Design of FIR Filterbank Transceivers with Effective Band Separation *

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ABSTRACT

The DMT (discrete multitone modulation) transceivers have been shown to be a very useful technique for data transmission over frequency selective channels. The DMT scheme is realized by a transceiver that divides the channel into subbands. The efficiency of the scheme depends on the frequency selectivity of the tranceiving filters. The filterbank transceiver or DWMT (discrete wavelet multitone) system, has been proposed as an implementation of DMT transceiver that has better frequency band separation. In this paper, we show how to design filterbank transceivers that have good frequency selectivity and at the same time cancel ISI completely.

1 Introduction

The discrete multitone modulation (DMT) is now a widely used technique for high speed transmission over channels such as digital subscriber loops [1]-[3]. In the DMT scheme, the channel is divided into subbands, each with a different frequency band. The transmission power and bits are judiciously allocated according to the SNR (signal to noise ratio) in each band [3]. This is similar to the water pouring scheme for discrete transmission channels. The realization of the DMT scheme relies on the design of a transceiver that effectively divides the channel into subbands of different frequency bands. Band separation is of particular importance when the channel is highly frequency selective and the SNRs of different frequency bands exhibit large differences.

The DFT based DMT system has been proposed as a practical implementation of DMT system [1]. Very good transmission rate can be accomplished. In the DFT based systems, the transmitter and receiver consists of DFT filters, which have limited frequency selectivity. Narrowband noise could induce serious impairment due to the poor stopband of the receiving filters [4]. For better frequency band separation, Sandberg and Tzannes [5] proposed the so called DWMT (discrete wavelet multitone) system, in which perfect reconstruction filter banks are used as the transceiver. The transmitting and receiving filters have excellent frequency separation property inherited from good filter bank designs. However, when the channel is not ideal, filterbank transceivers obtained from perfect reconstruction filter banks do not have ISI free property. Performance evaluation conducted in [6] shows that the resulting ISI can seriously degrade the system performance. To reduce the amount of ISI, inter- and intra-subband equalization are performed on the receiver outputs in [5][7]. So far, there is no methods for designing ISI free filterbank transceivers over frequency selective channels.

In this paper we will develop design methods for FIR filterbank transceiver with ISI free property using polyphase approach. We will use over-interpolated filter banks to introduce redundancy, which enabling us to cancel ISI completely. Two methods will be proposed for designing FIR transceivers with zero ISI.

2 Polyphase Representation of Filterbank Transceivers

Consider Fig. 1, where an M-subband filterbank transceiver is shown. The channel is represented by an FIR filter $P(z)$ and an additive noise $e(n)$. The channel filter $P(z)$ is assumed to be FIR of order L, which is a reasonable assumption after time domain equalization. The filters $F_k(z)$ and $H_k(z)$ are called transmitting and receiving filters respectively. When the interpolation ratio $N >$ the number of subbands M, we say it is over interpolated and redundancy is introduced in this case.

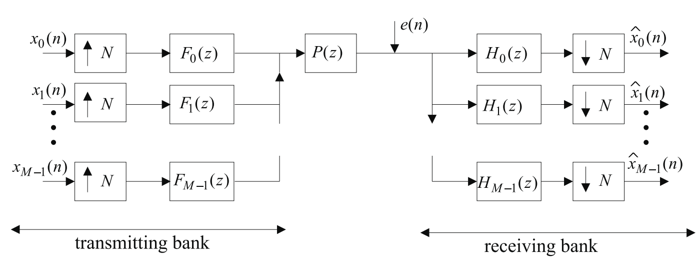

Figure 1: An M-subband filterbank transceiver over a non ideal channel $P(z)$.

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Using polyphase decomposition we can decompose the k-th transmitting filter $F_k(z)$ with respect to the integer N [10],

$$F_k(z) = \sum_{n=0}^{N-1} G_{n,k}(z^N)z^{-n}. \quad (1)$$

Writing the polyphase representation for all the M transmitting filters, we have

$$[F_0(z)\ F_1(z)\ \cdots\ F_{M-1}(z)] = [1\ z^{-1}\ \cdots\ z^{-N+1}]\mathbf{G}(z^N)$$
$$\text{where}\quad [\mathbf{G}(z)]_{n,k} = G_{n,k}(z), 0 \le k < M, 0 \le n < N.$$

The matrix $\mathbf{G}(z)$ is the polyphase matrix of the transmitter. Using the noble identity [10], we can interchange the expander and $\mathbf{G}(z^N)$. The transmitter can be implemented using its polyphase matrix as shown in Fig. 2. In a similar manner, we can decompose the receiving filters as

$$H_k(z) = \sum_{n=0}^{N-1} S_{k,n}(z^N)z^n, \quad (2)$$

Then by invoking the noble identity, the receiver can be redrawn as Fig. 2. The receiving filters $H_k(z)$ are related to the $M \times N$ polyphase matrix $\mathbf{S}(z)$ of the receiver as:

$$\begin{pmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{M-1}(z) \end{pmatrix} = \mathbf{S}(z^N) \underbrace{\begin{pmatrix} 1 \\ z^1 \\ \vdots \\ z^{N-1} \end{pmatrix}}_{\mathbf{d}(z)},$$

$$\text{where}[\mathbf{S}(z)]_{k,n} = S_{k,n}(z), 0 \le k < M, 0 \le n < N \quad (3)$$

Decomposition of the Channel

Using polyphase representation, we can decompose the channel as

$$P(z) = P_0(z^N) + P_1(z^N)z^{-1} + \cdots + P_{N-1}(z^N)z^{-N+1}. \quad (4)$$

Applying the polyphase identity from the multirate theory [10], it can shown that the $N \times N$ system from $\mathbf{y}(n)$ to $\widehat{\mathbf{y}}(n)$ in Fig. 2 is in fact an LTI system $\mathbf{C}(z)$. The transfer matrix $\mathbf{C}(z)$ is pseudo-circulant [10] with the first column given by,

$$\begin{pmatrix} P_0(z) & P_1(z) & \cdots & P_{N-1}(z) \end{pmatrix}^T. \quad (5)$$

Usually the interpolation ratio is chosen to be the same as the order L of $P(z)$. In this case, the N polyphases of $P(z)$ are constants and the last $N-L-1$ polyphases are zero. The matrix $\mathbf{C}(z)$ can be partitioned as an $N \times M$ constant matrix $\mathbf{C}_0$ and an $N \times L$ FIR causal matrix $\mathbf{C}_1(z)$ that is of order 1,

$$\mathbf{C}(z) = [\ \underbrace{\mathbf{C}_0}_{N\times(M)}\ \vdots\ \underbrace{\mathbf{C}_1(z)}_{N\times L}\ ]. \quad (6)$$

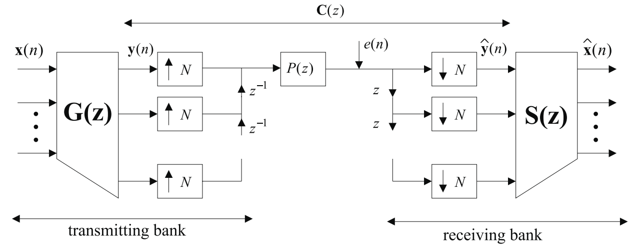


Figure 2: The polyphase representation of the transmitter and receiver in a filterbank transceiver.

3 Filterbank Transceivers with ISI Free Property

From the polyphase decomposition in Fig. 2, we see that even though multirate building blocks are used in a filterbank transceiver, it is in fact an LTI M-input M-output system. The transfer matrix $\mathbf{T}(z)$ of the overall system can be expressed as

$$\mathbf{T}(z) = \mathbf{S}(z)\mathbf{C}(z)\mathbf{G}(z). \quad (7)$$

The overall system is free from ISI if $\mathbf{T}(z)$ is the identity matrix except delays. In the absence of channel noise the outputs of an ISI free filterbank transceiver are identical to the inputs except delays and scalers.

Consider the transmitter $\mathbf{G}(z)$ of the form,

$$\mathbf{G}(z) = \begin{pmatrix} \mathbf{G}_0(z) \\ \mathbf{0} \end{pmatrix}. \quad (8)$$

Every input block of size M goes through an $M \times M$ transfer matrix, and L zeros are inserted between every two blocks before transmission. Then we have

$$\mathbf{C}(z)\mathbf{G}(z) = \mathbf{C}_0\mathbf{G}_0(z).$$

In this case the system is ISI free if

$$\mathbf{S}(z)\mathbf{C}_0\mathbf{G}_0(z) = \mathbf{I}. \quad (9)$$

Thus, the channel dependent term becomes a constant matrix $\mathbf{C}_0$. The receiver $\mathbf{S}(z)$ can be any left inverse for $\mathbf{C}_0\mathbf{G}_0(z)$. The following lemma gives us the condition for an ISI free FIR transceiver [9].

Lemma 1. Suppose the transmitter is as given in (8). Then there exist FIR solutions for $\mathbf{S}(z)$ if and only if the inverse of $\mathbf{G}_0(z)$ is FIR. In this case, the solution of the receiver is of the form

$$\mathbf{S}(z) = \mathbf{G}_0^{-1}(z)\mathbf{B}, \quad (10)$$

where the $M \times L$ matrix $\mathbf{B}$ is any left inverse of $\mathbf{C}_0$.

4 Design of FIR ISI Free Filterbank Transceivers

It is known that any causal FIR matrix with an FIR inverse can be factorized as [11]

$$\mathbf{H}(z)\mathbf{E}(z),$$

where $\mathbf{H}(z)$ is causal FIR orthogonal and $\mathbf{E}(z)$ is causal FIR unimodular. The class of FIR orthogonal matrices can be completely factorized into some basic building blocks [10]. There are also classes of unimodular matrices that have been shown to be very useful in filter bank designs [12]. We propose two design methods for FIR filterbank transceivers with ISI free property: one is based on FIR orthogonal matrices and the other is based on unimodular matrices.

4.1 Design based on Orthogonal Matrices

Let us consider the case where $\mathbf{G}_0(z)$ is FIR and $\mathbf{C}_0\mathbf{G}_0(z)$ is FIR orthogonal, i.e.,

$$\left(\mathbf{C}_0\mathbf{G}_0(e^{j\omega})\right)^\dagger \left(\mathbf{C}_0\mathbf{G}_0(e^{j\omega})\right) = \mathbf{I}.$$

Such a construction has the advantage that the receiver can be simply chosen as $\mathbf{S}(z) = \widetilde{\mathbf{G}}_0(z)\mathbf{C}_0^T$. Furthermore in the case of AWGN noise source, the channel noise will not be amplified by the receiver; the average receiver output noise power is the same as the receiver input noise power. Observe that matrix $\mathbf{C}_0$ can be decomposed using SVD (singular value decomposition),

$$\mathbf{C}_0 = \mathbf{U}\begin{pmatrix}\mathbf{\Lambda}\\ \mathbf{0}\end{pmatrix}_{N\times M}\mathbf{V},$$

where $\mathbf{U}$ and $\mathbf{V}$ are respectively $N\times N$ and $M\times M$ orthogonal matrices. The matrix $\mathbf{\Lambda}$ is diagonal and $[\mathbf{\Lambda}]^2_{k,k}$ for $k = 0, 1, \cdots, M-1$ are the eigenvalues of $\mathbf{C}_0^T\mathbf{C}_0$, which are nonzero as $\mathbf{C}_0$ has full rank. It can be shown that if $\mathbf{C}_0\mathbf{G}_0(z)$ is FIR and orthogonal, the matrix $\mathbf{G}_0(z)$ is necessarily of the form

$$\mathbf{G}_0(z) = \mathbf{V}^T\mathbf{\Lambda}^{-1}\mathbf{Q}(z), \qquad (11)$$

where $\mathbf{Q}(z)$ is an arbitrary $M\times M$ FIR orthogonal matrix. Partition $\mathbf{U}$ as

$$\mathbf{U} = [\underbrace{\mathbf{U}_0}_{N\times M}\ \underbrace{\mathbf{U}_1}_{N\times L}]. \qquad (12)$$

Then the product $\mathbf{C}_0\mathbf{G}_0(z)$ assumes the form

$$\mathbf{C}_0\mathbf{G}_0(z) = \mathbf{U}_0\mathbf{Q}(z).$$

In this case ISI free property can be obtained by choosing the receiver $\mathbf{S}(z)$ as

$$\mathbf{S}(z) = \widetilde{\mathbf{Q}}(z)\mathbf{U}_0^T.$$

However the above equation only gives one possible ISI free solution. To obtain all possible solutions, we note that the ISI free condition only requires that $\mathbf{S}(z)$ be a left inverse of $\mathbf{C}_0\mathbf{G}_0(z)$. As $\mathbf{C}_0\mathbf{G}_0(z)$ is of dimension $N\times M$, the receiver $\mathbf{S}(z)$ is not unique. In fact, we can incorporate the left null space of $\mathbf{U}_0$ and choose

$$\mathbf{S}(z) = \begin{pmatrix}\widetilde{\mathbf{Q}}(z) & \Xi(z)\end{pmatrix}\mathbf{U}^T, \qquad (13)$$

where $\Xi(z)$ is an arbitrary $M\times L$ FIR transfer matrix. The flexibility can be exploited to improve the frequency selectivity of the receiving filters or to minimize the total output noise power [8].

4.2 Design based on Unimodular Matrices

The FIR unimodular matrices, unlike orthogonal matrices, do not allow factorization in general. However, a particular class of unimodular has been shown to be very useful in designing M-subband filter banks. Using polyphase matrices that belongs to this class, we can design analysis and synthesis filters with sharp transition bands and good stopband attenuation. The unimodular matrices in this class can be written as a product of lower-triangular and upper-triangular matrices of the following form

$$\mathbf{\Phi}(z)\mathbf{\Psi}(z)$$

where the matrices $\mathbf{\Phi}(z)$ and $\mathbf{\Psi}(z)$ are respectively lower triangular and upper triangular FIR matrices given by,

$$\mathbf{\Phi}(z) = \begin{pmatrix} D_0 & 0 & \cdots & 0\\ \Phi_{1,0}(z) & D_1 & & \\ \Phi_{2,0}(z) & \Phi_{2,1}(z) & & \\ \vdots & & \ddots & \\ \Phi_{M-1,0}(z) & \Phi_{M-1,1}(z) & & D_{M-1}\end{pmatrix},$$

$$\mathbf{\Psi}(z) = \begin{pmatrix} 1 & \Psi_{0,1}(z) & \Psi_{0,2}(z) & \cdots & \Psi_{0,M-1}(z)\\ 0 & 1 & \Psi_{1,2}(z) & & \Psi_{1,M-1}(z)\\ 0 & 0 & 1 & & \\ \vdots & & & \ddots & \\ 0 & & & & 1\end{pmatrix},$$

where D_k are constants and, $\Phi_{i,j}(z)$ and $\Psi_{i,j}(z)$ are FIR filters. It can be immediately verified that such a product matrix $\mathbf{\Phi}(z)\mathbf{\Psi}(z)$ is a unimodular matrix as $\det\mathbf{\Phi}(z) = \Pi_{k=0}^{M-1}D_k$ and $\det\mathbf{\Psi}(z) = 1$. Therefore, its inverse is also FIR.

Consider the following choice of receiver and transmitter pair that is based on the above class of unimodular matrices,

$$\begin{aligned} \mathbf{S}(z) &= \left(\mathbf{\Phi}(z)\mathbf{\Psi}(z) \quad \Xi(z)\right)\mathbf{U}^T,\\ \text{and}\quad \mathbf{G}_0(z) &= \mathbf{V}^T\mathbf{\Lambda}^{-1}\left(\mathbf{\Phi}(z)\mathbf{\Psi}(z)\right)^{-1}, \end{aligned} \qquad (14)$$

where $\Xi(z)$ is an arbitrary $M\times L$ FIR transfer matrix. Using the partition of $\mathbf{U} = \begin{pmatrix}\mathbf{U}_0 & \mathbf{U}_1\end{pmatrix}$ in (12), the receiving filters $H_k(z)$ can be represented by

$$\begin{pmatrix} H_0(z)\\ H_1(z)\\ \vdots\\ H_{M-1}(z)\end{pmatrix} = \mathbf{\Phi}(z^N)\mathbf{\Psi}(z^N)\mathbf{U}_0^T\mathbf{d}(z) + \Xi(z^N)\mathbf{U}_1^T\mathbf{d}(z),$$

where $\mathbf{d}(z)$ is as given in (3). Let

$$\begin{pmatrix} \Theta_0(z)\\ \Theta_1(z)\\ \vdots\\ \Theta_{M-1}(z)\end{pmatrix} = \mathbf{\Psi}(z^N)\mathbf{U}_0^T\mathbf{d}(z).$$

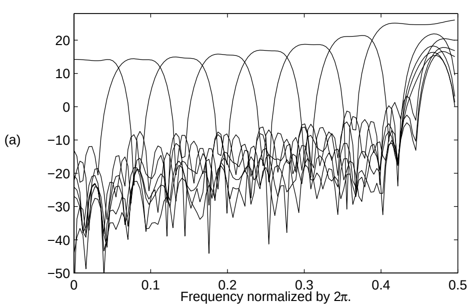
Then, we have $H_k(z)$ given by,

$$H_0(z) = D_0\Theta_0(z) + \boldsymbol{\xi}_0^T(z^N)\mathbf{U}_1^T\mathbf{d}(z)$$
$$H_1(z) = \Phi_{1,0}(z)\Theta_0(z) + D_1\Theta_1(z) + \boldsymbol{\xi}_1^T(z^N)\mathbf{U}_1^T\mathbf{d}(z)$$
$$\vdots$$
$$H_{M-1}(z) = \Phi_{M-1,0}(z)\Theta_0(z) + \Phi_{M-1,1}(z)\Theta_1(z) + \cdots + D_{M-1}\Theta_{M-1}(z) + \boldsymbol{\xi}_{M-1}^T(z^N)\mathbf{U}_1^T\mathbf{d}(z),$$

where $\boldsymbol{\xi}_k^T(z)$ is the k-th row of $\Xi(z)$. We can start the optimization process by designing D_0, $\Theta_0(z)$ and the 0-th row of $\Xi(z)$ to obtain $H_0(z)$. As $\Theta_0(z)$ is already determined in the design of $H_0(z)$, the filter $H_1(z)$ is designed by optimizing $\Phi_{1,0}(z)$, D_1, $\Theta_1(z)$ and $\boldsymbol{\xi}_1^T(z)$. In a similar manner we can continue on to the optimization of $H_2(z)$, $H_3(z)$, $\cdots$, and $H_{M-1}(z)$.

Note that in the design based on orthogonal matrices, the receiving filters are optimized simultaneously. Also all the transmitting filters have the same length and all the receiving filters have the same length. In the unimodular matrices based design, the filters are designed one by one. The filters that are designed earlier will not be affected by the optimization of other filters later. In this case, the filters can have different length. Also, as the filters are designed one by one, the optimization also converges faster than orthogonal design.

Design Example. *Design Using Unimodular Matrices.* The LTI channel to be used in the example is $P(z) = 1 + 0.8z^{-1}$. The order of $P(z)$ is $L = 1$. We choose $M = 8$ and $N = 9$. The transmitter and receiver are as given in (14). The matrices $\mathbf{\Phi}(z)$ and $\mathbf{\Psi}(z)$ are of order 3. The resulting the magnitude responses (dB) of the transmitting and receiving filters are shown in Fig. 3. The stopband attenuation of the receiving filters are around 22 dB.



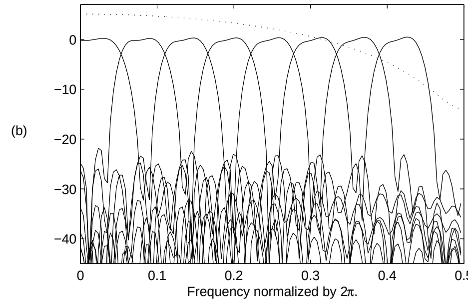


Figure 3: magnitude responses (dB) of (a) the transmitting filters and (b) the receiving filters. Also shown in (b) as a dotted line is the magnitude response of the channel $P(e^{j\omega})$.

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