PATTERN	Factor2	1.00	-0.0935	0.98	0.339	-0.10
29	RCORR	Factor1	1.00	-0.2019	.	.	.
30	RCORR	Factor2	-0.20	1.0000	.	.	.
31	REFERENC	Factor1	-0.08	0.8995	0.05	0.745	0.94
32	REFERENC	Factor2	0.98	-0.0916	0.96	0.332	-0.10
33	STRUCTUR	Factor1	0.12	0.8995	0.24	0.829	0.94
34	STRUCTUR	Factor2	0.99	0.0919	0.98	0.493	0.09

This output data set can be used for Harris-Kaiser rotation by deleting observations with `_TYPE_='PATTERN'` and `_TYPE_='FCORR'`, which are for the promax-rotated factors, and changing `_TYPE_='UNROTATE'` to `_TYPE_='PATTERN'`. In this way, the initial orthogonal factor pattern matrix is saved in the observations with `_TYPE_='PATTERN'`. The following factor analysis will then read in the factor pattern in the `fact2` data set as an initial factor solution, which will then be rotated by the Harris-Kaiser rotation with Cureton-Mulaik weights.

The following statements produce [Output 33.2.13](#):

```
data fact2(type=factor);
  set fact_all;
  if _TYPE_ in('PATTERN' 'FCORR') then delete;
  if _TYPE_='UNROTATE' then _TYPE_='PATTERN';

ods graphics on;

title3 'Harris-Kaiser Rotation with Cureton-Mulaik Weights';
proc factor rotate=hk norm=weight reorder
  plots=loadings;
run;

ods graphics off;
```

The results of the Harris-Kaiser rotation are displayed in [Output 33.2.13](#).

#### **Output 33.2.13** Harris-Kaiser Rotation

Five Socioeconomic Variables				
See Page 14 of Harman: Modern Factor Analysis, 3rd Ed				
Harris-Kaiser Rotation with Cureton-Mulaik Weights				
The FACTOR Procedure				
Rotation Method: Harris-Kaiser (hkpower = 0)				
Variable Weights for Rotation				
Population	School	Employment	Services	HouseValue
0.95982747	0.93945424	0.99746396	0.12194766	0.94007263
Oblique Transformation Matrix				
		1	2	
	1	0.73537	0.61899	
	2	-0.68283	0.78987	
Inter-Factor Correlations				
		Factor1	Factor2	
	Factor1	1.00000	0.08358	
	Factor2	0.08358	1.00000	

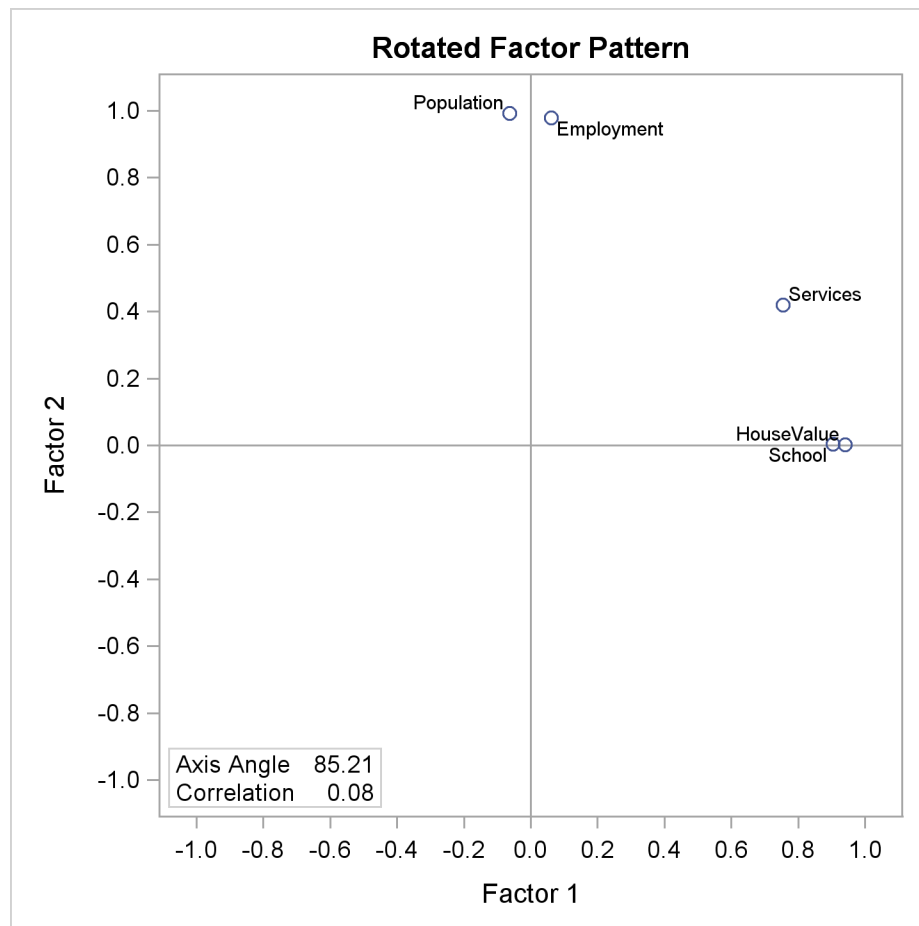
Output 33.2.13 *continued*

Rotated Factor Pattern (Standardized Regression Coefficients)				
	Factor1	Factor2		
HouseValue	0.94048	0.00279		
School	0.90391	0.00327		
Services	0.75459	0.41892		
Population	-0.06335	0.99227		
Employment	0.06152	0.97885		
Reference Axis Correlations				
	Factor1	Factor2		
Factor1	1.00000	-0.08358		
Factor2	-0.08358	1.00000		
Reference Structure (Semipartial Correlations)				
	Factor1	Factor2		
HouseValue	0.93719	0.00278		
School	0.90075	0.00326		
Services	0.75195	0.41745		
Population	-0.06312	0.98880		
Employment	0.06130	0.97543		
Variance Explained by Each Factor Eliminating Other Factors				
	Factor1	Factor2		
	2.2628537	2.1034731		
Factor Structure (Correlations)				
	Factor1	Factor2		
HouseValue	0.94071	0.08139		
School	0.90419	0.07882		
Services	0.78960	0.48198		
Population	0.01958	0.98698		
Employment	0.14332	0.98399		
Variance Explained by Each Factor Ignoring Other Factors				
	Factor1	Factor2		
	2.3468965	2.1875158		
Final Communality Estimates: Total = 4.450370				
Population	School	Employment	Services	HouseValue
0.97811334	0.81756387	0.97199928	0.79774304	0.88494998



A plot of the rotated loadings is shown in [Output 33.2.14](#).

**Output 33.2.14** Factor Pattern with Harris-Kaiser Rotation



In the results of the Harris-Kaiser rotation, the variable *Services* receives a small weight, and the axes are placed as desired.

### Example 33.3: Maximum Likelihood Factor Analysis

This example uses maximum likelihood factor analyses for one, two, and three factors. It is already apparent from the principal factor analysis that the best number of common factors is almost certainly two. The one- and three-factor ML solutions reinforce this conclusion and illustrate some of the numerical problems that can occur. The following statements produce [Output 33.3.1](#) through [Output 33.3.3](#):

```

title3 'Maximum Likelihood Factor Analysis with One Factor';
proc factor data=SocioEconomics method=ml heywood n=1;
run;

title3 'Maximum Likelihood Factor Analysis with Two Factors';
proc factor data=SocioEconomics method=ml heywood n=2;
run;

title3 'Maximum Likelihood Factor Analysis with Three Factors';
proc factor data=SocioEconomics method=ml heywood n=3;
run;

```

[Output 33.3.1](#) displays the results of the analysis with one factor.

#### Output 33.3.1 Maximum Likelihood Factor Analysis

Five Socioeconomic Variables				
See Page 14 of Harman: Modern Factor Analysis, 3rd Ed				
Maximum Likelihood Factor Analysis with One Factor				
The FACTOR Procedure				
Initial Factor Method: Maximum Likelihood				
Prior Communality Estimates: SMC				
Population	School	Employment	Services	HouseValue
0.96859160	0.82228514	0.96918082	0.78572440	0.84701921
Preliminary Eigenvalues: Total = 76.1165859 Average = 15.2233172				
	Eigenvalue	Difference	Proportion	Cumulative
1	63.7010086	50.6462895	0.8369	0.8369
2	13.0547191	12.7270798	0.1715	1.0084
3	0.3276393	0.6749199	0.0043	1.0127
4	-0.3472805	0.2722202	-0.0046	1.0081
5	-0.6195007		-0.0081	1.0000

Output 33.3.1 *continued*

1 factor will be retained by the NFACTOR criterion.

Iteration	Criterion	Ridge	Change	Communalities			
1	6.5429218	0.0000	0.1033	0.93828	0.72227	1.00000	0.71940
				0.74371			
2	3.1232699	0.0000	0.7288	0.94566	0.02380	1.00000	0.26493
				0.01487			

Convergence criterion satisfied.

#### Significance Tests Based on 12 Observations

Test	DF	Chi-Square	Pr > ChiSq
H0: No common factors	10	54.2517	<.0001
HA: At least one common factor			
H0: 1 Factor is sufficient	5	24.4656	0.0002
HA: More factors are needed			

Chi-Square without Bartlett's Correction	34.355969
Akaike's Information Criterion	24.355969
Schwarz's Bayesian Criterion	21.931436
Tucker and Lewis's Reliability Coefficient	0.120231

#### Squared Canonical Correlations

Factor1

1.0000000

Eigenvalues of the Weighted Reduced Correlation Matrix: Total = 0 Average = 0

	Eigenvalue	Difference
1	Infty	Infty
2	1.92716032	2.15547340
3	-.22831308	0.56464322
4	-.79295630	0.11293464
5	-.90589094	

#### Factor Pattern

	Factor1
Population	0.97245
School	0.15428
Employment	1.00000
Services	0.51472
HouseValue	0.12193

**Output 33.3.1** *continued*

Variance Explained by Each Factor		
Factor	Weighted	Unweighted
Factor1	17.8010629	2.24926004
Final Communality Estimates and Variable Weights		
Total Communality: Weighted = 17.801063    Unweighted = 2.249260		
Variable	Communality	Weight
Population	0.94565561	18.4011648
School	0.02380349	1.0243839
Employment	1.00000000	Infty
Services	0.26493499	1.3604239
HouseValue	0.01486595	1.0150903

The solution on the second iteration is so close to the optimum that PROC FACTOR cannot find a better solution; hence you receive this message:

**Convergence criterion satisfied.**

When this message appears, you should try rerunning PROC FACTOR with different prior communality estimates to make sure that the solution is correct. In this case, other prior estimates lead to the same solution or possibly to worse local optima, as indicated by the information criteria or the chi-square values.

The variable Employment has a communality of 1.0 and, therefore, an infinite weight that is displayed next to the final communality estimate as a missing/infinite value. The first eigenvalue is also infinite. Infinite values are ignored in computing the total of the eigenvalues and the total final communality.

**Output 33.3.2** displays the results of the analysis with two factors. The analysis converges without incident. This time, however, the Population variable is a Heywood case.

**Output 33.3.2** Maximum Likelihood Factor Analysis: Two Factors

Five Socioeconomic Variables				
See Page 14 of Harman: Modern Factor Analysis, 3rd Ed				
Maximum Likelihood Factor Analysis with Two Factors				
The FACTOR Procedure				
Initial Factor Method: Maximum Likelihood				
Prior Communality Estimates: SMC				
Population	School	Employment	Services	HouseValue
0.96859160	0.82228514	0.96918082	0.78572440	0.84701921

## Output 33.3.2 continued

Preliminary Eigenvalues: Total = 76.1165859    Average = 15.2233172				
	Eigenvalue	Difference	Proportion	Cumulative
1	63.7010086	50.6462895	0.8369	0.8369
2	13.0547191	12.7270798	0.1715	1.0084
3	0.3276393	0.6749199	0.0043	1.0127
4	-0.3472805	0.2722202	-0.0046	1.0081
5	-0.6195007		-0.0081	1.0000
2 factors will be retained by the NFACTOR criterion.				
Iteration	Criterion	Ridge	Change	Communalities
1	0.3431221	0.0000	0.0471	1.00000 0.89412
2	0.3072178	0.0000	0.0307	1.00000 0.92480
3	0.3067860	0.0000	0.0063	1.00000 0.92023
4	0.3067373	0.0000	0.0022	1.00000 0.92241
5	0.3067321	0.0000	0.0007	1.00000 0.92187
Convergence criterion satisfied.				
Significance Tests Based on 12 Observations				
	Test	DF	Chi-Square	Pr > ChiSq
H0:	No common factors	10	54.2517	<.0001
HA:	At least one common factor			
H0:	2 Factors are sufficient	1	2.1982	0.1382
HA:	More factors are needed			
	Chi-Square without Bartlett's Correction		3.3740530	
	Akaike's Information Criterion		1.3740530	
	Schwarz's Bayesian Criterion		0.8891463	
	Tucker and Lewis's Reliability Coefficient		0.7292200	
Squared Canonical Correlations				
	Factor1	Factor2		
	1.0000000	0.9518891		

Output 33.3.2 *continued*

Eigenvalues of the Weighted Reduced Correlation Matrix: Total = 19.7853157    Average = 4.94632893				
	Eigenvalue	Difference	Proportion	Cumulative
1	Infty	Infty		
2	19.7853143	19.2421292	1.0000	1.0000
3	0.5431851	0.5829564	0.0275	1.0275
4	-0.0397713	0.4636411	-0.0020	1.0254
5	-0.5034124		-0.0254	1.0000
Factor Pattern				
		Factor1	Factor2	
	Population	1.00000	0.00000	
	School	0.00975	0.90003	
	Employment	0.97245	0.11797	
	Services	0.43887	0.78930	
	HouseValue	0.02241	0.95989	
Variance Explained by Each Factor				
	Factor	Weighted	Unweighted	
	Factor1	24.4329707	2.13886057	
	Factor2	19.7853143	2.36835294	
Final Communality Estimates and Variable Weights				
Total Communality: Weighted = 44.218285    Unweighted = 4.507214				
	Variable	Communality	Weight	
	Population	1.00000000	Infty	
	School	0.81014489	5.2682940	
	Employment	0.95957142	24.7246669	
	Services	0.81560348	5.4256462	
	HouseValue	0.92189372	12.7996793	

The results of the three-factor analysis are shown in [Output 33.3.3](#).

ERROR: Converged, but not to a proper optimum.

## Output 33.3.3 continued

Try a different 'PRIORS' statement.

## Significance Tests Based on 12 Observations

Test	DF	Chi-Square	Pr > ChiSq
H0: No common factors	10	54.2517	<.0001
HA: At least one common factor			
H0: 3 Factors are sufficient	-2	0.0000	.
HA: More factors are needed			

Chi-Square without Bartlett's Correction	0.0000003
Akaike's Information Criterion	4.0000003
Schwarz's Bayesian Criterion	4.9698136
Tucker and Lewis's Reliability Coefficient	0.0000000

## Squared Canonical Correlations

Factor1	Factor2	Factor3
1.0000000	0.9751895	0.6894465

Eigenvalues of the Weighted Reduced Correlation  
Matrix: Total = 41.5254193 Average = 10.3813548

	Eigenvalue	Difference	Proportion	Cumulative
1	Infty	Infty		
2	39.3054826	37.0854258	0.9465	0.9465
3	2.2200568	2.2199693	0.0535	1.0000
4	0.0000875	0.0002949	0.0000	1.0000
5	-0.0002075		-0.0000	1.0000

## Factor Pattern

	Factor1	Factor2	Factor3
Population	0.97245	-0.11233	-0.15409
School	0.15428	0.89108	0.26083
Employment	1.00000	0.00000	0.00000
Services	0.51472	0.72416	-0.12766
HouseValue	0.12193	0.97227	-0.08473

## Variance Explained by Each Factor

Factor	Weighted	Unweighted
Factor1	54.6115241	2.24926004
Factor2	39.3054826	2.27634375
Factor3	2.2200568	0.11525433



**Output 33.3.3** *continued*

Final Communalities Estimates and Variable Weights		
Total Communality: Weighted = 96.137063    Unweighted = 4.640858		
Variable	Communality	Weight
Population	0.98201660	55.6066901
School	0.88585165	8.7607194
Employment	1.00000000	Infty
Services	0.80564301	5.1444261
HouseValue	0.96734687	30.6251078

In the results, a warning message is displayed:

**WARNING: Too many factors for a unique solution.**

The number of parameters in the model exceeds the number of elements in the correlation matrix from which they can be estimated, so an infinite number of different perfect solutions can be obtained. The criterion approaches zero at an improper optimum, as indicated by this message:

**Converged, but not to a proper optimum.**

The degrees of freedom for the chi-square test are  $-2$ , so a probability level cannot be computed for three factors. Note also that the variable Employment is a Heywood case again.

The probability levels for the chi-square test are 0.0001 for the hypothesis of no common factors, 0.0002 for one common factor, and 0.1382 for two common factors. Therefore, the two-factor model seems to be an adequate representation. Akaike's information criterion and Schwarz's Bayesian criterion attain their minimum values at two common factors, so there is little doubt that two factors are appropriate for these data.

### Example 33.4: Using Confidence Intervals to Locate Salient Factor Loadings

This example illustrates how you can use the standard errors and confidence intervals to understand the pattern of factor loadings under the maximum likelihood estimation. There are nine tests and you want a three-factor solution ( $N=3$ ) for a correlation matrix based on 200 observations. The following statements define the input data set and specify the desirable analysis by the FACTOR procedure:

```
data test(type=corr);
  title 'Quartimin-Rotated Factor Solution with Standard Errors';
  input _name_ $ test1-test9;
  _type_ = 'corr';
  datalines;
Test1      1   .561   .602   .290   .404   .328   .367   .179  -.268
Test2   .561      1   .743   .414   .526   .442   .523   .289  -.399
Test3   .602   .743      1   .286   .343   .361   .679   .456  -.532
Test4   .290   .414   .286      1   .677   .446   .412   .400  -.491
Test5   .404   .526   .343   .677      1   .584   .408   .299  -.466
Test6   .328   .442   .361   .446   .584      1   .333   .178  -.306
Test7   .367   .523   .679   .412   .408   .333      1   .711  -.760
Test8   .179   .289   .456   .400   .299   .178   .711      1  -.725
Test9  -.268  -.399  -.532  -.491  -.466  -.306  -.760  -.725      1
;

title2 'A nine-variable-three-factor example';
proc factor data=test method=ml reorder rotate=quartimin
  nobs=200 n=3 se cover=.45 alpha=.1;
run;
```

In the PROC FACTOR statement, you apply quartimin rotation with (default) Kaiser normalization. You define loadings with magnitudes greater than 0.45 to be salient (**COVER=0.45**) and use 90% confidence intervals (**ALPHA=0.1**) to judge the salience. The **REORDER** option is specified so that variables that have similar loadings with factors are clustered together.

After the quartimin rotation, the correlation matrix for factors is shown in [Output 33.4.1](#).

**Output 33.4.1** Quartimin-Rotated Factor Solution with Standard Errors

Quartimin-Rotated Factor Solution with Standard Errors A nine-variable-three-factor example  The FACTOR Procedure Rotation Method: Quartimin  Inter-Factor Correlations With 90% confidence limits Estimate/StdErr/LowerCL/UpperCL			
	Factor1	Factor2	Factor3
Factor1	1.00000	0.41283	0.38304
	0.00000	0.06267	0.06060
	.	0.30475	0.27919
	.	0.51041	0.47804
Factor2	0.41283	1.00000	0.47006
	0.06267	0.00000	0.05116
	0.30475	.	0.38177
	0.51041	.	0.54986
Factor3	0.38304	0.47006	1.00000
	0.06060	0.05116	0.00000
	0.27919	0.38177	.
	0.47804	0.54986	.

The factors are medium to highly correlated. The confidence intervals seem to be very wide, suggesting that the estimation of factor correlations might not be very accurate for this sample size. For example, the 90% confidence interval for the correlation between Factor1 and Factor2 is (0.30, 0.51), a range of 0.21. You might need a larger sample to get a narrower interval, or you might need a better estimation.

Next, coverage displays for factor loadings are shown in [Output 33.4.2](#).

**Output 33.4.2** Using the Rotated Factor Pattern to Interpret the Factors

Rotated Factor Pattern (Standardized Regression Coefficients)  
 With 90% confidence limits; Cover  $|*| = 0.45?$   
 Estimate/StdErr/LowerCL/UpperCL/Coverage Display

	Factor1	Factor2	Factor3
test8	0.86810	-0.05045	0.00114
	0.03282	0.03185	0.03087
	0.80271	-0.10265	-0.04959
	0.91286	0.00204	0.05187
	0*[]	*[0]	[0]*
test7	0.73204	0.27296	0.01098
	0.04434	0.05292	0.03838
	0.65040	0.18390	-0.05211
	0.79697	0.35758	0.07399
	0*[]	0[]*	[0]*
test9	-0.79654	-0.01230	-0.17307
	0.03948	0.04225	0.04420
	-0.85291	-0.08163	-0.24472
	-0.72180	0.05715	-0.09955
	[]*0	*[0]	*[]0
test3	0.27715	0.91156	-0.19727
	0.05489	0.04877	0.02981
	0.18464	0.78650	-0.24577
	0.36478	0.96481	-0.14778
	0[]*	0*[]	*[]0
test2	0.01063	0.71540	0.20500
	0.05060	0.05148	0.05496
	-0.07248	0.61982	0.11310
	0.09359	0.79007	0.29342
	[0]*	0*[]	0[]*
test1	-0.07356	0.63815	0.13983
	0.04245	0.05380	0.05597
	-0.14292	0.54114	0.04682
	-0.00348	0.71839	0.23044
	*[]0	0*[]	0[]*
test5	0.00863	0.03234	0.91282
	0.04394	0.04387	0.04509
	-0.06356	-0.03986	0.80030
	0.08073	0.10421	0.96323
	[0]*	[0]*	0*[]
test4	0.22357	-0.07576	0.67925
	0.05956	0.03640	0.05434
	0.12366	-0.13528	0.57955
	0.31900	-0.01569	0.75891
	0[]*	*[]0	0*[]
test6	-0.04295	0.21911	0.53183
	0.05114	0.07481	0.06905
	-0.12656	0.09319	0.40893
	0.04127	0.33813	0.63578
	*[0]	0[]*	0[*]

The coverage displays in [Output 33.4.2](#) show that Test8, Test7, and Test9 have salient relationships with Factor1. The coverage displays are either '0\*[ ]' or '[ ]\*0', indicating that the entire 90% confidence intervals for the corresponding loadings are beyond the salience value at 0.45. On the other hand, the coverage display for Test3 on Factor1 is '0[ ]\*'. This indicates that even though the loading estimate is significantly larger than zero, it is not large enough to be salient. Similarly, Test3, Test2, and Test1 have salient relationships with Factor2, while Test5 and Test4 have salient relationships with Factor3. For Test6, its relationship with Factor3 is a little bit ambiguous; the 90% confidence interval approximately covers values between 0.40 and 0.64. This means that the population value might have been smaller or larger than 0.45. It is marginal evidence for a salient relationship.

For oblique factor solutions, some researchers prefer to examine the factor structure loadings, which represent correlations, for determining salient relationships. In [Output 33.4.3](#), the factor structure loadings and the associated standard error estimates and coverage displays are shown.

**Output 33.4.3** Using the Factor Structure to Interpret the Factors

Factor Structure (Correlations)			
With 90% confidence limits; Cover  *  = 0.45?			
Estimate/StdErr/LowerCL/UpperCL/Coverage Display			
	Factor1	Factor2	Factor3
test8	0.84771	0.30847	0.30994
	0.02871	0.06593	0.06263
	0.79324	0.19641	0.20363
	0.88872	0.41257	0.40904
	0*[]	0[*]*	0[*]*
test7	0.84894	0.58033	0.41970
	0.02688	0.05265	0.06060
	0.79834	0.48721	0.31523
	0.88764	0.66041	0.51412
	0*[]	0*[]	0[*]
test9	-0.86791	-0.42248	-0.48396
	0.02522	0.06187	0.05504
	-0.90381	-0.51873	-0.56921
	-0.81987	-0.31567	-0.38841
	[]*0	[*]0	[*]0
test3	0.57790	0.93325	0.33738
	0.05069	0.02953	0.06779
	0.48853	0.86340	0.22157
	0.65528	0.96799	0.44380
	0*[]	0*[]	0[*]*
test2	0.38449	0.81615	0.54535
	0.06143	0.03106	0.05456
	0.27914	0.75829	0.44946
	0.48070	0.86126	0.62883
	0[*]	0*[]	0[*]
test1	0.24345	0.67351	0.41162
	0.06864	0.04284	0.05995
	0.12771	0.59680	0.30846
	0.35264	0.73802	0.50522
	0[*]*	0*[]	0[*]
test5	0.37163	0.46498	0.93132
	0.06092	0.04979	0.03277
	0.26739	0.37923	0.85159
	0.46727	0.54282	0.96894
	0[*]	0[*]	0*[]
test4	0.45248	0.33583	0.72927
	0.05876	0.06289	0.04061
	0.35072	0.22867	0.65527
	0.54367	0.43494	0.78941
	0[*]	0[*]*	0*[]
test6	0.25122	0.45137	0.61837
	0.07140	0.05858	0.05051
	0.13061	0.34997	0.52833
	0.36450	0.54232	0.69465
	0[*]*	0[*]	0*[]

The interpretations based on the factor structure matrix do not change much from that based on the factor loadings except for Test3 and Test9. Test9 now has a salient correlation with Factor3. For Test3, it has salient correlations with both Factor1 and Factor2. Fortunately, there are still tests that have salient correlations only with either Factor1 or Factor2 (but not both). This would make interpretations of factors less problematic.

---

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