ISI-Free FIR Filterbank Transceivers for Frequency-Selective Channels

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***Abstract*—Discrete multitone modulation transceivers (DMTs) have been shown to be very useful for data transmission over frequency selective channels. The DMT scheme is realized by a transceiver that divides the channel into subbands. The efficiency of the scheme depends on the frequency selectivity of the transmitting and receiving filters. The receiving filters with good stopband attenuation are also desired for combating narrowband noise. The filterbank transceiver or discrete wavelet multitone (DWMT) system has been proposed as an implementation of the DMT transceiver that has better frequency band separation, but usually, inrtersymbol interference (ISI) cannot be completely canceled in these filterbank transceivers, and additional equalization is required. In this paper, we show how to use over interpolated filterbanks to design ISI-free FIR transceivers. A finite impulse response (FIR) transceiver with good frequency selectivity can be designed, as will be demonstrated by design examples.**

I. Introduction

DISCRETE multitone modulation (DMT) is now a widely used technique for high-speed transmission over channels such as digital subscriber loops [1]–[5]. In the DMT scheme, the channel is divided into subbands, each with a different frequency band. The transmission power and bits are judiciously allocated according to the signal-to-noise ratio (SNR) in each band [4]. This is similar to the water pouring scheme for discrete transmission channels. The realization of the DMT scheme relies on the design of a transceiver that effectively divides the channel into subbands. Band separation is of particular importance when the SNR's of different bands exhibit large differences. This can happen when the channel or the channel noise is highly frequency selective or nonflat.

The DFT-based DMT system has been proposed as a practical implementation of DMT system [2], [5]. A certain redundancy known as cyclic prefix is added to allow complete removal of intersymbol interference (ISI). Very good transmission rate can be accomplished using DFT-based DMT systems for channels such as the asymmetric digital subscriber line (ADSL) and the high bit rate digital subscriber line (HDSL). In the DFT-based systems, the transmitting filters $F_k(z)$ and receiving filters $H_k(z)$ in Fig. 1 are DFT filters. The DFT filters have limited frequency selectivity (stopband attenuation around 13 dB). Narrowband noise could induce serious impairment due to poor stopband [6]. The DFT-based systems fall into the category of block-based DMT transceivers, where the transmitter and receiver consist of constant matrices. In this case, the filters have length $\leq$ the interpolation ratio N. The filter-length constraint imposes limits on the stopband attenuation of the filter in the block-based DMT transceivers.

For better band separation, Sandberg and Tzannes [7] proposed the so-called discrete wavelet multitone (DWMT) system, in which perfect reconstruction filter banks are used as the transceiver. The transmitting and receiving filters have excellent frequency separation property inherited from good filterbank designs. Connection between an M-band filterbank and an M-band transmultiplexer (an M-band filterbank transceiver or DWMT system) was first observed by Vetterli in [9]. When the analysis and synthesis bank banks of a perfect reconstruction filterbank are interchanged, the new structure becomes a transmultiplexer or a filterbank transceiver (see Fig. 1). The DMWT system in this case has interpolation ratio $N = M$, and it is called *minimally interpolated*. When the transmission channel is ideal, the minimally interpolated M-subband filterbank transceiver is ISI free if the corresponding filterbank has perfect reconstruction [8]. The ISI-free property means there is no intra-subband and inter-subband ISI. However, when the channel is not ideal, the perfect reconstruction property of the filterbank no longer translates to an ISI-free property of filterbank transceivers. Performance evaluation conducted in [9] and [10] shows that the resulting ISI can seriously degrade the system performance. To reduce the amount of ISI, inter-subband and intra-subband equalization are performed on the receiver outputs in [7]–[11].

When the interpolation ratio $N > M$, the filterbank transceiver is called *over interpolated*; in average every N output samples of the transmitter contains $K = N - M$ redundant samples. The cyclic prefix in DFT based DMT system is an example of such redundant samples. Advances to the non block-based FIR over interpolated system has been made in [12] and [13] for ISI cancellation using precoding. The development is made in the context of underdecimated filterbanks. It is shown therein that we can use redundancy $K = 1$, except in pathological cases. Fundamentals and many useful properties for over interpolated class are derived In [13]. The FIR DMT transceivers are considered in a more general framework in [14]. Time-varying systems are employed in designing FIR equalizers. Suppose the channel is of order L with distinct roots and that the interpolation ratio N and number of bands M satisfy $N, M > L$. It is shown that [14] we can always find a channel-independent

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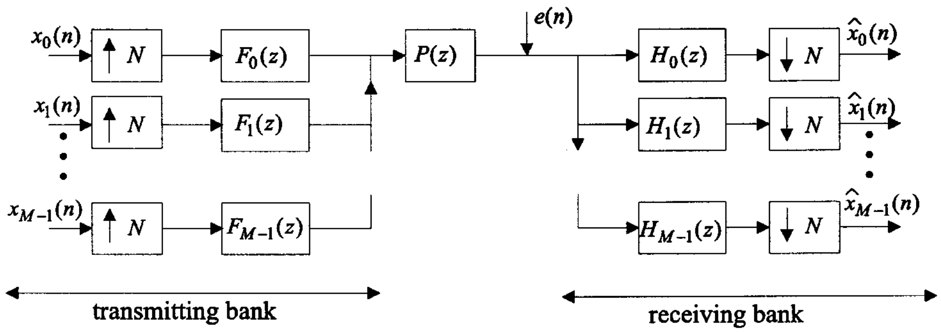

Fig. 1. M-subband filterbank transceiver over a fading channel $P(z)$.

time-varying transmitter such that FIR time-varying receivers exist. In particular, redundancy of one can be used as long as $N-1 > L$ and the time-varying receiving filters are sufficiently long.

In many cases, the statistics of the channel noise is incorporated in the design. For example, in [15], Kasturia *et al.* extend the DFT-based transceiver to a more general vector coding system. The transmitting filters or transmitting vectors are eigenvectors of an appropriately defined channel matrix. When the channel noise is AWGN, the vector coding is shown to be optimal in terms of bit rate maximization subject to a transmission power budget. Optimal DMT transceivers maximizing the total SNR are designed in [14]. Bit rate maximization for general noise sources is considered in [16] and [17]. Blind equalization for block-based DMT transceivers are developed in [18].

In this paper, we will develop design methods for ISI-free FIR filterbank transceivers with effective band separation. We will use overinterpolated filterbanks to introduce redundancy. The introduced redundancy enables us to cancel ISI *completely*. Two methods will be proposed for designing FIR transceivers with zero ISI. They are based on two classes of FIR systems with FIR inverses: the orthogonal matrices and unimodular matrices. For a given channel, the filters are optimized subject to the condition that ISI be canceled. The noise statistics are not considered; there is no need to estimate the noise spectrum. However, the ISI cancellation property and the band separation property provided by the transceivers facilitate the realization of the DMT scheme. Examples will be given to demonstrate that the performance of FIR filterbank transceivers is comparable to or better than that of DFT-based DMT systems. The FIR filterbank transceivers perform significantly better than the DFT-based system when the noise is narrowband.

The sections are organized as follows. In Section II, a polyphase framework of the filterbank transceiver is presented. Using the framework, we show that the transmitting and receiving filters can be interchanged, and the ISI free property is preserved. A class of FIR transceivers with an ISI-free property is developed in Section III using the polyphase approach. The development is based on FIR systems with FIR inverses. This class will be used in Section IV for designing FIR transceivers. Two types of FIR systems with FIR inverses are used: orthogonal matrices (Section IV-A) and unimodular matrices (Section IV-B). Receivers with minimum mean squared error for orthogonal transmitters are given in Section V.

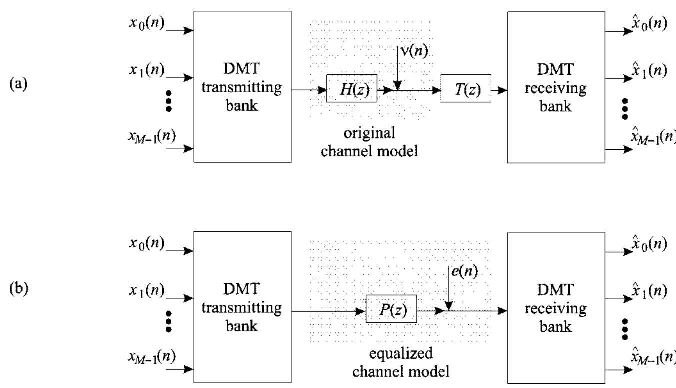

Fig. 2. (a) Block diagram of the filterbank transceiver, including a discrete time channel model and an equalizer $T(z)$. (b) Block diagram of the filterbank transceiver with an equalized channel model.

A. Notations and Preliminaries

- Boldfaced lower-case letters are used to represent vectors, and boldfaced upper case letters are reserved for matrices. The notations $\mathbf{A}^T$ and $\mathbf{A}^\dagger$ represent the transpose of $\mathbf{A}$ and transpose-conjugate of $\mathbf{A}$.
- The notation $\tilde{\mathbf{A}}(z)$ denotes $\mathbf{A}^\dagger(1/z^*)$. For matrices with real coefficients, $\tilde{\mathbf{A}}(z) = \mathbf{A}^T(z^{-1})$.
- The function $\mathcal{E}[y]$ denotes the expected value of the random variable y.
- The notation $\mathbf{I}_N$ is used to represent the $N \times N$ identity matrix. The subscript is omitted whenever the size is clear from the context. The notation $\mathbf{J}_N$ denotes the $N \times N$ reversal matrix. For example, a 3×3 reversal matrix is given by

$$\mathbf{J}_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

- *Unimodular Matrices.* An $N \times N$ matrix $\mathbf{A}(z)$ is called unimodular if $\det \mathbf{A}(z) = c$, which is a constant [20]. A causal unimodular FIR matrix $\mathbf{A}(z)$ has the property that $\mathbf{A}^{-1}(z)$ is also causal and FIR.

B. Channel Models

Fig. 2(a) shows the block diagram of a filterbank transceiver. The discrete time channel is modeled as an LTI filter $h(n)$ with additive noise $\nu(n)$, as shown in Fig. 2(a). A time domain equalizer (TEQ) $T(z)$ precedes the filterbank receiver. Typically, the

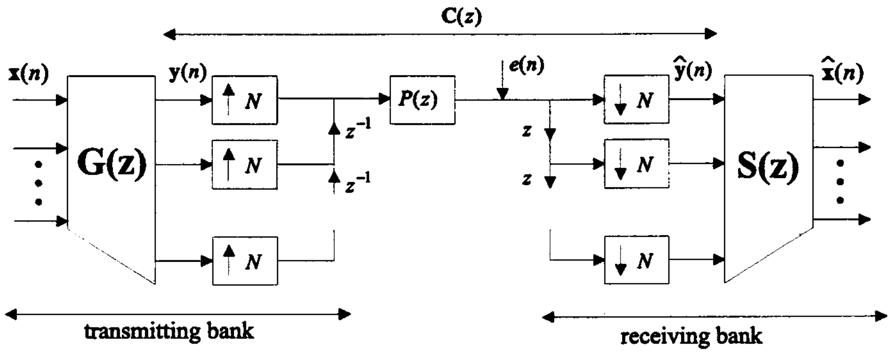

Fig. 3. Polyphase representation of the transmitter and receiver in a filterbank transceiver.

filter $H(z)$ can be further modeled as a rational transfer function $H(z) = P(z)/B(z)$. The equalizer $T(z)$ is usually designed to cancel the poles of $H(z)$, and the resulting overall transfer function becomes the FIR filter $P(z)$, as shown in Fig. 2(b). Suppose $P(z)$ is of order L and that

$$P(z) = p_0 + p_1 z^{-1} + \cdots + p_L z^{-L}.$$

The equalized impulse response of the channel is thus shortened to L. Each input sample will be spread to a duration of length $L+1$ as a result. The noise $e(n)$ shown in Fig. 2(b) is obtained by feeding the original noise $\nu(n)$ to the equalizer $T(z)$. The equalized channel model in Fig. 2(b) will be used throughout this paper; the channel refers to the equalized channel $P(z)$, and the channel noise refers to the noise $e(n)$ at the equalizer output in this paper.

II. Polyphase Representation of Filterbank Transceivers

Consider Fig. 1, where an M-subband filterbank transceiver is shown. The channel is represented by an FIR filter $P(z)$ with additive noise $e(n)$, as explained in Section I-B. The filters $F_k(z)$ and $H_k(z)$ are called transmitting and receiving filters, respectively. When $N > M$, we say the system is over interpolated and redundancy $K = N - M$.

Using polyphase decomposition, we can decompose the kth transmitting filter $F_k(z)$ with respect to the integer N [20]

$$F_k(z) = \sum_{n=0}^{N-1} G_{n,k}(z^N) z^{-n}. \tag{1}$$

Writing the polyphase representation for all the M transmitting filters, we have (2), shown at the bottom of the page, where the $N \times M$ matrix $\mathbf{G}(z)$ is the polyphase matrix of the transmitter. Using the noble identity [20], we can interchange the expander and $\mathbf{G}(z^N)$. The transmitter can be implemented using its polyphase matrix, as shown in Fig. 3. In a similar manner, we can decompose the receiving filters as

$$H_k(z) = \sum_{n=0}^{N-1} S_{k,n}(z^N) z^n. \tag{3}$$

Then, by invoking the noble identity, the receiver can be redrawn as Fig. 3. The receiving filters $H_k(z)$ are related to the $M \times N$ polyphase matrix $\mathbf{S}(z)$ of the receiver as

$$\begin{pmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{M-1}(z) \end{pmatrix} = \underbrace{\begin{pmatrix} S_{0,0}(z^N) & S_{0,1}(z^N) & \cdots & S_{0,N-1}(z^N) \\ S_{1,0}(z^N) & S_{1,1}(z^N) & \cdots & S_{1,N-1}(z^N) \\ \vdots & \vdots & \ddots & \vdots \\ S_{M-1,0}(z^N) & S_{M-1,1}(z^N) & \cdots & S_{M-1,N-1}(z^N) \end{pmatrix}}_{\mathbf{S}(z^N)} \times \begin{pmatrix} 1 \\ z \\ \vdots \\ z^{N-1} \end{pmatrix}. \tag{4}$$

A. Decomposition of the Channel

Using polyphase representation, we can decompose the channel as

$$P(z) = P_0(z^N) + P_1(z^N) z^{-1} + \cdots + P_{N-1}(z^N) z^{-N+1}. \tag{5}$$

$$[F_0(z) \quad F_1(z) \quad \cdots \quad F_{M-1}(z)] = [1 \quad z^{-1} \quad \cdots \quad z^{-N+1}] \underbrace{\begin{pmatrix} G_{0,0}(z^N) & G_{0,1}(z^N) & \cdots & G_{0,M-1}(z^N) \\ G_{1,0}(z^N) & G_{1,1}(z^N) & \cdots & G_{1,M-1}(z^N) \\ \vdots & \vdots & \ddots & \vdots \\ G_{N-1,0}(z^N) & G_{N-1,1}(z^N) & \cdots & G_{N-1,M-1}(z^N) \end{pmatrix}}_{\mathbf{G}(z^N)} \tag{2}$$

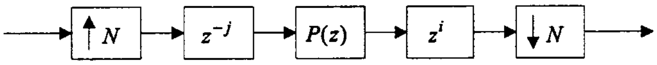

Fig. 4. Polyphase identity.

In order to further simplify Fig. 3, we need to apply an identity from the multirate theory. It is shown in [20] that the multirate system in Fig. 4 is, in fact, equivalent to an LTI system with transfer function $A(z)$ given by

$$A(z) = \begin{cases} P_{i-j}(z), & \text{for } i \geq j \\ z^{-1}P_{N+i-j}(z), & \text{for } i < j \end{cases}$$

where $P_k(z)$ is defined in (5). We see that the $N \times N$ system from $\mathbf{y}(n)$ to $\hat{\mathbf{y}}(n)$ in Fig. 3 is in fact an LTI system with transfer matrix $\mathbf{C}(z)$ given by (6), shown at the bottom of the page. Matrices in the above form are known as pseudocirculant matrices [20]. A first detailed study of pseudocirculant matrices was made in [21]. Many useful properties, as well as applications of pseudocirculant matrices in QMF banks and block filtering, are given therein.

Usually, the interpolation ratio N is chosen to be larger than the order L of $P(z)$. In this case, the N polyphases $P_i(z)$ in (5) are constants, and the last $N-L-1$ polyphases are zero. The matrix $\mathbf{C}(z)$ is causal, and of order one

$$\mathbf{C}(z) = \mathbf{C}_0 + z^{-1}\mathbf{C}_1 \tag{7}$$

where

$$\mathbf{C}_0 = \begin{pmatrix} p_0 & 0 & \cdots & \cdots & 0 \\ p_1 & p_0 & & & 0 \\ \vdots & & \ddots & & \vdots \\ p_L & p_{L-1} & & & \\ 0 & p_L & & & \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & & & p_0 \end{pmatrix} \text{ and}$$

$$\mathbf{C}_1 = \begin{pmatrix} 0 & \cdots & 0 & p_L & p_{L-1} & \cdots & p_1 \\ 0 & & 0 & 0 & p_L & \cdots & p_2 \\ \vdots & & \vdots & \vdots & & \ddots & \vdots \\ & & & & & & p_L \\ & & & & & & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}.$$

The matrices $\mathbf{C}_0$ and $\mathbf{C}_1$ are both $N \times N$ and Toeplitz; $\mathbf{C}_0$ is lower triangular, and $\mathbf{C}_1$ is upper triangular. Equivalently, the

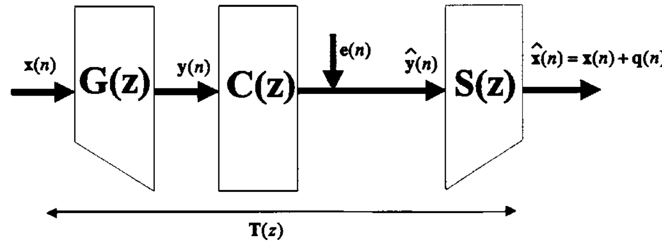

Fig. 5. Polyphase representation of a filterbank transceiver.

matrix $\mathbf{C}(z)$ can be partitioned as an $N \times (N-L)$ constant matrix $\mathbf{P}_0$ and an $N \times L$ FIR causal matrix $\mathbf{P}_1(z)$ that is of order 1

$$\mathbf{C}(z) = \begin{bmatrix} \underbrace{\mathbf{P}_0}_{N\times(N-L)} & \vdots \underbrace{\mathbf{P}_1(z)}_{N\times L} \end{bmatrix}. \tag{8}$$

Using the channel matrix $\mathbf{C}(z)$, we can redraw Fig. 3 as Fig. 5. As we will see later, the polyphase representation in Fig. 5 will facilitate a systematic study of filterbank transceivers.

Zero ISI Condition: From the polyphase decomposition in Fig. 5, we see that even though multirate building blocks are used in a filterbank transceiver, it is in fact an LTI system with M inputs and M outputs. The transfer matrix $\mathbf{T}(z)$ of the overall system can be expressed as

$$\mathbf{T}(z) = \mathbf{S}(z)\mathbf{C}(z)\mathbf{G}(z). \tag{9}$$

The overall system is free from inter-subband ISI if $\mathbf{T}(z)$ is a diagonal matrix. It is free from intra-subband ISI when the diagonal elements of $\mathbf{T}(z)$ are merely delays. If it is free from both inter-subband and intra-subband ISI, we say that the filterbank transceiver is ISI free; in the absence of channel noise, the outputs of an ISI-free filterbank transceiver are identical to the inputs except delays and scalars. Without much loss of generality, we can use the ISI-free condition

$$\mathbf{S}(z)\mathbf{C}(z)\mathbf{G}(z) = \mathbf{I}. \tag{10}$$

B. Interchange of the Transmitting and Receiving Filters

Using the polyphase framework, we can immediately show that the transmitting and receiving filters can be exchanged, and ISI-free property is preserved. To see this, observe that the matrix $\mathbf{C}(z)$ is Toeplitz, and it satisfies

$$\mathbf{C}^T(z) = \mathbf{J}_N\mathbf{C}(z)\mathbf{J}_N \tag{11}$$

$$\mathbf{C}(z) = \begin{pmatrix} P_0(z) & z^{-1}P_{N-1}(z) & z^{-1}P_{N-2}(z) & \cdots & z^{-1}P_1(z) \\ P_1(z) & P_0(z) & z^{-1}P_{N-1}(z) & \cdots & z^{-1}P_2(z) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{N-1}(z) & P_{N-2}(z) & P_{N-3}(z) & \cdots & P_0(z) \end{pmatrix} \tag{6}$$

where $\mathbf{J}_N$ is the $N \times N$ reversal matrix defined in Section I. Taking transpose of the both sides of (9) and using (11), we have

$$\mathbf{T}^T(z) = \underbrace{z^{-(N-1)}\mathbf{G}^T(z)\mathbf{J}_N}_{\mathbf{S}'(z)} \mathbf{C}(z) \underbrace{\mathbf{J}_N \mathbf{S}^T(z) z^{N-1}}_{\mathbf{G}'(z)}. \quad (12)$$

From the above equation, we can conclude the following: If the filterbank transceiver with $\mathbf{G}(z)$ and $\mathbf{S}(z)$ as the transmitter and receiver, respectively, is ISI free, then the filterbank transceiver with $\mathbf{G}'(z)$ and $\mathbf{S}'(z)$ as the transmitter and receiver, respectively, will also be ISI free, where $\mathbf{G}'(z)$ and $\mathbf{S}'(z)$ are as given in (12). Using the polyphase representation, the new transmitting filters can be expressed as

$$\begin{aligned}
&(F'_0(z) \quad F'_1(z) \quad \cdots \quad F'_{M-1}(z)) \\
&\quad = (1 \quad z^{-1} \quad \cdots \quad z^{-(N-1)}) z^{N-1} \mathbf{J}_N \mathbf{S}^T(z^N) \\
&\quad = (1 \quad z \quad \cdots \quad z^{N-1}) \mathbf{S}^T(z^N) \\
&\quad = (H_0(z) \quad H_1(z) \quad \cdots \quad H_{M-1}(z)).
\end{aligned}$$

Therefore, we have new transmitting filters $F'_k(z) = H_k(z)$. Similarly, we can show that the new receiving filters $H'_k(z) = F_k(z)$. We conclude that the *ISI-free property is preserved if we interchange the transmitting and receiving filters*.

Theorem 2.1: Suppose the transmitting filters $F_k(z)$ and receiving filters $H_k(z)$ in Fig. 1 form an ISI-free filterbank transceiver. Then, using $H_k(z)$ as the transmitting filters and $F_k(z)$ as the receiving filters, the resulting filterbank transceiver is still ISI free.

Remarks and Applications of Theorem 2.1:

1) The stopband attenuation of the receiving filters determine the receiver's ability to reject out-of-band noise. If the receiving filters have poor stopband attnuation, all the neighboring bands will be affected when there is strong narrowband noise. For example, in the DFT based DMT system, the stopband attenuation of the receiving filters is around 13 dB; the receiver cannot reject out-of-band noise effectively. Therefore, in the DFT-based systems, there is usually a design margin of around 6 dB. When the receiving filters have better frequency capability, a smaller design margin can be used. In view of Theorem 2.1, we can always choose the better one [from the two sets of filters $F_k(z)$ and $H_k(z)$] as the receiving filters.
2) On the other hand, it is desired that the transmitter have smaller gain (for a fixed error probability and bit rate) so that the energy needed in transmission is less. Therefore, we can choose the filters with smaller 2-norm between the two sets of $H_k(z)$ and $F_k(z)$ as the transmitter.

III. Overinterpolated Filterbank Transceivers

In an overinterpolated transceiver, there are more samples at the output of the transmitter than the input. There are $K = N - M$ redundant samples in every N samples of the transmitter output. If we allow the transmitting and receiving filters to be FIR with length longer than the interpolation ratio N, then the transmitter and receiver become transfer matrices $\mathbf{G}(z)$ and $\mathbf{S}(z)$. The systems are non block based. Consider the case $N = M + L$; the transmitter is in the form of trailing zeros

$$\mathbf{G}(z) = \begin{pmatrix} \mathbf{G}_0(z) \\ \mathbf{0}_{(L\times M)} \end{pmatrix} \quad (13)$$

where $\mathbf{G}_0(z)$ is an $M \times M$ matrix. Here, redundancy is in the form of zero padding. Every input block of size M goes through an $M \times M$ transfer matrix, and L zeros are inserted between every two blocks before transmission. In this case, the constant matrix $\mathbf{P}_0$ in (8) is of dimension $N \times M$ and

$$\mathbf{C}(z)\mathbf{G}(z) = \mathbf{P}_0 \mathbf{G}_0(z).$$

The system is ISI free if

$$\mathbf{S}(z)\mathbf{P}_0\mathbf{G}_0(z) = \mathbf{I}. \quad (14)$$

Thus, the channel-dependent term becomes a constant matrix $\mathbf{P}_0$. For a given transmitter $\mathbf{G}_0(z)$, the receiver $\mathbf{S}(z)$ can be any left inverse for $\mathbf{P}_0\mathbf{G}_0(z)$. The following lemma gives us the condition for an FIR transceiver.

Lemma 3.1: Suppose the transmitter is given by (13). Then, there exist FIR solutions for $\mathbf{S}(z)$ if and only if the inverse of $\mathbf{G}_0(z)$ is FIR. In this case, the solution of the receiver is of the form

$$\mathbf{S}(z) = \mathbf{G}_0^{-1}(z)\mathbf{B} \quad (15)$$

where the $M \times L$ matrix $\mathbf{B}$ is any left inverse of $\mathbf{P}_0$.

Proof: Sufficiency. Pre-multiplying $\mathbf{G}_0(z)$ and post-multiplying $\mathbf{G}_0^{-1}(z)$ with both sides of (14), we get $(\mathbf{G}_0(z)\mathbf{S}(z))\mathbf{P}_0 = \mathbf{I}$. This means that $\mathbf{G}_0(z)\mathbf{S}(z)$ is a left inverse of $\mathbf{P}_0$. Therefore, we have

$$\mathbf{G}_0(z)\mathbf{S}(z) = \mathbf{B}$$

where $\mathbf{B}$ is a left inverse of $\mathbf{P}_0$. Pre-multiplying $\mathbf{G}_0^{-1}(z)$ of the above equation with $\mathbf{G}_0(z)$, we obtain the receiver $\mathbf{S}(z)$ in (15). If $\mathbf{G}_0^{-1}(z)$ is FIR, the receiver $\mathbf{S}(z)$ in (15) is also FIR. Furthermore, the solution of $\mathbf{S}(z)$ is not unique as $\mathbf{B}$ is not unique.

Necessity. From (14), we see that $\mathbf{S}(z)\mathbf{P}_0$ is the left inverse of $\mathbf{G}_0(z)$. Therefore, for the FIR transceiver solutions, it is necessary that $\mathbf{G}_0(z)$ has an FIR inverse. $\Delta\Delta\Delta$

From Lemma 3.1 we know that as long as $\mathbf{G}_0(z)$ is FIR and it has an FIR inverse, we can obtain an ISI-free FIR transceiver. Based on Lemma 3.1, we will design the FIR transceiver using classes of FIR matrices that are known to have FIR inverses.

Left Inverses of $\mathbf{P}_0$: Suppose $\mathbf{B}_0$ is a left inverse of $\mathbf{P}_0$. Let $\mathbf{N}$ be an $N \times L$ matrix whose column vectors span the null space of $\mathbf{P}_0$. Any left inverse of $\mathbf{P}_0^T$ can be written as $\mathbf{B}_0 + \mathbf{A}\mathbf{N}^T$. Two left inverses of $\mathbf{P}_0$ can be found easily, as follows.

1) *Pseudo Inverse.* It is given by $\mathbf{B} = (\mathbf{P}_0^T\mathbf{P}_0)^{-1}\mathbf{P}_0^T$. This was used in the block-based DMT system in [14] to obtain ISI-free solutions.
2) It is mentioned in [18] that the matrix $\mathbf{P}_0$ admits a left inverse in the form of lower triangular Toeplitz. In fact, such a left inverse can be found in closed form, as we see next. Let $q(n) = \mathcal{Z}^{-1}\{1/P(z)\}$, where $\mathcal{Z}^{-1}\{\cdot\}$ denotes

inverse Z transform. The filter $q(n)$ can be unstable, depending the zeros of $P(z)$. In particular, if $P(z)$ does not have minimum phase, then $q(n)$ is not causal and stable. Regardless of whether the causal $q(n)$ is stable or not, we can use the first M coefficients of $q(n)$ to form an $M \times N$ lower triangular Toeplitz matrix $\mathbf{B}_0$

$$\mathbf{B}_0 = \begin{pmatrix} q(0) & 0 & \cdots & 0 & 0 & \cdots & 0 \\ q(1) & q(0) & & 0 & 0 & & 0 \\ \vdots & & \ddots & \vdots & & & \vdots \\ q(M-1) & q(M-2) & \cdots & q(0) & 0 & \cdots & 0 \end{pmatrix}. \tag{16}$$

It can be verified that $\mathbf{B}_0$ is a left inverse of $\mathbf{P}_0$. Due to the Toeplitz nature of the left inverse $\mathbf{B}_0$ in (16), it can be implemented using the scalar filter $1/P(z)$. Note that the memory of $1/P(z)$ should be cleared for every input block of length N.

Remarks: The use of a zero padding transmitter means that the last K polyphases of the transmitting filters $F_k(z)$ are zero, but the receiver $\mathbf{S}(z)$ in (15) does not necessarily have some polyphases equal to 0. Using the theorem in Section II, we can exchange the transmitting filters $F_k(z)$ and the receiving filters $H_k(z)$. In this case, the redundancy no longer takes the form of zero padding. The new receiving filters now have K polyphases equal to 0. The matrix $\mathbf{S}(z)$ is of the form $\mathbf{S}(z) = (\mathbf{0} \quad \mathbf{S}_0(z))$, where $\mathbf{S}_0(z)$ is an $M \times M$ matrix; K samples are discarded from every input N samples of the receiver.

Redundancy $K = \lceil L/2 \rceil$: It is shown in [17] that when the system is block based, under some condition, we can use redundancy $K = \lceil L/2 \rceil$, where the notation $\lceil y \rceil$ denotes the smallest integer greater or equal to y. We will see that the result holds for non block-based systems as well. Suppose the redundancy is $K = \lceil L/2 \rceil$ and the transmitter $\mathbf{G}(z)$ is in the trailing zero form

$$\mathbf{G}(z) = \begin{pmatrix} \mathbf{G}_0(z) \\ \mathbf{0}_{(K \times M)} \end{pmatrix}. \tag{17}$$

We partition the $\mathbf{P}_0$ matrix in (8) as

$$\mathbf{P}_0 = \begin{pmatrix} \mathbf{C}_{00} \\ \mathbf{C}_{10} \end{pmatrix} \tag{18}$$

where $\mathbf{C}_{00}$ is of dimension $(L-K) \times M$, and $\mathbf{C}_{10}$ is of dimension $(N+K-L) \times M$.

Lemma 3.2: We can use redundancy $K = \lceil L/2 \rceil$ to obtain FIR ISI-free transceivers if the matrix $\mathbf{C}_{10}$ in (18) has full rank.

Proof: First, let us consider the case where L is even and $K = L/2$. Suppose the transmitter is as in (17) and that the receiver is given by

$$\mathbf{S}(z) = (\mathbf{0} \quad \mathbf{S}_0(z))$$

where $\mathbf{S}_0(z)$ is an $M \times M$ matrix. Then, the transceiver is ISI free if

$$\mathbf{S}_0(z)\mathbf{C}_{10}\mathbf{G}_0(z) = \mathbf{I}.$$

All three matrices in the above equation have dimensions $M \times M$. Therefore, solutions for FIR $\mathbf{G}_0(z)$ and $\mathbf{S}_0(z)$ can be obtained if $\mathbf{C}_{10}$ is nonsingular or has full rank. The case that L is odd can be verified in a similar way. In this case, $\mathbf{C}_{10}$ has dimension $(M+1) \times M$, and the condition is that $\mathbf{C}_{10}$ has full rank. $\Delta\Delta\Delta$

Remark: In most of our experiments, the matrix $\mathbf{C}_{10}$ has full rank. The problem of conditioning the channel $P(z)$ such that $\mathbf{C}_{10}$ has full rank is still open.

IV. Design of FIR ISI-Free Filterbank Transceivers

In Section III, we have seen that there always exist FIR ISI-free transceivers when redundancy $K = L$. In this case, if zero padding is used at the transmitter, then the top matrix $\mathbf{G}_0(z)$ of the transmitter can be any FIR matrix with an FIR inverse. The design becomes a lot more tractable. It is known that any causal FIR matrix with an FIR inverse can be factorized as [22]

$$\mathbf{H}(z)\mathbf{E}(z)$$

where $\mathbf{H}(z)$ is causal FIR orthogonal, and $\mathbf{E}(z)$ is causal FIR unimodular. The class of FIR orthogonal matrices can be completely factorized into some basic building blocks [20]. There are also classes of unimodular matrices that have been shown to be very useful in filterbank designs [24]. We propose two design methods for FIR filterbank transceivers with the ISI-free property: One is based on FIR orthogonal matrices, and the other is based on unimodular matrices.

A. Design Based on Orthogonal Matrices

In the context of filterbank theory and design, FIR orthogonal matrices have been shown to be a very useful class. In this section, we consider the case where $\mathbf{G}_0(z)$ is FIR and $\mathbf{P}_0\mathbf{G}_0(z)$ is FIR orthogonal, i.e.,

$$\left(\mathbf{P}_0\mathbf{G}_0(e^{j\omega})\right)^\dagger \left(\mathbf{P}_0\mathbf{G}_0(e^{j\omega})\right) = \mathbf{I}.$$

Such a construction has the advantage that the receiver can be simply chosen as $\mathbf{S}(z) = \tilde{\mathbf{G}}_0(z)\mathbf{P}_0^T$. Furthermore, in the case of AWGN noise source, the channel noise will not be amplified by the receiver; the average receiver output noise power is the same as the receiver input noise power. Observe that matrix $\mathbf{P}_0$ can be decomposed using singular value decomposition (SVD)

$$\mathbf{P}_0 = \mathbf{U} \begin{pmatrix} \mathbf{\Lambda} \\ \mathbf{0} \end{pmatrix}_{N \times M} \mathbf{V}$$

where $\mathbf{U}$ and $\mathbf{V}$ are, respectively, $N \times N$ and $M \times M$ orthogonal matrices. The matrix $\mathbf{\Lambda}$ is diagonal and $[\mathbf{\Lambda}]_{k,k}^2$ for $k = 0, 1, \ldots, M-1$ are the eigenvalues of $\mathbf{P}_0^T\mathbf{P}_0$, which are nonzero as $\mathbf{P}_0$ has full rank. It can be shown that if $\mathbf{P}_0\mathbf{G}_0(z)$ is FIR and orthogonal, the matrix $\mathbf{G}_0(z)$ is necessarily of the form

$$\mathbf{G}_0(z) = \mathbf{V}^T\mathbf{\Lambda}^{-1}\mathbf{Q}(z) \tag{19}$$

where $\mathbf{Q}(z)$ is an arbitrary $M \times M$ FIR orthogonal matrix. Partition $\mathbf{U}$ as

$$\mathbf{U} = \Big[\underbrace{\mathbf{U}_0}_{N \times M} \underbrace{\mathbf{U}_1}_{N \times L}\Big]. \tag{20}$$

Then, the product $\mathbf{P}_0\mathbf{G}_0(z)$ assumes the form

$$\mathbf{P}_0\mathbf{G}_0(z) = \mathbf{U}_0\mathbf{Q}(z).$$

In this case, the ISI-free property can be obtained by choosing the receiver $\mathbf{S}(z)$ as

$$\mathbf{S}(z) = \widetilde{\mathbf{Q}}(z)\mathbf{U}_0^T.$$

However, the above equation only gives one possible ISI-free solution. To obtain all possible solutions, we note that the ISI-free condition only requires that $\mathbf{S}(z)$ be a left inverse of $\mathbf{P}_0\mathbf{G}_0(z)$. As $\mathbf{P}_0\mathbf{G}_0(z)$ is of dimension $N \times M$, the receiver $\mathbf{S}(z)$ is not unique. We can incorporate the left null space of $\mathbf{U}_0$ and choose

$$\mathbf{S}(z) = (\tilde{\mathbf{Q}}(z) \quad \mathbf{\Xi}(z))\mathbf{U}^T \tag{21}$$

where $\mathbf{\Xi}(z)$ is an arbitrary $M \times L$ FIR transfer matrix. The flexibility can be exploited to improve the frequency selectivity of the receiving filters. It can also be used to minimize the total output noise power, as we will see in Section V.

To maximize band separation, we minimize the stopband energy of the transmitting and receiving filters. The objective function is

$$\phi = \alpha\phi_s + (1-\alpha)\phi_g \tag{22}$$

where

$$\phi_g = \int_{k\text{th}\, stopband} |F_k(e^{j\omega})|^2 \, d\omega$$
$$\phi_s = \int_{k\text{th}\, stopband} |H_k(e^{j\omega})|^2 \, d\omega.$$

Design Example 1—Design Using Orthogonal Matrices: The channel to be used in the example is $P(z) = 1 + 0.8z^{-1}$. The order of $P(z)$ is $L = 1$. We choose $M = 8$ and $N = 9$. The transmitter $\mathbf{G}_0(z)$ is as given in (13), and the receiver is as given by (21). Using the factorization theorem of orthogonal matrices, the orthogonal matrix $\tilde{\mathbf{Q}}(z)$ can be parameterized using degree-one building blocks [20]. We optimize $\tilde{\mathbf{Q}}(z)$ and $\mathbf{\Xi}(z)$ to minimize the stopband energy of the receiving filters. In the optimization, $\tilde{\mathbf{Q}}(z)$ contains four degree-one building blocks, and $\mathbf{\Xi}(z)$ has the same order. Fig. 6 shows the magnitude responses (in decibels) of the transmitting and receiving filters. The stopband attenuation of the receiving filters are around 19 dB. The magnitude response of $P(e^{j\omega})$ is also shown in Fig. 6(b) as a dotted line.

B. Design Based on Unimodular Matrices

The FIR unimodular matrices, unlike orthogonal matrices, do not allow factorization in general. However, a particular class of unimodular has been shown to be very useful in designing M-subband filter banks. Using polyphase matrices that belong to this class, we can design analysis and synthesis filters with sharp transition bands and good stopband attenuation. The unimodular matrices in this class can be written as a product of

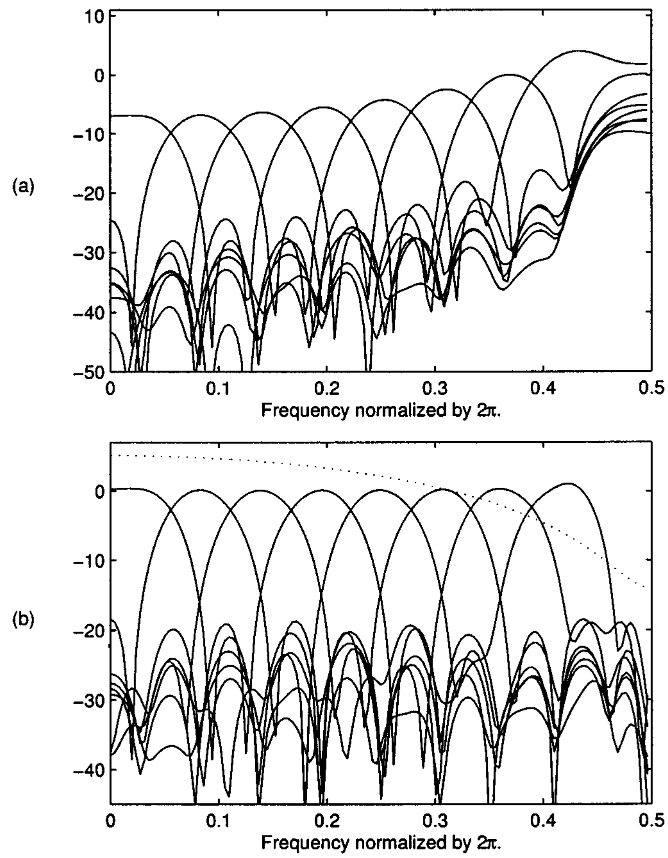

Fig. 6. *Design Example 1. Design Using Orthogonal Matrices.* The magnitude responses (in decibels) of (a) the transmitting filters and (b) the receiving filters. The magnitude response of the channel $P(e^{j\omega})$ is also shown in (b) as a dotted line.

lower triangular and upper triangular matrices of the following form:

$$\mathbf{\Phi}(z)\mathbf{\Psi}(z)$$

where the matrices $\mathbf{\Phi}(z)$ and $\mathbf{\Psi}(z)$ are, respectively, lower triangular and upper triangular FIR matrices given by the equation shown at the bottom of the next page, where D_k are constants, and $\Phi_{i,j}(z)$ and $\Psi_{i,j}(z)$ are FIR filters. It can be immediately verified that such a product matrix $\mathbf{\Phi}(z)\mathbf{\Psi}(z)$ is a unimodular matrix as $\det\ \mathbf{\Phi}(z) = \Pi_{k=0}^{M-1} D_k$ and $\det\ \mathbf{\Psi}(z) = 1$. Therefore, its inverse is also FIR.

Consider the following choice of receiver and transmitter pair that is based on the above class of unimodular matrices

$$\mathbf{S}(z) = (\mathbf{\Phi}(z)\mathbf{\Psi}(z) \quad \mathbf{\Xi}(z))\mathbf{U}^T, \text{ and}$$
$$\mathbf{G}_0(z) = \mathbf{V}^T\mathbf{\Lambda}^{-1}(\mathbf{\Phi}(z)\mathbf{\Psi}(z))^{-1} \tag{23}$$

where $\mathbf{\Xi}(z)$ is an arbitrary $M \times L$ FIR transfer matrix. The receiving filters $H_k(z)$ can be represented by

$$\begin{pmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{M-1}(z) \end{pmatrix} = \mathbf{S}(z^N) \underbrace{\begin{pmatrix} 1 \\ z^1 \\ \vdots \\ z^{N-1} \end{pmatrix}}_{\mathbf{d}(z)}$$
$$= (\, \mathbf{\Phi}(z^N)\mathbf{\Psi}(z^N) \quad \mathbf{\Xi}(z^N)\,)\, \mathbf{U}^T\mathbf{d}(z)$$

where $\mathbf{d}(z)$ is the delay chain vector, as given above. Using the partition of $\mathbf{U} = (\mathbf{U}_0 \quad \mathbf{U}_1)$ in (20), the above equation can be rewritten as

$$\begin{pmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{M-1}(z) \end{pmatrix} = \mathbf{\Phi}(z^N)\mathbf{\Psi}(z^N)\mathbf{U}_0^T\mathbf{d}(z) + \mathbf{\Xi}(z^N)\mathbf{U}_1^T\mathbf{d}(z).$$

Let

$$\begin{pmatrix} \Theta_0(z) \\ \Theta_1(z) \\ \vdots \\ \Theta_{M-1}(z) \end{pmatrix} = \mathbf{\Psi}(z^N)\mathbf{U}_0^T\mathbf{d}(z).$$

Then, we have $H_k(z)$, which is given by

$$\begin{aligned} H_0(z) &= D_0\Theta_0(z) + \boldsymbol{\xi}_0^T(z^N)\mathbf{U}_1^T\mathbf{d}(z) \\ H_1(z) &= \Phi_{1,0}(z)\Theta_0(z) + D_1\Theta_1(z) + \boldsymbol{\xi}_1^T(z^N)\mathbf{U}_1^T\mathbf{d}(z) \\ &\vdots \\ H_{M-1}(z) &= \Phi_{M-1,0}(z)\Theta_0(z) + \Phi_{M-1,1}(z)\Theta_1(z) + \cdots \\ &\quad + D_{M-1}\Theta_{M-1}(z) + \boldsymbol{\xi}_{M-1}^T(z^N)\mathbf{U}_1^T\mathbf{d}(z) \end{aligned}$$

where $\boldsymbol{\xi}_k^T(z)$ is the kth row of $\mathbf{\Xi}(z)$. We can start the optimization process by designing D_0, $\Theta_0(z)$, and the 0th row of $\mathbf{\Xi}(z)$ to obtain $H_0(z)$. As $\Theta_0(z)$ is already determined in the design of $H_0(z)$, the filter $H_1(z)$ is designed by optimizing $\Phi_{1,0}(z)$, D_1, $\Theta_1(z)$, and $\boldsymbol{\xi}_1^T(z)$. In a similar manner, we can continue on to the optimization of $H_2(z)$, $H_3(z)$, $\cdots$, and $H_{M-1}(z)$.

Note that in the design based on orthogonal matrices, the receiving filters are optimized simultaneously. In addition, all the transmitting filters have the same length, and all the receiving filters have the same length. In the unimodular matrices-based design, the filters are designed one by one. The filters that are designed earlier will not be affected by the optimization of other filters later. In this case, the filters can have different length. The objective function is as in (22).

Design Example 2—Design Using Unimodular Matrices: The LTI channel used in this example is the same as in Example 1: $P(z) = 1 + 0.8z^{-1}$. The values of L, M, and N are the same as well, and $L = 1$, $M = 8$, and $N = 9$. The transmitter and receiver are as given in (23). The matrices $\mathbf{\Phi}(z)$ and $\mathbf{\Psi}(z)$ are of order 3. The resulting magnitude responses (in decibels) of the transmitting and receiving filters are shown in Fig. 7. The stopband attenuation of the receiving filters are around 22 dB.

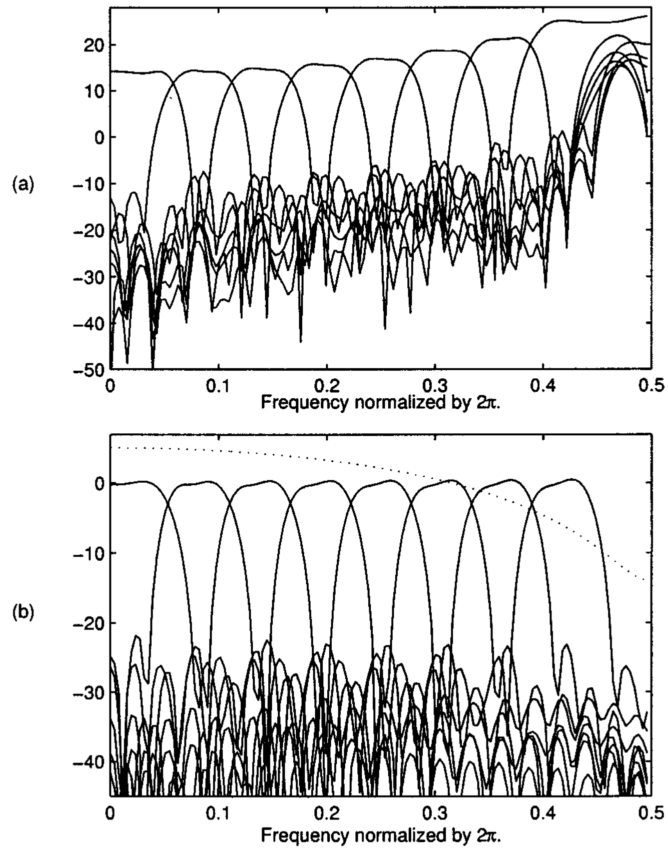

Fig. 7. *Design Example 2. Design Using Unimodular Matrices.* The magnitude responses (in decibels) of (a) the transmitting filters and (b) the receiving filters.

Simulation Example: Consider the LTI channel in Design Example 2. In this experiment, we will apply the transceiver designed in Example 2 and compare the performance with that of DFT-based DMT transceivers. The average number of bits per output sample of the transmitter is $8/9$ bits. Two cases of channel noise will be used: i) white noise with variance $=$ 0.0125 and ii) white noise plus narrowband noise with power spectrum as shown in Fig. 8. The results for these two cases

$$\mathbf{\Phi}(z) = \begin{pmatrix} D_0 & 0 & 0 & \cdots & 0 \\ \Phi_{1,0}(z) & D_1 & 0 & & \\ \Phi_{2,0}(z) & \Phi_{2,1}(z) & D_2 & & \\ \vdots & & & \ddots & \\ \Phi_{M-1,0}(z) & \Phi_{M-1,1}(z) & \Phi_{M-1,2}(z) & & D_{M-1} \end{pmatrix}$$

$$\mathbf{\Psi}(z) = \begin{pmatrix} 1 & \Psi_{0,1}(z) & \Psi_{0,2}(z) & \cdots & \Psi_{0,M-1}(z) \\ 0 & 1 & \Psi_{1,2}(z) & & \Psi_{1,M-1}(z) \\ 0 & 0 & 1 & & \\ \vdots & & & \ddots & \\ 0 & & & & 1 \end{pmatrix}$$

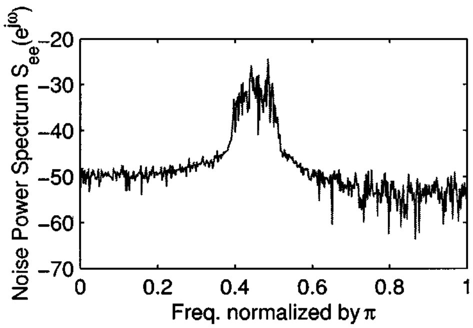

Fig. 8. Power spectrum of the channel noise for case ii). White noise plus narrowband noise.

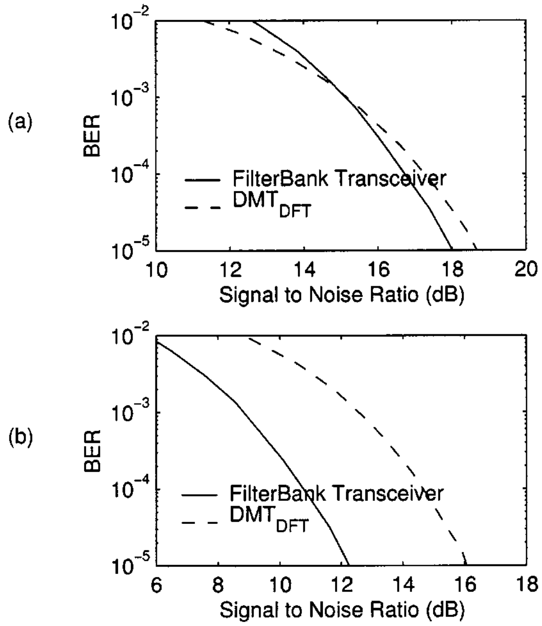

Fig. 9. *Simulation Example.* Bit error rate of filterbank transceiver and DFT-based DMT transceiver for two cases of channel noise. (a) White noise and (b) white noise plus narrowband noise with spectrum, as shown in Fig. 8.

of channel noise are shown, respectively, in Fig. 9(a) and (b). In case i), the performance of the filterbank transceiver is comparable with that of the DFT-based DMT system. In case ii), where the noise is of a narrowband nature, the filterbank transceiver achieves the same bit error rate with a much lower signal-to-noise ratio.

V. Minimum Mean Squared Error Receivers for Orthogonal Transmitters

A. ISI-Free Transceivers with MMSE Receiver

In the design of FIR transceivers using zero padding in Section III, the receiver solution is not unique for a given transmitter. The flexibility can be used to minimize the output noise power. Suppose the channel noise $e(n)$ is a zero mean WSS random process and that it is not correlated with the input. We define the output noise power E_N as

$$E_N = \mathcal{E}\left[\sum_{k=0}^{M-1}\left(\hat{x}_k(n) - x_k(n)\right)^2\right]$$

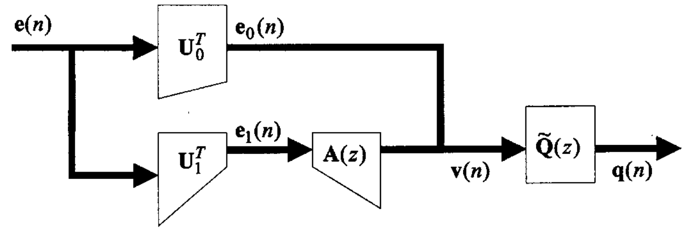

Fig. 10. MMSE receiver for ISI free filterbank transceivers.

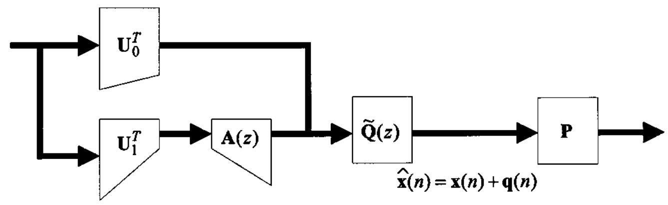

Fig. 11. MMSE Wiener solution of the receiver.

where the function $\mathcal{E}[y]$ denotes the expected value of the random variable y. When the filterbank transceiver is ISI free, the output noise comes entirely from the channel noise. Now, we use the transmitter as given in (19), and we rewrite the receiver in (21) as

$$\mathbf{S}(z) = \tilde{\mathbf{Q}}(z)(\mathbf{I} \quad \mathbf{A}(z))\mathbf{U}^T. \tag{24}$$

The receiver $\mathbf{S}(z)$ is drawn in Fig. 10 for noise analysis. As $\mathbf{Q}(e^{j\omega})$ is orthogonal, the output noise power $\mathcal{E}\left[||\mathbf{q}(n)||_2^2\right] = \mathcal{E}[||\mathbf{v}(n)||_2^2]$. Suppose the order of $\mathbf{A}(z)$ is J and $\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1} + \cdots + \mathbf{A}_J z^{-J}$. Then

$$\begin{aligned}\mathbf{v}(n) =& \mathbf{e}_0(n) + \mathbf{A}_0\mathbf{e}_1(n) + \mathbf{A}_1\mathbf{e}_1(n-1) + \cdots \\ &+ \mathbf{A}_J\mathbf{e}_1(n-J) \\ =& \mathbf{e}_0(n) + \underbrace{(\mathbf{A}_0 \quad \mathbf{A}_1 \quad \cdots \quad \mathbf{A}_J)}_{\mathbf{B}} \underbrace{\begin{pmatrix}\mathbf{e}_1(n) \\ \mathbf{e}_1(n-1) \\ \vdots \\ \mathbf{e}_1(n-J)\end{pmatrix}}_{\mathbf{s}(n)}.\end{aligned} \tag{25}$$

The minimization of $\mathcal{E}[||\mathbf{v}(n)||_2^2]$ becomes a linear estimation problem: estimation of $\mathbf{e}_0(n)$ based on the observations $\mathbf{e}_1(n), \mathbf{e}_1(n-1), \ldots, \mathbf{e}_1(n-J)$. By the orthogonality principle, the optimal $\mathbf{B}$ that minimizes $\mathcal{E}[||\mathbf{v}(n)||_2^2]$ is such that $\mathcal{E}[\mathbf{v}(n)\mathbf{s}^T(n)] = \mathbf{0}$, where $\mathbf{s}(n)$ is as indicated in (25). Therefore, $\mathbf{B}$ should be chosen so that

$$\mathcal{E}\left[\mathbf{e}_0(n)\mathbf{s}^T(n)\right] = -\mathbf{B}\mathcal{E}\left[\mathbf{s}(n)\mathbf{s}^T(n)\right]$$

is satisfied. Note that when the noise $e(n)$ is white, the vectors $\mathbf{e}_0(n)$ and $\mathbf{e}_1(m)$ are uncorrelated for all n and m. In this case, we have $\mathcal{E}[\mathbf{e}_0(n)\mathbf{s}^T(n)] = 0$, and the optimal $\mathbf{B}$ is the $\mathbf{0}$ matrix.

When the order $\mathbf{A}(z)$ is J, the order of the receiving filters is increased by NJ. To avoid increasing the order of the receiving filters, we can choose $\mathbf{A}(z)$ to be a constant matrix $\mathbf{A}_0$. Then, we have $\mathbf{v}(n) = \mathbf{e}_0(n) + \mathbf{A}_0\mathbf{e}_1(n)$. The orthogonality principle requires that $\mathcal{E}[\mathbf{v}(n)\mathbf{e}_1^T(n)] = 0$. Solving for $\mathbf{A}_0$, we obtain optimal solution of $\mathbf{A}_0$

$$\mathbf{A}_0 = -\mathcal{E}\left[\mathbf{e}_0(n)\mathbf{e}_1^T(n)\right]\left(\mathcal{E}\left[\mathbf{e}_1(n)\mathbf{e}_1^T(n)\right]\right)^{-1}.$$

Using $\mathbf{e}_0(n) = \mathbf{U}_0^T\mathbf{e}(n)$ and $\mathbf{e}_1(n) = \mathbf{U}_1^T\mathbf{e}(n)$, the above equation can be rewritten as

$$\mathbf{A}_0 = -\mathbf{U}_0\mathbf{R}_{ee}\mathbf{U}_1^T\left(\mathbf{U}_1\mathbf{R}_{ee}\mathbf{U}_1^T\right)^{-1}$$

where $\mathbf{R}_{ee}$ is the $N \times N$ autocorrelation matrix of the noise $e(n)$.

B. Wiener Solution of the Receiver

The output noise power can be further reduced by adding a Wiener matrix to the end of the receiver solution in (24). Consider the receiver $\mathbf{S}(z)$ of the form

$$\mathbf{S}(z) = \mathbf{P}\tilde{\mathbf{Q}}(z)(\mathbf{I} \quad \mathbf{A}(z))\mathbf{U}^T. \tag{26}$$

The receiver can be drawn as in Fig. 11. By the orthogonality principle, the final output power noise is minimized if

$$\mathcal{E}\left[(\mathbf{P}\hat{\mathbf{x}}(n) - \mathbf{x}(n))\,\hat{\mathbf{x}}^T(n)\right] = \mathbf{0}$$

i.e.,

$$\mathbf{P}\mathcal{E}\left[\hat{\mathbf{x}}(n)\hat{\mathbf{x}}^T(n)\right] = \mathcal{E}\left[\mathbf{x}(n)\hat{\mathbf{x}}^T(n)\right].$$

Assuming that $\mathbf{x}(n)$ and the noise vector $\mathbf{q}(n)$ are uncorrelated, which is usually true, we have

$$\begin{aligned}\mathcal{E}\left[\mathbf{x}(n)\hat{\mathbf{x}}^T(n)\right] &= \mathbf{R}_{\mathbf{xx}}\\ \mathcal{E}\left[\hat{\mathbf{x}}(n)\hat{\mathbf{x}}^T(n)\right] &= \mathbf{R}_{\mathbf{xx}} + \mathbf{R}_{\mathbf{qq}}.\end{aligned} \tag{27}$$

Therefore, the optimal $\mathbf{P}$ is given by

$$\mathbf{P} = \mathbf{R}_{\mathbf{xx}}\left(\mathbf{R}_{\mathbf{xx}} + \mathbf{R}_{\mathbf{qq}}\right)^{-1}.$$

Note that the above MMSE receiver solution gives us output identical to the input in the absence of noise although the design of the receiver itself depends on the noise statistics. The Wiener solution in (26) does not yield an ISI-free transceiver in the absence of noise.

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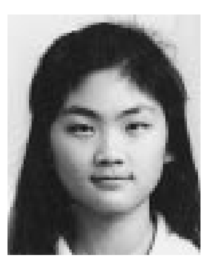

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