Design of Causal Stable IIR Filter Banks with Powers-of-Two Coefficients *

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ABSTRACT

We introduce an efficient method for the design of causal stable IIR filter bank (FB) with Powers-of-two coefficients. Ladder structure is used to construct the IIR FBs with perfect reconstruction (PR). In the proposed method, FBs with real coefficients are first designed and an iterative procedure is then employed to discretize the coefficients. A sensitivity measure is introduced to determine the order of coefficients to be discretized. To ensure the stability of the IIR filters, the stability triangle is used. Even though the method is suboptimal, it avoids integer programing and yields satisfactory results.

1 Introduction And Previous Work

Fig. 1 shows a two-channel filter bank (FB). It has been widely applied to various areas of signal processing [1]. In FIR FBs, all the four filters $H_0(z)$, $H_1(z)$, $F_0(z)$ and $F_1(z)$ are FIR filters while in IIR FBs, some or all of these filters are IIR filters. The theory and design of FIR FBs have been studied by many researchers while there are relatively few studies on the IIR case until recently. Though IIR filters have the advantage of low cost, it is not easy to obtain satisfactory perfect reconstruction (PR) FB with causal stable IIR filters. The earliest good designs for IIR FBs were such that the analysis bank was paraunitary and the filters have allpass polyphase components (see p. 201 of [1]). Even though the filters are causal stable, such IIR FBs suffer from phase distortion. IIR PR FBs typically have noncausal stable filters [2] [3].

In recent years, PR FBs with causal stable IIR filters have been successfully constructed [4] [5] [6]. By constraining the determinant of the stable IIR polyphase matrix to be a minimum phase transfer function, the authors in [4] are able to obtain PR IIR FBs with causal stable filters. FBs with good frequency separation can be obtained by optimizing the filter responses. In [5] and [6], a different approach, namely the ladder structure (or so-called lifting scheme), was proposed as a framework for IIR FBs. Using such structure, causal stable IIR FBs with good frequency responses can easily be obtained by designing a single transfer function. Moreover these 1D FBs can be mapped into nonseparable 2D case using a simple mapping that preserves all the useful properties [5] [6].

Filter banks with powers-of-two coefficients have the advantage that no multiplication is needed in their implementation. In such a FB, the coefficients are in general represented by a radix-2 canonic-signed-digit (CSD) code. The radix-2 signed-digit representation of a fractional number c is given by

$$c = \sum_{k=1}^{L} s_k 2^{-p_k}, \tag{1}$$

where $s_k = \pm 1$ and $p_k \in \{0,\ 1,\ \ldots,\ M\}$. The number of adders/subtractors needed to implement such a coefficient is $L - 1$. In general, the radix-2 signed-digit representation is not unique. The CSD code is the representation that has the minimum number L of nonzero digit. In this paper the class of CSD code which can be expressed as (1) will be denoted as CSD(M,L). FIR FBs with such discrete coefficients have been successfully designed by a number of researchers [7] [8] [9] [10]. In [7], multiplierless linear-phase FIR FBs with CSD code was designed. The resulting FBs do not achieve PR and the magnitude distortion is minimized. In [8], FIR lattice structure for orthonormal FBs was used for the design. The lattice coefficients were optimally represented using CSD code. PR FBs with CSD coefficients are obtained. In [9] and [10], a weighted least-square algorithm was proposed for the design of uniform and nonuniform FBs. The continuous coefficients are then discretized using the CSD code. FBs with very sharp filters can be obtained using the proposed method.

In this paper, we introduce an efficient method for the design of causal stable IIR PR FBs with CSD coefficients. In Sec. 2, we will briefly discuss the ladder structure, which will be used as a framework for the construction of IIR FBs in this paper. Stability triangle will be described in Sec. 3. In Sec. 4, we will introduce an iterative algorithm to discretize the coefficients and

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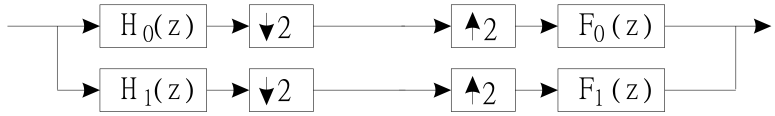

Figure 1: A two-channel filter bank.

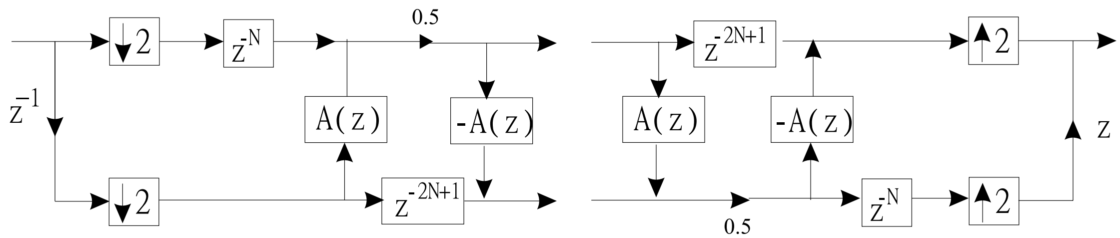

Figure 2: The ladder structure for two-channel filter bank.

a design example will be given.

2 Two-channel Ladder Structure

Fig. 2 shows the ladder structure for two-channel FBs. Such a structure is also well-known as the lifting scheme[11]. In these FBs, the analysis filters are related to the transfer function $A(z)$ as:

$$H_0(z) = \frac{z^{-2N} + z^{-1}A(z^2)}{2}, \tag{2}$$

$$H_1(z) = -A(z^2)H_0(z) + z^{-4N+1}. \tag{3}$$

The synthesis filters are $F_0(z) = -zH_1(-z)$ and $F_1(z) = zH_0(-z)$. These FBs enjoy many advantages[6]. In this paper, we will exploit the following properties of ladder structure FBs:

1. *Structurally PR:* The FBs have PR for any choice of $A(z)$, provided that the analysis and synthesis banks use the same implementation of $A(z)$. In particular, the FBs continue to have PR even when the coefficients of $A(z)$ are quantized.

2. *Low design cost:* As PR is structurally guaranteed, we can optimize the responses of $H_0(z)$ and $H_1(z)$ by choosing any $A(z)$ while preserving the PR at the same time. Two approximation methods are given in [6], namely FIR linear-phase PR FBs and causal stable IIR PR FBs. In this paper, we will consider only the IIR case. To get a good IIR lowpass filter $H_0(z)$, $A(z)$ can be chosen as an Nth-order allpass function. The phase response of $A(z^2)$ can be optimized so that it approximates the desired phase response:

$$\angle\phi_{DA}(e^{2j\omega}) = \begin{cases} (-2N+1)\omega & \text{if } \omega \in [0, \pi/2]; \\ (-2N+1)\omega \pm \pi & \text{if } \omega \in (\pi/2, \pi]. \end{cases} \tag{4}$$

It was shown[6] that if the lowpass filter $H_0(z)$ obtained by such method has an attenuation of x dB, then $H_1(z)$ will be a highpass filter with an attenuation of around $x - 10$ dB. Moreover $F_0(z)$ and $F_1(z)$ will be lowpass and highpass filters with an attenuation of $x - 10$ dB and x dB respectively. Therefore the design of an IIR FB reduces to the design of a single allpass function with the phase constraint in (4).

3. *Low implementational cost:* To implement such FBs, we need to implement only one transfer function $A(z)$. Note that all the computations are done at half the input data rate. Suppose that the allpass function $A(z)$ has order N, we need a total of only $2N$ multiplications and $4N + 4$ additions per input sample to implement both analysis and synthesis banks.

In addition to the above advantages, ladder structure FBs have many other merits. For examples[6], (i) IIR FBs designed using the above phase constraint are guaranteed to be causal stable; (ii) zeros of arbitrary multiplicity at $\omega = \pi$ of $H_0(z)$ can be easily imposed in the design, for the purpose of generating wavelets with regularity property; (iv) nonseparable 2D PR FB with IIR filters can be obtained by applying a simple 1D to 2D mapping.

3 Stability Triangular for Second-Order IIR Transfer Functions

Given a general Nth-order polynomial, the necessary and sufficient conditions on the coefficients so that the roots of the polynomial lie within the unity circle are still unknown (to our knowledge). Fortunately this is not true for the special case of second-order polynomial. Consider a second-order polynomial of the form

$$D(z) = 1 + a_1z^{-1} + a_2z^{-2}.$$

It was shown in Homework Problem 2-13 of [1] that the zeros of $D(z)$ lie strictly inside the unit circle if and only if the coefficients a_1 and a_2 are strictly inside the triangular region shown in Fig. 3.

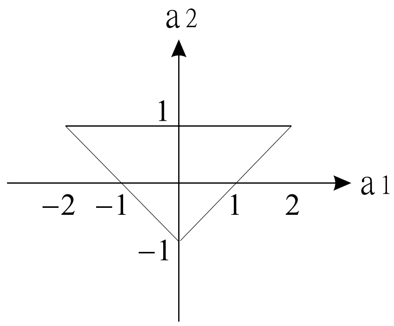

Figure 3: The stability triangle.

4 IIR FBs with CSD Coefficients

As we have mentioned in Sec. 2, using the ladder structure, the design of IIR FBs reduces to the design of a single allpass transfer function:

$$A(z) = \frac{a_N + a_{N-1}z^{-1} + \ldots + z^{-N}}{1 + a_1 z^{-1} + \ldots + a_N z^{-N}}.$$

There are many efficient methods for the design of allpass functions with real coefficients satisfying the phase constraint given in (4) [6] [12]. In this paper, we assume that we are given a real coefficient allpass function that satisfies (4) approximately. To minimize the effect of coefficient quantization on the response, we will consider the cascade form implementation. Suppose that $A(z)$ is factorized as

$$A(z) = A_0(z)A_1(z)\ldots A_{K-1}(z), \qquad (5)$$

where K is the smallest integer larger than $N/2$ and $A_i(z)$ are real-coefficient second-order allpass function:

$$A_i(z) = \frac{a_{i2} + a_{i1}z^{-1} + z^{-2}}{1 + a_{i1}z^{-1} + a_{i2}z^{-2}}.$$

Using the cascade form, one can quantize the coefficients of $A_i(z)$ by directly finding the CSD(M,L) code in (1) that is closest to the original coefficient. In the process of quantization, one can restrict the CSD code of a_{i1} and a_{i2} so that they lie strictly inside the stability triangle in Fig. 3. By doing so, we can ensure the stability of the resulting discretized filter. However such direct quantization method will in general result in filters with unsatisfactory response, as we will see in the design exmple. To improve the response of the transfer function, we introduce the following iterative method.

Iterative Method for Coefficient Discretization

Instead of discretizing all the factors $A_i(z)$ simultaneously, we discretize them sequencially. Suppose that $A_J(z)$ is the first factor to be discretized using CSD code and $\hat{A}_J(z)$ is the corresponding discretized factor. Let the phase response of $A_J(z^2)$ and $\hat{A}_J(z^2)$ be $\angle A_J(e^{2j\omega})$ and $\angle \hat{A}_J(e^{2j\omega})$ respectively. Define

$$B_J(z) = A_0(z)\ldots A_{J-1}(z)A_{J+1}(z)\ldots A_{K-1}(z). \qquad (6)$$

That is, $B_J(z)$ is the $(N-2)$th-order allpass function obtained from $A(z)$ by removing the factor $A_J(z)$. Given $\hat{A}_J(z)$, the real coefficient $(N-2)$th order allpass function $B_J(z)$ is no longer optimal. In other words, the product $B_J(e^{2j\omega})\hat{A}_J(e^{2j\omega})$ may not approximate $\phi_{DA}(e^{2j\omega})$ well enough. Therefore we re-optimize $B_J(z)$. The desired phase response for $B_J(z^2)$ is:

$$\phi_{DB_J}(e^{2j\omega}) = \phi_{DA}(e^{2j\omega}) - \angle \hat{A}_J(e^{2j\omega}). \qquad (7)$$

We can optimize the $(N-2)$th-order allpass function $B_J(z^2)$ so that its phase approximates $\phi_{DB_J}(e^{2j\omega})$. This can be easily achieved using any methods described in [6] [12]. We can again apply the above discretization procedure on the re-optimized $B_J(z)$ and one second-order factor from $B_J(z)$ is discretized. By repeating the above procedure, we will arrive at a cascade form of $A(z)$ with discretized second-order factors.

To decide the ordering of the factors in the discretization process, we introduce a sensitivity measure. The sensitivity of the factor $A_i(z)$ is defined as:

$$S(A_i) = \frac{\int_0^{\pi} |\angle A_i(e^{j\omega}) - \angle \hat{A}_i(e^{j\omega})| d\omega}{\sqrt{(a_{i1} - \hat{a}_{i1})^2 + (a_{i2} - \hat{a}_{i2})^2}}, \qquad (8)$$

where $\hat{a}_{i1}$ and $\hat{a}_{i2}$ are the CSD codes for the coefficients a_{i1} and a_{i2} respectively. In this paper, the factor with the highest sensitivity is chosen as the factor to be discretized.

Remark: During the design simulation, we found that if the factor with the lowest sensitivity is chosen, then the resulting filter response may be worse than that of direct discretization.

Example. In this example, the transfer function $A(z)$ is taken as an 6th order allpass function. The integer N in Fig. 2 is therefore equal to 6. Therefore to implement both the analysis and synthesis banks, we need a total of 12 multiplications and additions per input sample. For the CSD code expressed in (2), we take $M = 10$ and $L = 3$. So we need only 2 additions/subtractions to implement each coefficient. The results of quantized $|H_0(e^{j\omega}))|$ for direct quantization and our method are shown in Fig. 3 and 4 respectively. As we can see from the figures that the proposed method is much better than the direct quantization. The stopband attenuations of the original filter, filter discretized with direct quantization, and filter discretized with the proposed method are respectively 38.3 dB, 31.4 dB, and 37.4 dB.

The denominators of the factors discretized using our method are

$$1 + (2^0 - 2^{-2} + 2^{-4})z^{-1}$$
$$1 + (-2^{-1} + 2^{-3} + 2^{-5})z^{-1}$$
$$1 + (-2^{-1} + 2^{-3} + 2^{-8})z^{-1} + (2^{-3} + 2^{-8})z^{-2}$$
$$1 + (2^{-1} - 2^{-3} + 2^{-6})z^{-1} + (2^{-2} - 2^{-4} - 2^{-6})z^{-2}.$$

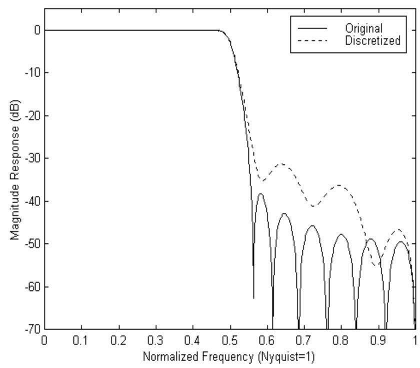

Figure 4: IIR filter quantized directly using CSD code.

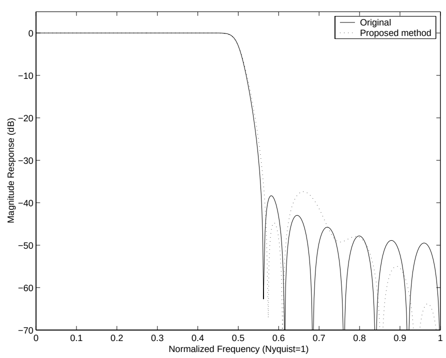

Figure 5: IIR filter quantized by our proposed method using CSD code.

5 Conclusions

In this paper, we introduce an iterative method for the design of IIR PR FBs with powers-of-two coefficients. The method does not need any integer programming and converges very fast. Note that the same iterative procedure can also be applied to design FIR PR FBs with powers-of-two coefficients [13].

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