involves solving a semidefinite programming problem. The new detector offers additional robustness compared to a detector that assumes a known signature, and is a relevant alternative whenever there exists a possible mismatch between the actual signature and the presumed one.

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ISI-Free Block Transceivers for Unknown Frequency Selective Channels

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***Abstract*—The orthogonal frequency-division multiplexing (OFDM) transceiver has enjoyed great success in many wideband communication systems. It has low complexity and robustness against channel-induced intersymbol interference (ISI). When the channel order does not exceed the length of cyclic prefix, any frequency-selective channel is converted to a set of frequency-nonselective subchannels. This channel-independent ISI-free property is useful for many applications. In this correspondence, we study general block transceiver with such a property. We will show that the solutions of channel-independent ISI-free block transceivers are given in a closed form. It is found that except for some special cases, the solutions are identical to the Lagrange–Vandermonde and Vandermonde–Lagrange transceivers.**

***Index Terms*—Filter bank, multicarrier, multitone, orthogonal frequency-division multiplexing (OFDM), transceiver, transmultiplexer.**

I. INTRODUCTION

In recent years, the orthogonal-frequency-division-multiplexing (OFDM) system has been widely adopted for wideband communications [1]. One of the advantages of OFDM systems is their ability to combat channel-induced intersymbol interference (ISI). In an OFDM system, the transmitter and receiver perform respectively M point inverse discrete Fourier transform (IDFT) and discrete Fourier transform (DFT) operations. By adding a cyclic prefix of length L, any frequency-selective channel of order L is converted to a set of M parallel frequency-nonselective subchannels. Symbol recovery can be obtained by using simple one-tap equalizers at the receiver. Such a channel-independent ISI-free property is useful for many applications.

Recently, there has been some interest in finding other transceivers with channel-independent ISI-free property [2]–[8]. The first non-DFT-based transceiver with such a property was proposed [2]. By judiciously selecting the zeros of the transmit filters, the authors showed that when the number of trailing zeros is larger than or equal to the channel order, ISI can be eliminated completely by using a channel-independent receiver. The transmit filters are M Lagrange interpolation polynomials, whereas the receive filters are M Vandermonde filters, and therefore such a transceiver is called a Lagrange–Vandermonde (LV) transceiver. A dual system called a Vandermonde–Lagrange (VL) transceiver, where the transmit filters are Vandermonde filters and the receive filters are Lagrange filters, was derived in [3]. The LV and VL systems were generalized to the so-called mutually orthogonal user code receiver (AMOUR) system [4]. In [5] and [6], these LV and VL systems were studied using a different framework. Using the multirate technique, it was demonstrated that given an exponential vector input, the output of the Toeplitz channel matrix is also an exponential vector. Exploiting this

Manuscript received November 14, 2005; revised June 27, 2006. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Zoran Cvetkovic. This work was supported by the National Science Council, Taiwan, R.O.C., under Grant NSC94-2752-E-002-006-PAE, NSC94-2213-E-002-075, and NSC94-2213-E-009-038.

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Digital Object Identifier 10.1109/TSP.2006.890826

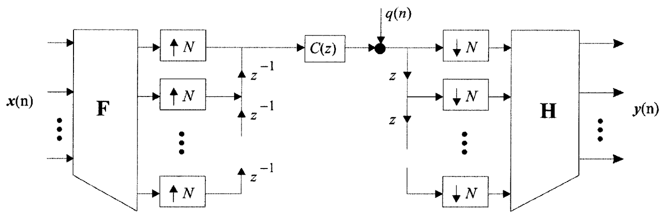

Fig. 1. Block transceiver.

property, the channel-independent LV and VL systems were derived. In [7], the authors extended the idea of channel-independent ISI-free transceivers to the case where the filter length can be longer than the block size. It was showed that for cyclic prefixed systems where the redundant samples are added in the form of cyclic prefix at the transmitter, the only channel-independent ISI-free transceiver is the OFDM system with possibly a different diagonal scaling matrix. In [8], using a filter bank formulation, the authors derived the necessary and sufficient conditions for channel-independent ISI-free filter bank transceivers. However, the solutions to these necessary and sufficient conditions are not tractable, and the filters are optimized so that the signal-to-interference ratio (SIR) is maximized. Another technique for combating the channel ISI is to use the polynomial ambiguity resistant modulated codes (PAMRC) [9]–[11]. It was shown that by incorporating the PAMRC, we can blindly identify the input signal without knowing the channel.

In this correspondence, we focus on the block transmission scheme shown in Fig. 1. In a block transmission system, the transmit and receive matrices $\mathbf{F}$ and $\mathbf{H}$ are memoryless matrices (constant matrices independent of z). Under the assumption that the first column of $\mathbf{H}$ is not a zero vector, we will derive the most general channel-independent ISI-free transceiver. The solution is given in a closed form. It is found that except for some special cases, the solution is the LV system. Using a similar approach, we can also derive the transceiver under the assumption that the last row of $\mathbf{F}$ is not zero. The solution is also given in a closed form, and except for some special cases, the solution is the VL system.

II. Problem Formulation

Consider Fig. 1, where a block transceiver is shown. The input vector $\mathbf{x}(n)$ and the output vector $\mathbf{y}(n)$ are $M \times 1$ vectors. The matrices $\mathbf{F}$ and $\mathbf{H}$ are, respectively, $N \times M$ and $M \times N$. To avoid the degenerated case of scalar system, we assume $M \geq 2$. In this correspondence, we assume that the channel does not vary during the transmission of one data block so that it can be modelled as an linear time-invariant (LTI) system. We will further assume that the channel is finite impulse response (FIR) with an order less than or equal to L, as follows:

$$C(z) = c_0 + c_1 z^{-1} + \ldots + c_L z^{-L}. \tag{1}$$

As we focus only on the ISI-free solution, the channel noise $q(n)$ does not affect our solution. For convenience, we set $q(n) = 0$ in the rest of the correspondence. The transceiver is ISI free for unknown channels if for any $C(z)$ of the form (1), the output is related to the input by

$$\mathbf{y}(n) = \mathbf{\Lambda}\mathbf{x}(n) \tag{2}$$

for some $M \times M$ diagonal matrix $\mathbf{\Lambda}$. Transceivers possessing such a property are said to be *channel-independent ISI free*. The orthogonality of data is not affected by any LTI channel, provided that the channel order does not exceed L. Any frequency-selective channel of order L is converted into a set of M parallel frequency nonselective subchannels. In this case, zero-forcing solution can be obtained using a set of M scalar multipliers known as the frequency-domain equalizers at the end of the receiver. The OFDM system is an example of channel-independent ISI-free transceivers. In the following, we will find the necessary and sufficient conditions on $\mathbf{F}$ and $\mathbf{H}$ so that (2) is satisfied for any $C(z)$ of the form (1).

First note that if $N < M$ or rank$(\mathbf{F}) < M$, we can never fully recover the input vector $\mathbf{x}(n)$ no matter what the channel $C(z)$ is. Thus, we will assume that $N \geq M$ and rank$(\mathbf{F}) \geq M$. Furthermore, one can show that if the transceiver is ISI free for any $C(z)$; then, $N > L$ (in fact, we will show that $N \geq M + L$ in Section III). When $N > L$, the input vector $\mathbf{x}(n)$ and the output vector $\mathbf{y}(n)$ are related as

$$\mathbf{y}(n) = \mathbf{H}\mathbf{C}_0\mathbf{F}\mathbf{x}(n) + \mathbf{H}\mathbf{C}_1\mathbf{F}\mathbf{x}(n-1)$$

where the $N \times N$ matrix $\mathbf{C}_0$ is a lower triangular Toeplitz matrix whose first column is given by $[c_0\ c_1\ \ldots\ c_L\ 0\ \ldots\ 0]^T$ and the $N \times N$ by matrix $\mathbf{C}_1$ is an upper triangular Toeplitz matrix whose first row is given by $[0\ \ldots\ 0\ c_L\ \ldots\ c_1]$. The ISI-free conditions become

$$\mathbf{H}\mathbf{C}_0\mathbf{F} = \mathbf{\Lambda}, \quad \mathbf{H}\mathbf{C}_1\mathbf{F} = \mathbf{0}_M.$$

For $0 \leq i \leq N-1$, define two sets of $N \times N$ matrices $\mathbf{S}_i$ and $\mathbf{R}_i$ as

$$[\mathbf{S}_i]_{kj} = \begin{cases} 1, & k = j + i \\ 0, & \text{otherwise} \end{cases}$$
$$[\mathbf{R}_i]_{kj} = \begin{cases} 1, & j = k + N - i \\ 0, & \text{otherwise} \end{cases}.$$

For example, when $N = 4$, we have

$$\mathbf{S}_0 = \mathbf{I}_4, \quad \mathbf{S}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$
$$\mathbf{S}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$
$$\mathbf{S}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{R}_0 = \mathbf{0}_4, \quad \mathbf{R}_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{R}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\mathbf{R}_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Multiplication with these matrices only involves simple row or column operations. To be more specific, we have

$$[\mathbf{a}_0\ \mathbf{a}_1\ \ldots\ \mathbf{a}_{N-1}]\mathbf{S}_i = [\mathbf{a}_i\ \mathbf{a}_{i+1}\ \ldots\ \mathbf{a}_{N-1}\ \mathbf{0}\ \ldots \mathbf{0}]$$

$$\mathbf{R}_i \begin{bmatrix} \mathbf{b}_0^T \\ \mathbf{b}_1^T \\ \vdots \\ \mathbf{b}_{N-1}^T \end{bmatrix} = \begin{bmatrix} \mathbf{b}_{N-i}^T \\ \vdots \\ \mathbf{b}_{N-1}^T \\ \mathbf{0}^T \\ \vdots \\ \mathbf{0}^T \end{bmatrix}$$

for any $N \times N$ matrices $[\mathbf{a}_0\ \mathbf{a}_1\ \ldots\ \mathbf{a}_{N-1}]$ and $[\mathbf{b}_0\ \mathbf{b}_1\ \ldots\ \mathbf{b}_{N-1}]^T$. These properties will be used later in the derivation of ISI-free block transceivers. Using $\mathbf{S}_i$ and $\mathbf{R}_i$, we can rewrite the ISI-free conditions as

$$\sum_{i=0}^{L} c_i \mathbf{H}\mathbf{S}_i\mathbf{F} = \mathbf{\Lambda}, \quad \sum_{i=1}^{L} c_i \mathbf{H}\mathbf{R}_i\mathbf{F} = \mathbf{0}.$$

For a channel-independent ISI-free transceiver, the condition should be satisfied for arbitrary coefficients c_i. Therefore, the above conditions are equivalent to

$$\mathbf{H}\mathbf{S}_i\mathbf{F} = \mathbf{\Lambda}_i, \quad \mathbf{H}\mathbf{R}_i\mathbf{F} = \mathbf{0} \tag{3}$$

for $0 \le i \le L$ and for some diagonal matrices $\mathbf{\Lambda}_i$. In the next section, we will find the transmit matrix $\mathbf{F}$ and the receive matrix $\mathbf{H}$ such that the above conditions are satisfied.

Remarks:

1. Note that the diagonal matrices $\mathbf{\Lambda}_i$ do not depend the channel impulse response c_i. If the transceiver is an OFDM system, one can verify that the matrix $\mathbf{\Lambda}_i = \text{diag}[1\ e^{-j2\pi i/M}\ \ldots e^{-j2\pi(M-1)i/M}]$. For any channel-independent ISI-free transceivers, the diagonal entries of the sum $c_i\mathbf{\Lambda}_i$ are the subchannel gains.
2. It is emphasized that the ISI-free condition in (3) does not guarantee symbol recovery. In fact, for any $\mathbf{H}$ and $\mathbf{F}$ that satisfy (3), one can find channel coefficients c_i so that one or more subchannel gains are equal to zero.
3. From [8], it is known that when a transceiver satisfies the ISI-free conditions in (3), it is also ISI-free for both multiuser and multiple-input multiple-output (MIMO) transmissions, provided that the orders of all the transmission channels do not exceed L.

III. Channel-Independent ISI-Free Block Transceivers

In the following, we will derive the ISI-free solution under the assumption that the first column of $\mathbf{H}$ is not a zero vector. The following lemma is important for the derivation of the main result.

Lemma 1: Suppose the first column of $\mathbf{H}$ is not a zero vector. Then, the last L rows of the transmit matrix $\mathbf{F}$ are zero.

Proof: As the first column of $\mathbf{H}$ is not a zero vector, it has at least one nonzero entry, say $h_{i0} \neq 0$. The condition $\mathbf{H}\mathbf{R}_1\mathbf{F} = \mathbf{0}$ implies that the last row of $\mathbf{F}$ is a zero row. This fact and the condition $\mathbf{H}\mathbf{R}_2\mathbf{F} = \mathbf{0}$ imply that the last two rows of $\mathbf{F}$ are zero. Continuing the process, one can show that the last L rows of $\mathbf{F}$ are zero rows.

In other words, the transmitter appends L zeros to every block of M data samples. Such a transmission scheme is also known as the zero-padding transmission. Using the above lemma, we see that the rank of the matrix $\mathbf{F}$ is smaller than $N - L + 1$. As the rank of $\mathbf{F}$ should be at least M, we have the following inequality:

$$M \le \text{rank}(\mathbf{F}) < N - L + 1$$

which implies that $N \ge M + L$. That is, the number of redundant samples should be at least L, the channel order. In what follows, we will show that ISI-free solution exists for the case of minimum redundancy $N = M + L$. For convenience, we define an $M \times M$ matrix $\hat{\mathbf{F}}$ and an $N \times M$ matrix $\hat{\mathbf{S}}_i$, respectively, as

$$\mathbf{F} = \begin{bmatrix} \hat{\mathbf{F}} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{S}_i = [\hat{\mathbf{S}}_i \quad \times].$$

In other words, $\hat{\mathbf{F}}$ is obtained by deleting the last L rows of $\mathbf{F}$, and $\hat{\mathbf{S}}_i$ is obtained by deleting the last L columns of $\mathbf{S}_i$. Then, the ISI-free condition in (3) reduces to $\mathbf{H}\hat{\mathbf{S}}_i\hat{\mathbf{F}} = \mathbf{\Lambda}_i$, for $i = 0, 1, \ldots, L$. Note that the square matrix $\hat{\mathbf{F}}$ is invertible, and let

$$\hat{\mathbf{F}}^{-1} = \mathbf{G}.$$

Therefore, we can write $\mathbf{H}\hat{\mathbf{S}}_i = \mathbf{\Lambda}_i\mathbf{G}$. Looking at the kth row of these matrix equations for $0 \le i \le L$, we have

$$\begin{bmatrix} h_{k,0} & h_{k,1} & \ldots & h_{k,M-1} \\ h_{k,1} & h_{k,2} & \ldots & h_{k,M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{k,L} & h_{k,L+1} & \ldots & h_{k,L+M-1} \end{bmatrix} = \begin{bmatrix} \lambda_{k,0} & 0\ldots & 0 & \\ 0 & \lambda_{k,1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \ldots & 0 & \lambda_{k,L} \end{bmatrix} \begin{bmatrix} g_{k,0} & g_{k,1} & \ldots & g_{k,M-1} \\ g_{k,0} & g_{k,1} & \ldots & g_{k,M-1} \\ \vdots & \vdots & \ddots & \vdots \\ g_{k,0} & g_{k,1} & \ldots & g_{k,M-1} \end{bmatrix} \tag{4}$$

where $\lambda_{k,i}$ is the kth diagonal entry of $\mathbf{\Lambda}_i$. First note that $\lambda_{k,i}$ should be nonzero for at least one i; otherwise $h_{k,i} = 0$ for $0 \le i \le M + L - 1$. That is, the k row of $\mathbf{H}$ is a zero row; the symbol transmitted through the kth subchannel can never be recovered no matter what the channel $C(z)$ is. Using this fact, we can prove the following lemma (see the Appendix for a proof).

Lemma 2: Consider (4). The solutions $h_{k,i}$ and $g_{k,i}$ are either of the form

$$[h_{k,0}\ h_{k,1}\ \ldots\ h_{k,N-1}] = h_{k,N-1}[0\ \ldots\ 0\ 1], \text{ for some nonzero } h_{k,N-1} \tag{5}$$

$$[g_{k,0}\ g_{k,1}\ \ldots\ g_{k,M-1}] = g_{k,M-1}[0\ \ldots\ 0\ 1], \text{ for some nonzero } g_{k,M-1} \tag{6}$$

or they have the form

$$[h_{k,0}\ h_{k,1}\ \ldots\ h_{k,N-1}] = h_{k,0}\left[1\ \rho_k\ \rho_k^2 \ldots\ \rho_k^{N-1}\right], \text{ for some nonzero } h_{k,0} \tag{7}$$

$$[g_{k,0}\ g_{k,1}\ \ldots\ g_{k,M-1}] = g_{k,0}\left[1\ \rho_k\ \rho_k^2 \ldots\ \rho_k^{M-1}\right], \text{ for some nonzero } g_{k,0}. \tag{8}$$

Note that both the receive matrix $\mathbf{H}$ and the transmit matrix $\hat{\mathbf{F}} = \mathbf{G}^{-1}$ are independent of the channel coefficients c_i. Because the ranks of $\mathbf{H}$ and $\mathbf{G}$ are M, $h_{k,i}$, and $g_{k,i}$ can have the form shown in (5) and (6) for at most one k. In fact, if one is allowed to choose $\rho = +\infty$, then (7) and (8) reduce as (5) and (6), respectively. To see this, let us rewrite (7) and (8), respectively, as

$$[h_{k,0}\ h_{k,1}\ \ldots\ h_{k,N-1}] = h_{k,N-1}\left[\rho_k^{-N+1}\ \ldots\ \rho_k^{-1}\ 1\right]$$

$$[g_{k,0}\ g_{k,1}\ \ldots\ g_{k,M-1}] = g_{k,M-1}\left[\rho_k^{-M+1}\ \ldots\ \rho_k^{-1}\ 1\right].$$

As ρ approaches infinity, the above equations become those in (5) and (6), respectively. In summary, we have proved the following results.

Assume that $N = M + L$ and the first column of the receive matrix $\mathbf{H}$ is not a zero vector. Then, the block transceiver in Fig. 1 achieves channel-independent ISI-free for any FIR channel of order L if the following conditions are met:

a) the transmit matrix is given by

$$\mathbf{F} = \begin{bmatrix} \hat{\mathbf{F}} \\ \mathbf{0} \end{bmatrix} \tag{9}$$

where $\hat{\mathbf{F}}$ is an $M \times M$ matrix whose inverse is

$$\hat{\mathbf{F}}^{-1} = \mathbf{G} = \begin{bmatrix} g_{0,0} & 0 & \dots & 0 \\ 0 & g_{1,0} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & g_{M-1,0} \end{bmatrix} \begin{bmatrix} 1 & \rho_0 & \dots & \rho_0^{M-1} \\ 1 & \rho_1 & \dots & \rho_1^{M-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \rho_{M-1} & \dots & \rho_{M-1}^{M-1} \end{bmatrix};$$

b) the receive matrix is given by

$$\mathbf{H} = \begin{bmatrix} h_{0,0} & 0 & \dots & 0 \\ 0 & h_{1,0} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & h_{M-1,0} \end{bmatrix} \begin{bmatrix} 1 & \rho_0 & \rho_0^2 & \dots & \rho_0^{N-1} \\ 1 & \rho_1 & \rho_1^2 & \dots & \rho_1^{N-1} \\ \vdots & \vdots & & & \vdots \\ 1 & \rho_{M-1} & \rho_{M-1}^2 & \dots & \rho_{M-1}^{N-1} \end{bmatrix}. \tag{10}$$

The scalars $h_{k,0}$ and $g_{k,0}$ are nonzero and the parameters ρ_k are distinct. Conversely, if the transceiver achieves channel-independent ISI-free for any FIR channel of order L, then, except for the extreme case of one of ρ_k approaching infinity, the transmit and receive matrices have the closed-form expressions given in (9) and (10), respectively. In other words, the receive matrix $\mathbf{H}$ is a scaled Vandermonde matrix, and the matrix $\hat{\mathbf{F}} = \mathbf{G}^{-1}$ is the inverse of a scaled Vandemonde matrix. Note that the matrix $\hat{\mathbf{F}}$ can also be obtained by using the Lagrange interpolation formula. Thus, except for the extreme case of (5) and (6), the most general channel-independent ISI-free block transceiver is the Lagrange–Vandermonde transceiver proposed in [2].

Similarly, by assuming that the last row of $\mathbf{F}$ is not a zero row, we can show that the first L columns of $\mathbf{H}$ are zero columns. Such a transmission scheme is known as the zero jamming transmission [5]. Solving the ISI-free conditions, it can be found that, except for some special cases, the resulting solution is identical to the Vandermonde–Lagrange transceiver proposed in [3].

IV. Conclusion

In this correspondence, the most general channel-independent ISI-free block transceivers are derived. It is found that, except for some special cases, the solutions are given by the LV and VL systems in [3] and [2], respectively. For transceivers with a longer filter length, which corresponds to the nonblock transmission case, the most general channel-independent ISI-free solutions are still unknown.

Appendix
A Proof of Lemma 2

For convenience, we will drop the index "k" in the following discussion. We know that there is at least one $\lambda_i \neq 0$. Let the first nonzero diagonal entry be λ_m. We will show that m is either 0 or L. When $m = L$, the solution has the form given in (5) and (6), and when $m = 0$, the solution has the form given in (7) and (8).

Suppose that $m > 0$. Then $\lambda_0 = \ldots = \lambda_{m-1} = 0$ and $\lambda_m \neq 0$. The equation in (4) implies that $h_0 = \cdots = h_{m+M-2} = 0$. As $M \geq 2$, from the equation $[h_m \ \ldots \ h_{m+M-1}] = \lambda_m [g_0 \ \ldots \ g_{M-1}]$, we get $g_0 = \ldots = g_{M-2} = 0$ and $g_{M-1} \neq 0$ (because g_i cannot be all zero). Moreover, $h_{m+M-1} = \lambda_m g_{M-1} \neq 0$. If $m \neq L$, then from the relation

$$[h_{m+1} \ \ldots \ h_{m+M-1} \ h_{m+M}] = \lambda_{m+1}[g_0 \ \ldots \ g_{M-2} \ g_{M-1}]$$

we will get $h_{m+M-1} = \lambda_{m+1} g_{M-2}$, which is zero as $g_{M-2} = 0$. This contradicts the fact that h_{m+M-1} is a nonzero number. Thus, we can conclude that if $m \neq 0$, m must be L.

Suppose that $m = L$. Substituting $\lambda_0 = \cdots = \lambda_{L-1} = 0$ into (4), we immediately get the solution given in (5) and (6). When $m = 0, \lambda_0 \neq 0$. The first two rows of (4) imply that $[h_1 \ \ldots \ h_M] = \lambda_1/\lambda_0 [h_0 \ \ldots \ h_{M-1}]$. From this, we get $h_i = \rho^i h_0$, and $g_i = \rho^i g_0$ for $0 \leq i \leq M-1$ and $\rho = \lambda_1/\lambda_0$. Substituting these results into (4), we will get a set of equations

$$[h_i \ \ldots h_{i+M-1}] = \lambda_i g_0 [1 \ \rho \ \ldots \ \rho^{M-1}]$$

for $0 \leq i \leq L$. From these equations, we can get $h_i = \rho^i h_0$ for $0 \leq i \leq M + L - 1$.

Note that when $\lambda_0 \neq 0$, λ_i for $i \neq 0$ can still be zero. It can be verified from (4) that if any $\lambda_i = 0$ for some $i > 0$, then $\rho = 0$, which implies $\lambda_1 = \ldots = \lambda_L = 0$.

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