A New Iterative Algorithm for Finding the Minimum Sampling Frequency of MultiBand Signals

Yuan-Pei Lin, Yi-De Liu, and See-May Phoong

***Abstract*—In this correspondence, we propose a new iterative algorithm for finding the minimum sampling frequency of a signal that consists of multiple bandpass signals. This finds important application in software radio where it is desirable to downconvert multiple bandpass signals simultaneously. We will derive a new set of conditions for alias-free sampling. The minimum sampling frequency can be found by iteratively increasing the sampling frequency to meet the alias-free conditions. We will show how the algorithm can be generalized to find alias-free sampling frequency intervals. Simulations will be given to demonstrate the usefulness of the proposed method.**

***Index Terms*—Bandpass sampling, minimum sampling frequency, multiband signal.**

I. INTRODUCTION

Bandpass sampling has important applications in downcoverting radio frequency (RF) signals. In the application of software defined radio systems, it is desirable to downconvert multiple RF signals simultaneously to save cost [1], [2]. The signal to be sampled may consist of more than one bandpass signal. An example of spectrum that contains two bandpass signals (four passbands) is shown in Fig. 1. Sampling theorem for a bandpass signal (two passbands) is well known [3]. The minimum frequency for alias-free sampling can be found in a closed form [4]. The minimum sampling frequency is usually significantly lower than the carrier frequency of the bandpass signal.

For signals with more than two passbands, the minimum sampling frequency can not be found in a closed from due to the nonlinear nature of spectrum folding in the process of sampling. Conditions for alias-free sampling can be stated in different ways in terms of the band edges and bandwidths of the bandpass signals. The conditions that are used for finding the minimum sampling frequency affect the complexity of the algorithms. Sampling for multiband signals is extended in [2] and conditions for alias-free sampling of multiband signals are derived [2]. A systematic algorithm for finding valid sampling frequencies is developed in [5]. In [6]–[8], the complexity for finding valid sampling frequency is reduced by imposing constraints on the ordering of the bands in the folded spectrum. Simulation results in [8] have shown that the minimum alias-free sampling frequency with the ordering constraint can be found very efficiently. In [9], an efficient algorithm for finding valid sampling frequency range for multiband signals is proposed. By exhausting all possible orderings of the bands in the folded spectrum and categorizing all possible cases, the computational complexity can be reduced. An algorithm for finding the minimum sampling frequency is developed in [10] by finding the intersection of valid sampling frequencies for every two signal bands. A downconversion function is proposed in [11] to simplify the downconversion of multiband signals when the member bandpass signals are separated by a frequency larger than the sampling frequency. In [12], the authors considered the sampling of an RF signal located among a group of RF signals that have contiguous spectrum. The minimum sampling frequency range is derived for the desired RF signal.

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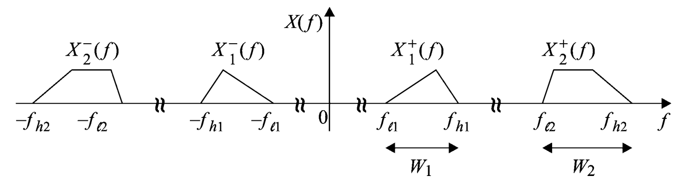

Fig. 1. Example of spectrum that consists of two bandpass signals.

On the other hands, sampling of multiband signals has also been studied extensively using nonuniform sampling, particularly periodic nonuniform sampling [13]–[17]. With nonuniform sampling, there is more freedom in sampling and continuous-time signals can be reconstructed from samples using a lower sampling frequency than that is possible with uniform sampling [13]. In the application of sampling for multiband signals, the discrete-time sequence obtained from sampling is usually filtered so that individual bandpass signals can be extracted and further processed. With nonuniform sampling the samples are obtained in a nonuniform manner. As filtering is typically performed on uniform samples, restoration of uniform samples [17] may be needed before filtering can be applied.

In this correspondence, we propose a new algorithm for finding the minimum sampling frequency for a signal consisting of two or more bandpass signals using uniform sampling. We will first derive a set of conditions for alias-free sampling of signals that consist of two bandpass signals (four bands). These conditions can be checked with few computations. When one of these conditions is not satisfied, the sampling frequency can be adjusted with minimum increment so that the condition becomes satisfied. By iteratively increasing the sampling frequency to meet the conditions for alias-free sampling, an algorithm for finding the minimum sampling frequency can be developed. There is no need to consider ordering of the signal bands in the folded spectrum. The algorithm can be generalized to find the minimum sampling frequency for multiband signals. We can also extend the result to find alias-free sampling frequency intervals. Simulation examples will be given to demonstrate usefulness of the proposed algorithm.

The rest of the correspondence is organized as follows. We derive conditions for alias-free sampling of signals that contain two bandpass signals (four bands) in Section II and an algorithm for finding the minimum sampling frequency is given in Section III. In Section IV, we will extend the results to the case of multiband signals. Simulation examples are presented in Section V and a conclusion is given in Section VI. Some preliminary results on sampling two-bandpass signals have been published in [18].

II. CONDITIONS FOR ALIAS-FREE SAMPLING OF TWO-BANDPASS SIGNALS

Conditions for alias-free sampling can be stated in various ways in terms of the band edges and bandwidths of the member bandpass signals. The conditions that are employed affect the complexity of ensuing algorithms. In this section, we derive a new set of conditions for alias-free sampling of two bandpass-signals.

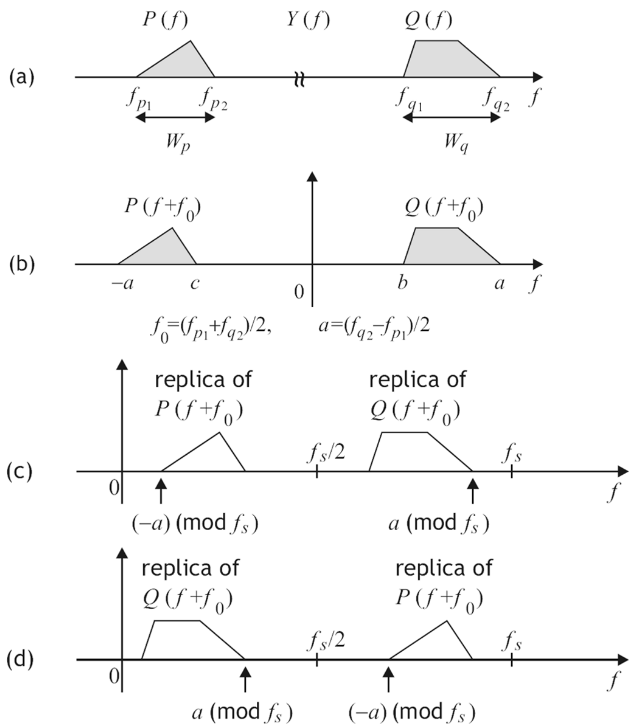

Fig. 2. (a) The spectrum of a hypothetical two-band signal $Y(f)$. (b) $Y(f+f_0)$, where $f_0=(f_{p_1}+f_{q_2})/2$. (c) An example of the folded spectrum for the interval $[0, f_s)$ when $a(\mathrm{mod} f_s) \geq (-a)(\mathrm{mod} f_s)$. (d) An example of the folded spectrum for the interval $[0, f_s)$ when $a(\mathrm{mod} f_s) < (-a)(\mathrm{mod} f_s)$.

Suppose we are to sample a signal $X(f)$ that consists of two bandpass signals $X_1(f)$ and $X_2(f)$ as shown in Fig. 1. Assume $X_i(f)$ is nonzero only in the passbands, i.e., $X_i(f)=0$, for $|f| \leq f_{l_i}$, and $|f| \geq f_{h_i}, i=1,2$, where f_{ℓ_i} and f_{h_i} are band edges, and $W_i=f_{h_i}-f_{\ell_i}$ are the one-sided bandwidths (Fig. 1). Let $X_i^+(f)$, and $X_i^-(f)$ denote respectively the positive frequency part and negative frequency part of $X_i(f)$. There are four signal bands, $X_1^+(f)$, $X_1^-(f)$, $X_2^+(f)$, and $X_2^-(f)$. Since the replicas of any two bands may overlap and result in aliasing after sampling, there are a total of $C_2^4=6$ cases. Note that $X_1^+(f)$ and $X_1^-(f)$ are symmetric with respect to 0, and so are $X_2^+(f)$ and $X_2^-(f)$. If $X_1^+(f)$ and $X_2^+(f)$ are not aliasing after sampling, then $X_1^-(f)$ and $X_2^-(f)$ will not be aliasing by symmetry. Similarly, if $X_1^-(f)$ and $X_2^+(f)$ are not aliasing after sampling, then $X_1^+(f)$ and $X_2^-(f)$ will not be aliasing. Thus, we need to consider only four cases:

$$\begin{aligned}&\text{(a) } \{X_1^+(f), X_1^-(f)\}\\ &\text{(b) } \{X_2^+(f), X_2^-(f)\}\\ &\text{(c) } \{X_1^+(f), X_2^+(f)\}\\ &\text{(d) } \{X_1^-(f), X_2^+(f)\}.\end{aligned} \tag{1}$$

To discuss the above different cases in a more general setting, we first consider the sampling of a hypothetical two-band signal $Y(f)$ as shown in Fig. 2(a). $Y(f)$ consists of $P(f)$ and $Q(f)$, where $P(f) \neq 0$, only for $f_{p_1} < f < f_{p_2}$ and $Q(f) \neq 0$, only for $f_{q_1} < f < f_{q_2}$. The bandedges f_{p_1}, f_{p_2}, f_{q_1}, and f_{q_2} can be positive or negative, $W_p=f_{p_2}-f_{p_1}$, and $W_q=f_{q_2}-f_{q_1}$.

Lemma 1: For the two-band signal $Y(f)$ in Fig. 2(a), there is no aliasing for a given sampling frequency f_s if and only if

$$\begin{aligned}&(f_{q_2}-f_{p_1})(\mathrm{mod} f_s)=0, \quad \text{or}\\ &(f_{q_2}-f_{p_1})(\mathrm{mod} f_s) \geq W_p+W_q.\end{aligned} \tag{2}$$

Proof: We observe that there is no aliasing in sampling $Y(f)$ if and only if there is no aliasing when we sample a shifted version $Y(f+f_0)$, where f_0 is the shift. For convenience we will consider the condition for alias-free sampling of $Y(f+f_0)$. Suppose we choose f_0 as $f_0=(f_{p_1}+f_{q_2})/2$, then the shifted pair is as shown in Fig. 2(b), where $a=(f_{q_2}-f_{p_1})/2, b=f_{q_1}-(f_{p_1}+f_{q_2})/2, c=f_{p_2}-(f_{p_1}+f_{q_2})/2$. If we consider the folded spectrum in the $[0, f_s)$ interval, the band edges $a(\mathrm{mod} f_s)$ and $(-a)(\mathrm{mod} f_s)$ are equal-distanced from $f_s/2$. We now discuss two possible scenarios: i) $a(\mathrm{mod} f_s) \geq (-a)(\mathrm{mod} f_s)$ and ii) $a(\mathrm{mod} f_s) < (-a)(\mathrm{mod} f_s)$. Examples of these two possible cases are shown respectively in Fig. 2(c) and (d).

i) When $a(\mathrm{mod} f_s) \geq (-a)(\mathrm{mod} f_s)$ there will be no aliasing if and only if $(-a)(\mathrm{mod} f_s)=a(\mathrm{mod} f_s)$ or if the interval $((-a)(\mathrm{mod} f_s), a(\mathrm{mod} f_s))$ is large enough to accommodate the two replicas. That is, $x=0$, or $x \geq W_p+W_q$, where $x=a(\mathrm{mod} f_s)-((-a)(\mathrm{mod} f_s))$. The equivalent conditions are

$$2a(\mathrm{mod} f_s)=0, \text{ or } 2a(\mathrm{mod} f_s) \geq W_p+W_q. \tag{3}$$

ii) When $a(\mathrm{mod} f_s) < (-a)(\mathrm{mod} f_s)$ as shown in Fig. 2(d), there is some space between the two replicas and the space is of length $((-a)(\mathrm{mod} f_s)-a(\mathrm{mod} f_s))$. There will be no aliasing if and only if the remaining part of the $[0, f_s)$ interval is large enough to take in the two replicas. That is, $f_s-((-a)(\mathrm{mod} f_s)-a(\mathrm{mod} f_s)) \geq W_p+W_q$, Or equivalently $2a(\mathrm{mod} f_s) \geq W_p+W_q$. This is the same as the second condition in (3).

Substituting $a=(f_{q_2}-f_{p_1})/2$ to (3), we obtain the necessary and sufficient condition for alias-free sampling of $Y(f)$ in (2). ■

The result in Lemma 1 is for the sampling of a two-band signal with arbitrary band locations. We can apply it to each of the cases in (1). Then we can obtain sufficient and necessary conditions for alias-free sampling of two-bandpass signals. For a given sampling frequency f_s, there will not be aliasing if and only if the following are true:

$$2f_{h_1}(\mathrm{mod} f_s)=0, \quad \text{or} \quad 2f_{h_1}(\mathrm{mod} f_s) \geq 2W_1. \tag{4}$$

$$2f_{h_2}(\mathrm{mod} f_s)=0, \quad \text{or} \quad 2f_{h_2}(\mathrm{mod} f_s) \geq 2W_2. \tag{5}$$

$$\begin{aligned}&(f_{h_2}-f_{\ell_1})(\mathrm{mod} f_s)=0, \quad \text{or}\\ &(f_{h_2}-f_{\ell_1})(\mathrm{mod} f_s) \geq W_1+W_2.\end{aligned} \tag{6}$$

$$\begin{aligned}&(f_{h_1}+f_{h_2})(\mathrm{mod} f_s)=0, \quad \text{or}\\ &(f_{h_1}+f_{h_2})(\mathrm{mod} f_s) \geq W_1+W_2.\end{aligned} \tag{7}$$

III. Finding Minimum Sampling Frequency: Two-Bandpass Case

For a given sampling frequency f_s, there will be no aliasing if all four conditions in (4)–(7) are met. If any one of the conditions is not satisfied, we can make minimum increment to the sampling frequency so that the condition becomes satisfied for each case. Let us first go back to the hypothetical two-band signal $Y(f)$ that is useful in previous section.

Lemma 2: Consider the sampling of the signal $Y(f)$ in Fig. 2. Suppose there is aliasing for a given sampling frequency f_s. Then the smallest $f_{s,\mathrm{new}} > f_s$ that yields alias-free sampling of $Y(f)$ is

$$f_{s,\mathrm{new}}=\frac{f_{q_2}-f_{p_1}}{\lfloor (f_{q_2}-f_{p_1})/f_s \rfloor}. \tag{8}$$

Proof: Consider the folded spectrum in the interval $[0, f_s)$ as shown in Fig. 2(c) and (d). We discuss the two cases: i) $a(\mathrm{mod} f_s) < f_s/2$ and ii) $f_s/2 < a(\mathrm{mod} f_s) < f_s$, separately. i) $0 < a(\mathrm{mod} f_s) < f_s/2$: When we gradually increase the sampling frequency the band edge $a(\mathrm{mod} f_s)$ of replica $Q(f)$ moves towards 0 while the band edge

$(-a)(\mathrm{mod} f_s)$ of replica $P(f)$ moves towards f_s. When the sampling frequency is increased such that $a(\mathrm{mod} f_s)$ decreases to 0, then the condition in (2) becomes satisfied. ii) $f_s/2 < a(\mathrm{mod} f_s) < f_s$: Similarly the condition in (2) becomes satisfied when $a(\mathrm{mod} f_s)$ decreases to $f_s/2$. Therefore, we can conclude that the alias-free condition in (2) can be satisfied by increasing the sampling frequency such that a becomes an integer multiple of $f_s/2$. The smallest new sampling $f_{s,\mathrm{new}}$ for this to happen can be computed as follows. Let us write a as $a = n_a f_s/2 + r_a$, where $r_a = a(\mathrm{mod} f_s/2)$ and $n_a = \lfloor a/(f_s/2) \rfloor$. Then we have $a = n_a f_{s,\mathrm{new}}/2$, or $f_{s,\mathrm{new}} = 2a/n_a$. Using the fact that $n_a = \lfloor 2a/f_s \rfloor$, we obtain the expression of $f_{s,\mathrm{new}}$ in (8). ■

We can apply Lemma 2 to the cases in (1). Then for each case in (1), we can obtain a formula for adjusting the sampling frequency so that the corresponding alias-free conditions in (4)–(7) is satisfied:

$$f_{s,\mathrm{new}} = 2f_{h_1}/\lfloor 2f_{h_1}/f_s \rfloor \tag{9}$$
$$f_{s,\mathrm{new}} = 2f_{h_2}/\lfloor 2f_{h_2}/f_s \rfloor \tag{10}$$
$$f_{s,\mathrm{new}} = (f_{h_2} - f_{\ell_1})/\lfloor (f_{h_2} - f_{\ell_1})/f_s \rfloor \tag{11}$$
$$f_{s,\mathrm{new}} = (f_{h_1} + f_{h_2})/\lfloor (f_{h_1} + f_{h_2})/f_s \rfloor. \tag{12}$$

Proposed Iterative Algorithm for Two-Bandpass Signals: Using the conditions for alias-free sampling in Section II and the methods for computing new sampling frequency for each case, we have the following iterative algorithm for finding the minimum sampling frequency. To start off, let $f_s = 2(W_1 + W_2)$, which is the lowest possible sampling frequency for no aliasing.

1. Examine the conditions for alias-free sampling in (4)–(7) one by one. If any one of the condition is not satisfied, go to the next step. If all the conditions are satisfied, then we have found the minimum sampling frequency.
2. For the condition that is violated in Step 1, compute the corresponding new sampling frequency using (9)–(12). Go to Step 1.

There is no need considering the ordering of signal bands in the folded spectrum. The conditions in (4)–(7) can be easily examined and frequency adjusted in (9)–(12). Usefulness of the algorithm will be verified numerically in Section V.

Intervals for Alias-Free Sampling: In practice it is of interest to obtain an interval for alias-free sampling instead of just the minimum sampling frequency. To do this, let us go back to the hypothetical two-band signal $Y(f)$ in Fig. 2(a). Suppose there is no aliasing for a given sampling frequency f_s^*. When we gradually increase the sampling frequency, the bandedges of the replicas will touch. Let the corresponding frequency be $f_{s,\mathrm{new}}$. There will be aliasing if we increase the sampling frequency further. Following a procedure similar to that in Lemma 2, we can express $f_{s,\mathrm{new}}$ as

$$f_{s,\mathrm{new}} = (f_{q_1} - f_{p_2})/\lfloor (f_{q_1} - f_{p_2})/f_s \rfloor.$$

Applying this result to each of the four cases using $f_s^* = f_{s,\min}$, we can obtain four sampling frequencies, say f_{u_1}, f_{u_2}, f_{u_3}, f_{u_4}. When we choose sampling frequency from the interval

$$f_{s,\min} \le f_s \le f_{s,\max}$$

where $f_{s,\max} = \min\{f_{u_1}, f_{u_2}, f_{u_3}, f_{u_4}\}$, there will not be aliasing. We can see that once $f_{s,\min}$ is known, an alias-free sampling interval can be easily obtained by computing $f_{s,\max}$, which requires only the computation of f_{u_i} for $i = 1, 2, 3, 4$ and the minimum of these four frequencies. We can also use $f_{s,\max}$ to find the next interval for alias-free sampling. In particular using $f_{s,\max}$ plus a small ϵ as the starting frequency in the proposed iterative algorithm, we can find the next alias-free sampling frequency and alias-free sampling interval.

Sampling With Guard Bands: In practice it is desirable to have guard bands between different bandpass signals after sampling. Suppose the minimum guard band is GB. Then every 2 replicas from different bandpass signals should be spaced apart by at least GB after sampling. Let us make the following adjustment of band edges for $X_1(f)$ and $X_2(f)$

$$f'_{h_i} = f_{h_i} + GB/2,$$
$$f'_{\ell_i} = f_{\ell_i} - GB/2, \quad \text{for} \quad i = 1, 2. \tag{13}$$

Then $X_1(f)$ and $X_2(f)$ have expanded one-sided bandwidths $W'_1 = W_1 + GB$, and $W'_2 = W_2 + GB$, respectively. The iterative algorithm in Section III can be modified accordingly. The original band edges and bandwidths are used to check (4)–(5) and to increase the sampling frequency in (9)–(10) when the conditions are not satisfied. The new band edges in (13) and the expanded bandwidths W'_i are used to check (6)–(7) and to increase the sampling frequency in (11)–(12) when the conditions are not satisfied. As a result there will be a spacing of at least GB between replicas of different bandpass signals.

IV. Multiband Signals

We can extend the proposed algorithm to find the minimum sampling frequency for signals that contain multiple bandpass signals. Suppose we are to sample a signal consisting of N bandpass signals ($2N$ bands). Since every two of the passbands may cause aliasing, we need to consider C_2^{2N} cases. However, we will see that due to symmetry these C_2^{2N} cases result in only N^2 conditions. First consider the pair $X_i^+(f)$ and $X_i^-(f)$. There is no aliasing if

$$2f_{h_i}(\mathrm{mod} f_s) = 0, \quad \text{or} \quad 2f_{h_i}(\mathrm{mod} f_s) \ge 2W_i, \tag{14}$$

for $i = 1, 2, \ldots, N$. Thus, we have N alias-free conditions. Consider the other $C_2^{2N} - N$ cases. If replica of $X_i^+(f)$ and $X_j^+(f)$ are not aliasing after sampling, then replicas of $X_i^-(f)$ and $X_j^-(f)$ will not be aliasing due to symmetry. The corresponding condition is

$$(f_{h_j} - f_{\ell_i})(\mathrm{mod} f_s) = 0, \quad \text{or}$$
$$(f_{h_j} - f_{\ell_i})(\mathrm{mod} f_s) \ge W_i + W_j,$$
$$\text{for} \quad 1 \le i < j \le N. \tag{15}$$

Similarly, if $X_i^+(f)$ and $X_j^-(f)$ are not aliasing after sampling, then $X_i^-(f)$ and $X_j^+(f)$ will not be aliasing. This requires

$$(f_{h_i} + f_{h_j})(\mathrm{mod} f_s) = 0, \quad \text{or}$$
$$(f_{h_i} + f_{h_j})(\mathrm{mod} f_s) \ge W_i + W_j,$$
$$\text{for} \quad 1 \le i < j \le N. \tag{16}$$

There are N conditions in (14), $N(N-1)/2$ conditions in (15) and $N(N-1)/2$ conditions in (16). Combining (14)–(16), we have a total of N^2 conditions. We can examine each of the N^2 conditions. If one condition is not satisfied, we can always increase the sampling frequency so that the condition becomes satisfied. By iteratively examining the conditions and increasing the sampling frequency, we can find the minimum frequency for alias-free sampling. Similar to the two-bandpass case, we can find alia-sfree sampling intervals for multiband signals by considering every pair of bandpass signals. Moreover, we can leave guard bands between different bandpass signals after sampling as in the two-bandpass case. We can make the adjustment of band edges and bandwidths and use the conditions in (15)–(16).

Ordering Constraint: We can also extend our algorithm to the case when the replicas are constrained to have a certain ordering [6]–[8] in the folded spectrum. For example, suppose it is desirable that the

TABLE I
COMPLEXITY FOR FINDING THE MINIMUM SAMPLING FREQUENCY WITHOUT AN ORDERING CONSTRAINT FOR DIFFERENT COMBINATIONS OF BANDPASS SIGNALS

	$f_{s,min}$ (MHz)	Method in [9]			Method in [10]			Proposed Method		
		ADD	MUL	iter	ADD	MUL	iter	ADD	MUL	iter
GSM900, GSM1800	240	29	50	32	38	50	27	17	28	7
DAB, 802.11g	46.788	65	122	80	161	296	150	42	78	18
GSM900, WCDMA	64.3636	35	62	40	101	176	90	21	29	7
DAB, WCDMA	13.9737	119	230	152	452	878	441	77	141	34
GSM900, GSM1800, 802.11g	320	105	186	120	87	109	59	60	42	8
DAB, GSM1800, 802.11g	209.6139	75	126	80	99	133	71	41	36	6
GSM900, DAB, WCDMA	77.2364	183	342	224	198	331	170	77	84	15

TABLE II
COMPLEXITY FOR FINDING THE MINIMUM SAMPLING FREQUENCY WHEN THE SIGNALS BANDS IN THE FOLDED SPECTRUM ARE CONSTRAINED TO HAVE THE SAME ORDERING AS THE ORIGINAL BANDPASS SIGNALS

	$f_{s,min}$ (MHz)	Method in [8]			Proposed Method		
		ADD	MUL	iter	ADD	MUL	iter
GSM900, GSM1800	417.78	16	24	3	13	26	4
DAB, 802.11g	58.60	30	42	10	33	82	11
GSM900, WCDMA	77.24	18	18	4	12	34	5
DAB, WCDMA	14.02	26	34	8	27	66	9
GSM900, GSM1800, 802.11g	4864	8	16	4	37	62	8
DAB, GSM1800, 802.11g	4864	16	32	8	54	94	12
GSM900, DAB, WCDMA	4248	30	60	15	79	168	21

replicas follow the same ordering of the original bandpass signals. There is no aliasing if the sampling frequency f_s satisfies

$$(f_{h_i}(\mathrm{mod} f_s)) - (f_{h_{i-1}}(\mathrm{mod} f_s)) \geq W_i, \quad i = 1, 2, \cdots, N, \quad \text{and} \quad f_{h_N}(\mathrm{mod} f_s) \leq f_s/2 \tag{17}$$

where we have used $f_{h_0} = 0$ for convenience. This implies $f_{h_i}(\mathrm{mod} f_s) \leq f_s/2 - \sum_{j=i+1}^{N} W_j, \quad i = 1, 2, \ldots, N$. Combining this condition with that in (17), we have the following necessary and sufficient condition for alias-free sampling with ordering constraint:

$$f_{h_{i-1}}(\mathrm{mod} f_s) + W_i \leq f_{h_i} \cdot (\mathrm{mod} f_s) \leq f_s/2 - \sum_{j=i+1}^{N} W_j, \quad i = 1, 2, \ldots, N. \tag{18}$$

Using a procedure similar to the case without ordering constraint, we can examine the conditions in (18) one by one and increase f_s until all the conditions are satisfied. For instance, for a given f_s suppose the first $(i-1)$ equations are satisfied but the ith equation is not. Further suppose the first inequality of the ith equation does not hold and we are to increase the sampling frequency. Notice that the first inequality involves both $f_{h_{i-1}}(\mathrm{mod} f_s)$ and $f_{h_i}(\mathrm{mod} f_s)$ and it is much easier to make minimum increment in f_s to satisfy the second inequality. Such a new sampling frequency $f_{s,\mathrm{new}}$ can be computed using a procedure similar to that in Lemma 2 and we have

$$f_{s,\mathrm{new}} = \left(f_{h_i} + \sum_{j=i+1}^{N} W_j\right) / (\lfloor f_{h_i}/f_s \rfloor - 1/2). \tag{19}$$

Now consider the case that the first inequality of the ith condition in (18) holds, but the second inequality does not. We can increase the sampling frequency such that the second inequality in (18) becomes an equality. The new sampling frequency can be computed using

$$f_{s,\mathrm{new}} = \left(f_{h_i} + \sum_{j=i+1}^{N} W_j\right) / (\lfloor f_{h_i}/f_s \rfloor + 1/2). \tag{20}$$

The above discussion leads to the following iterative algorithm for finding the minimum sampling frequency with a constrained ordering. Let $f_s = 2\sum_{j=1}^{N} W_j$.

1. Examine the conditions in (18) one by one. If any one of the condition is not satisfied, go to the next step. If all the conditions are satisfied, then we have found the solution.
2. For the condition that is violated in Step 1, compute the corresponding new sampling frequency using (19) or (20). Go to Step 1.

By modifying above algorithm slightly, we can find the minimum sampling frequency for an arbitrary ordering. This can be done by re-naming the bandpass signals. We can rename the replicas from left to right as replicas of $X'_1(f), X'_2(f), \ldots, X'_N(f)$, the algorithm can then be applied on $X'_i(f)$.

V. SIMULATIONS

In this section, we apply the proposed algorithm to wireless applications. The bandpass signals considered in the simulations are GSM 900 (935–960 MHz, one-sided bandwidth 25 MHz), GSM 1800 (1805–1880 MHz, one-sided bandwidth 75 MHz) [19], DAB Eureka-147 L-Band (1472.286–1473.822 MHz, one-sided bandwidth 1536 KHz) [20], IEEE 802.11g (2412–2432 MHz, one-sided bandwidth 20 MHz) [21], and WCDMA (2119–2124 MHz, one-sided bandwidth 5 MHz).

Table I lists the complexity of finding $f_{s,\min}$ for different combinations of two-bandpass signals and three-bandpass signals. The complexity is given in terms of numbers of multiplications (MUL), additions (ADD) and iterations (iter) counted in actual implementation of the algorithms. The simulation result demonstrates that the proposed method can reduce the number of additions, multiplications, and iterations considerably. Table II lists the minimum sampling frequency and complexity when there is an ordering constraint among the replicas. The constraint is such that in the $[0, f_s)$ frequency range the replica of $X_i^+(f)$ is at the left of $X_{i+1}^+(f)$. Although our method can also compute the minimum sampling frequency with constraint, the method in

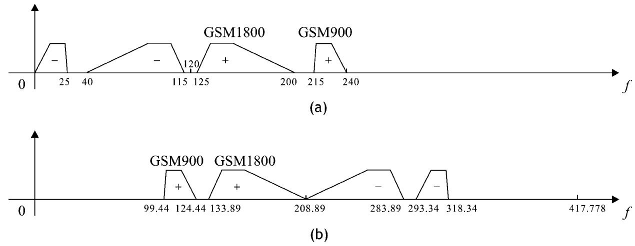

Fig. 3. An example of the replica in $[0, f_s)$ for (a) $f_{s,\min} = 240$ MHz without an ordering constraint, and (b) $f_{s,\min} = 417.778$ MHz with an ordering constraint.

[8] is more efficient. (The method in [8] is not compared in Table I as it does not compute minimum sampling frequency without ordering restriction.) Comparing Tables I and II, we can see that the minimum sampling frequency without a constraint can be much smaller than that with a constraint. Fig. 3 shows the replica in $[0, f_s)$ with a constraint ($f_{s,\min} = 417.78$ MHz) and without a constraint ($f_{s,\min} = 240$ MHz) when the bandpass signals are GSM 900 and GSM1800 as in the first case of Table II.

VI. Conclusion

We have proposed a new algorithm for finding the minimum sampling frequency for multiband signals. We have derived a new set of conditions for alias-free sampling. These conditions lead to an iterative algorithm for finding the minimum sampling frequency. There is no need to consider ordering of the signal bands in the folded spectrum in the implementation of algorithm. The method can be generalized to find alias-free sampling frequency intervals and to find the minimum sampling frequency when the ordering of replicas is constrained.

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Cooperative Interference Management With MISO Beamforming

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***Abstract*—In this correspondence, we study the downlink transmission in a multi-cell system, where multiple base stations (BSs) each with multiple antennas cooperatively design their respective transmit beamforming vectors to optimize the overall system performance. For simplicity, it is assumed that all mobile stations (MSs) are equipped with a single antenna each, and there is one active MS in each cell at one time. Accordingly, the system of interests can be modeled by a multiple-input single-output (MISO) Gaussian interference channel (IC), termed as MISO-IC, with interference treated as noise. We propose a new method to characterize different rate-tuples for active MSs on the Pareto boundary of the achievable rate region for the MISO-IC, by exploring the relationship between the MISO-IC and the cognitive radio (CR) MISO channel. We show that each Pareto-boundary rate-tuple of the MISO-IC can be achieved in a decentralized manner when each of the BSs attains its own channel capacity subject to a certain set of interference-power constraints (also known as interference-temperature constraints in the CR system) at the other MS receivers. Furthermore, we show that this result leads to a new decentralized algorithm for implementing the multi-cell cooperative downlink beamforming.**

***Index Terms*—Beamforming, cooperative multi-cell system, interference channel, multi-antenna, Pareto optimal, rate region.**

I. Introduction

Conventional wireless mobile networks are designed with a cellular architecture, where base stations (BSs) from different cells control communications for their associated mobile stations (MSs) independently. The resulting inter-cell interference is treated as

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