PERFORMANCE EVALUATION OF OPTIMAL DMT TRANSCEIVERS FOR ADSL APPLICATION

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ABSTRACT

Optimal transceivers that minimize the transmission power for a given probability of error and transmission bit rate will be evaluated for ADSL applications. We will compare the performance of the optimal DMT systems to that of DFT based DMT systems. The experiments show that the optimal DMT system usually has a gain of 3 dB or more over that of DFT based system. For comparable performances, the optimal DMT requires a smaller number of subchannels and hence has a smaller system delay.

1. INTRODUCTION

Fig. 1 shows the block diagram of the DFT based DMT transceiver [1]. It has been shown to have very good transmission rate for ADSL application [2][3]. Kasturia et. al [4] makes an extension to the so-called vector coding system by using a general constant orthogonal transmitting matrix instead of the IDFT matrix. The transmitter is an orthogonal matrix followed by zero padding. For AWGN channels, the optimal transmitting matrix consists of eigen vectors associated with an appropriately defined channel matrix. However, dominating noise sources that arise in ADSL channels are crosstalks from adjacent loops in the same wire bundle and these are typically colored noise.

The design of transceiver that incorporates both the underlying channel and channel noise is considered in [5]. The transmitter also contains a constant orthogonal matrix with zero padding. For a given probability of error and a target bit rate, the transceiver is optimized to achieve a minimum transmitted power. It is shown that the optimal transceiver can be obtained from the vector coding system by pre- and post- multiplying unitary matrices. It is also shown in [5] that when the number of subchannels M approaches infinity, the performance of the DFT based approaches that of the optimal DMT systems. However for moderate number of subchannels, optimal DMT can provide significant gain over the DFT based DMT system.

In this paper we will perform an evaluation of optimal DMT system developed in [5] for ADSL application. The simulation is performed on two CSA test loops with three types of commonly used noise environments. We will see that, for the same number of subchannels, the optimal DMT is better than the DFT based case by around 3 dB. We also compare the performance of the optimal DMT transceivers with that of DFT based transceivers for different number of subchannels. For comparable performance, the optimal DMT requires a smaller number of subchannels, hence a smaller system latency can be achieved.

2. OPTIMAL DMT TRANSCEIVER

A general transceiver with zero padding is as shown in Fig. 2, where $\mathbf{G}$ is $M \times M$ and $\mathbf{S}$ is $M \times N$. Suppose the order of equalized channel $C(z)$ is L. For each input block of size M, the output of $C(z)$ has block size $N = M + L$. The equivalent transform matrix from $\mathbf{y}$ to $\mathbf{r}$ is a constant Toeplitz matrix $\mathbf{C}$ given by,

$$\mathbf{C} = \begin{pmatrix} c_0 & 0 & \dots & 0 & \dots & 0 \\ c_1 & c_0 & & & & 0 \\ \vdots & & \ddots & & & \vdots \\ c_L & c_{L-1} & & & & 0 \\ 0 & c_L & & & & 0 \\ \vdots & \vdots & \ddots & & \ddots & \vdots \\ 0 & 0 & & c_L & & c_0 \\ \vdots & \vdots & & & \ddots & \vdots \\ 0 & 0 & & 0 & \dots & c_L \end{pmatrix}_{N \times M} . \quad (1)$$

The system is ISI free if $\mathbf{SCG} = \mathbf{I}$. Using singular value decomposition (SVD), we can decompose $\mathbf{C}$ as

$$\mathbf{C} = \underbrace{[\mathbf{U}_0 | \mathbf{U}_1]_{N \times N}}_{\mathbf{U}} \begin{pmatrix} \mathbf{\Lambda} \\ \mathbf{0} \end{pmatrix}_{N \times M} \mathbf{V}^T_{M \times M}, \quad (2)$$

where $\mathbf{U}_0$ is of dimensions $N \times M$, $\mathbf{U}_1$ is of dimensions $N \times L$, and $\mathbf{\Lambda}$ is an $M \times M$ diagonal matrix. To have ISI free property, $\mathbf{G}$ and $\mathbf{S}$ can be chosen as

$$\mathbf{G}: \text{ unitary matrix and } \quad \mathbf{S} = \mathbf{G}^T \mathbf{V} \mathbf{\Lambda}^{-1} (\mathbf{I}\ \mathbf{A}) \mathbf{U}^T, \quad (3)$$

where $\mathbf{A}$ is an arbitrary $M \times L$ matrix [5].

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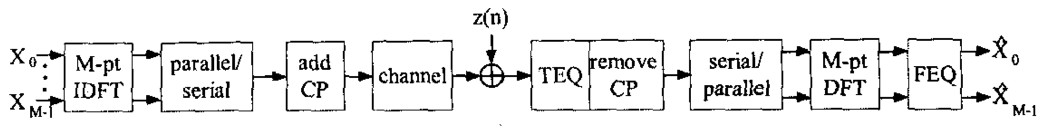

Figure 1: Block diagram of the DFT based DMT transceiver.

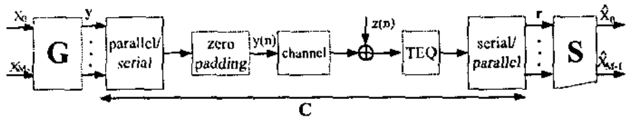

Figure 2: Block diagram of a general DMT transceiver with zero padding.

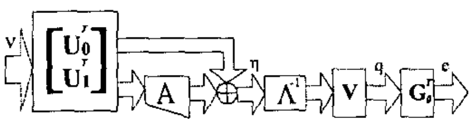

Figure 3: The noise path at the receiver.

Optimal Bit Allocation. The actual transmitted signal is $y(n)$ as shown in Fig. 2. The transmission power $\mathcal{P} = \sigma_y^2$. As the unitary transmitter $\mathbf{G}$ preserves energy, we have

$$\mathcal{P} = \frac{1}{N} \sum_{k=0}^{M-1} \sigma_{x_k}^2, \qquad (4)$$

where $\sigma_{x_k}^2$ is the variance of the input modulation symbol in the $k-th$ subchannel. Using (3), the noise path at the receiver can be redrawn as Fig. 3, where the noise vector ν is the blocked version of the equalized noise after TEQ. Suppose the input are PAM symbols X_k carrying b_k bits, the probability of error is given by

$$P_e = 2(1 - 2^{-b_k})Q\left(\sqrt{\frac{3\sigma_{x_k}^2}{(2^{2b_k} - 1)\sigma_{e_k}^2}}\right), \qquad (5)$$

where $\sigma_{e_k}^2$ is the output noise variance in the $k-th$ subchannel and $Q(y) = \frac{1}{\sqrt{2\pi}}\int_y^\infty e^{-t^2/2}dt,\ y \geq 0$. Using (5) and the approximation $1 - 2^{-b_k} \approx 1$, we have

$$\sigma_{x_k}^2 = c(2^{2b_k} - 1)\sigma_{e_k}^2, \text{ where } c = \frac{1}{3}\left(Q^{-1}(\frac{P_e}{2})\right)^2. \qquad (6)$$

Suppose the average number of bits is b, then

$$b = \frac{1}{M}\sum_{k=0}^{M-1} b_k.$$

For a given transceiver pair $(\mathbf{G}, \mathbf{S})$ as in (3), a fixed probability of error P_e in each subchannel, and an average bit rate b, the required transmission power $\mathcal{P}$ depends on bit allocation. It can be shown that [5], the bit allocation that minimizes $\mathcal{P}$ is given by

$$b_k = b + \frac{1}{2}\log_2 E_0 - \frac{1}{2}\log_2 \sigma_{e_k}^2, \qquad (7)$$

where

$$E_0 = \left(\prod_{k=0}^{M-1} \sigma_{e_k}^2\right)^{\frac{1}{M}}.$$

In this case, the transmission power is

$$\mathcal{P} = \frac{M}{N}c2^{2b}E_0 \equiv \mathcal{P}_{opt-bit}.$$

For a given rate b, $\mathcal{P}_{opt-bit}$ depends on the product $\prod_{k=0}^{M-1}\sigma_{e_k}^2$, which is determined by the transceiver design. According to the ISI free solution given in (3), two things remain to be determined: The matrix $\mathbf{A}$ and the transmitting matrix $\mathbf{G}$. It is shown in [5] that, the optimal transceiver that minimizes $\mathcal{P}_{opt-bit}$ is such that

$$\mathbf{A} = -\mathbf{U}_0^T\mathbf{R}_{\nu\nu}\mathbf{U}_1(\mathbf{U}_1^T\mathbf{R}_{\nu\nu}\mathbf{U}_1)^{-1}. \qquad (8)$$

Furthermore, $\mathbf{G}$ should be the unitary matrix that decorrelates the noise vector $\mathbf{q}$. That is, the autocorrelation matrix of the output noise vector $\mathbf{e}$ is diagonal. The minimum transmitted power is [5],

$$\mathcal{P}_{min} = \frac{M}{N}c2^{2b}\left[\det(\mathbf{\Lambda}^{-2})\frac{\det(\mathbf{R}_{\nu\nu})}{\det(\mathbf{U}_1^T\mathbf{R}_{\nu\nu}\mathbf{U}_1)}\right]^{\frac{1}{M}}, \qquad (9)$$

which is a fixed quantity independent of the choice of the transceiver. Notice that when $\mathbf{G} = \mathbf{V}$ and $\mathbf{A} = \mathbf{0}$, the solution becomes the vector coding system in [4].

3. DESIGN PROCEDURE

Channel estimation. We first obtain an estimate of the channel using training blocks. The estimate will be used for the design of the transmitter and receiver. To obtain noise autocorrelation, one might use the quiet state of initialization [2], when both transmitter and receiver do not send signals.

Time-domain equalization (TEQ). Many TEQ design techniques can be used here, e.g., adaptive design and non-iterative procedure [6]. Having designed TEQ, the autocorrelation matrix $\mathbf{R}_{\nu\nu}$ of the noise vector ν in Fig. 3 can be determined.

Transceiver (G, S). The equalized channel $c(n)$ is obtained by convolving the estimated channel with TEQ. The resulting $c(n)$ may have small non zero coefficients for $n > L$. We use $c(n)$ to obtain the Toeplitz channel matrix $\mathbf{C}$, which has first column given by

$$(c_0\ c_1\ \ldots\ c_{M-1}\ 0 \ldots 0)^T.$$

With $\mathbf{R}_{\nu\nu}$ and SVD of $\mathbf{C}$ in (2), we can compute the matrix $\mathbf{A}$ in (8). Then $\mathbf{G}_0$ can be determined: It should be chosen so that the output noise vector is decorrelated.

Bit allocation. We can use greedy algorithm by allocating one bit at a time. For each bit to be allocated, we compute $\sigma_{x_k}^2 = c(2^{2b_k} - 1)\sigma_{e_k}^2$ as in (6). We give one bit to the subchannel that yields the smallest transmitted power. That is, each time we allocate one bit to the $k\!-\!th$ subchannel that has the smallest $\sigma_{x_k}^2$ until all the bits are allocated. As there are a total of Mb bits in each block of N samples, the transmission bit rate R_b is give by

$$R_b = \frac{M}{N} f_s b,$$

where f_s is the sampling frequency.

4. PERFORMANCE EVALUATION

Two CSA (carrier serviced area) test loops will be used in our simulation: CSA #6 Loop and CSA #7 Loop as shown in Fig. 4. The magnitude responses are shown in Fig. 5. The sampling rate is assumed to be $2.208MHz$. The input and output resistances are assumed to be 100 Ω. We use the TEQ design procedure in [6]. The TEQ has 10 taps. The SIR's (Signal to interference ratio) are 58.6 dB for CSA #6 and 69.7 dB for CSA #7. The magnitude responses of the two equalized channels are shown in Fig. 6.

Example 1. Three types of noise environments will be used in our evaluation [3]: AWGN, AWGN+(10-ISDN NEXT) (10 ISDN NEXT crosstalkers), AWGN+(10-ADSL FEXT) (10 ADSL FEXT crosstalkers). The noise spectrums are shown in Fig. 7. We use $M = 512$ and $L = 32$ and transmission rate $R_b = 6.23Mbps$. For CSA #6 with AWGN and AWGN+10-ADSL FEXT environments (Fig. 8), the optimal transceiver requires a transmission power about 3 dB less than that of the DFT based transceiver. For CSA #7 Loop (Fig. 9), the gain ranges from 1.5 dB to 4 dB.

Example 2. In Fig. 10 and Fig. 11, we compare the performance of the optimal transceivers and DFT-based transceivers for different M. One observes from Fig. 10 that for $P_e = 10^{-5}$, the DFT based system with $M = 512$ is worse than the optimal system with $M = 128$ for CSA #6. In CSA #7

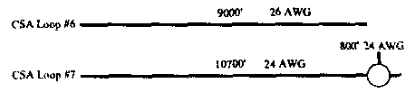

Figure 4: Test loops

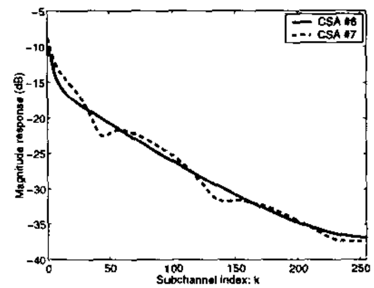

Figure 5: Magnitude responses of test loops.

(Fig. 11), for $P_e = 10^{-5}$, the DFT based system needs at least 2 times the number of subchannels. For comparable performances, the optimal DMT system requires a smaller number of subchannels. As the overall delay of the system is directly related to the number of subchannels M, a smaller M means a smaller system delay.

We also observe from Fig. 10 and Fig. 11 that, when M increases, the performance of the two systems become closer. This is because when M approaches infinity, the performance of DFT based transceiver approaches that of the optimal transceiver [5]. This result is consistent with the asymptotic behavior of DFT based DMT.

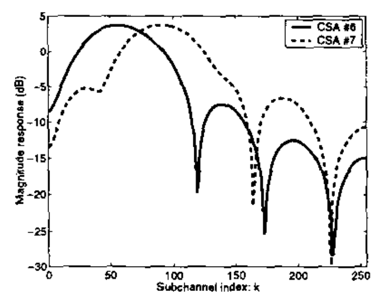

Figure 6: Magnitude responses of equalized channels.

5. REFERENCES

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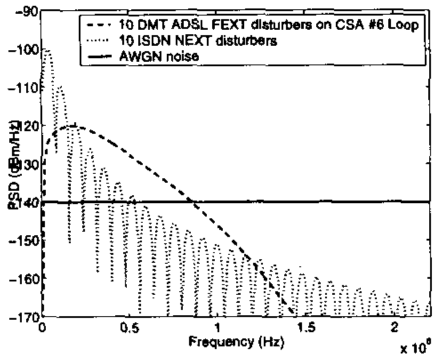

Figure 7: Power spectrums of 10-ISDN NEXT, 10-ADSL FEXT noise and AWGN

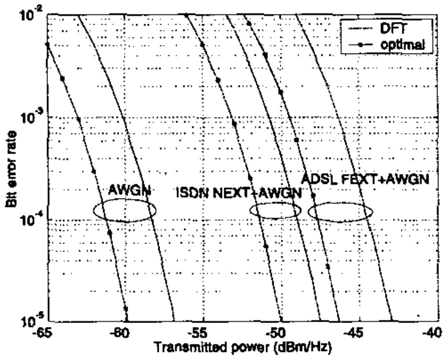

Figure 8: Performance over CSA #6 Loop with three types of noise environments: AWGN, ISDN NEXT+AWGN and ADSL FEXT+AWGN ($R_b = 6.23Mbps, M = 512$).

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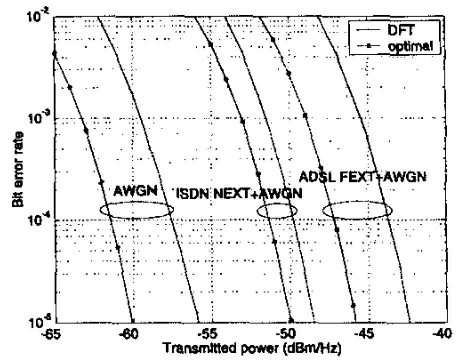

Figure 9: Performance over CSA #7 Loop with three kinds of noise environments: AWGN,, ISDN NEXT+AWGN and ADSL FEXT+AWGN ($R_b = 6.23Mbps, M = 512$).

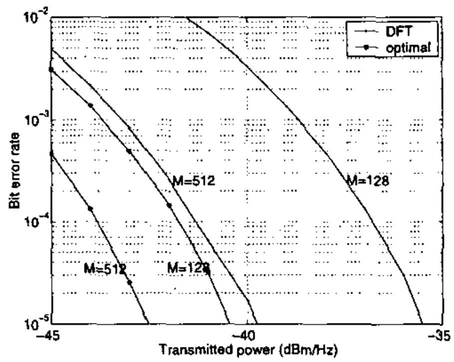

Figure 10: Performance over CSA #6 Loop with ISDN NEXT+AWGN noise environment ($R_b = 8.83Mbps$).

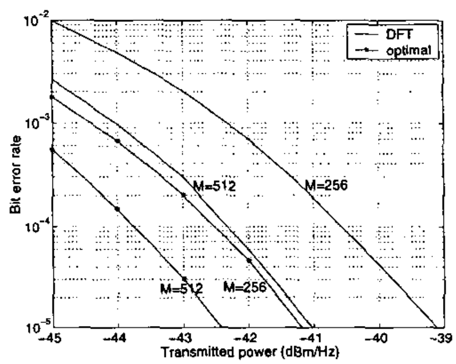

Figure 11: Performance over CSA #7 Loop with ISDN NEXT+AWGN noise environment ($R_b = 8.83Mbps$).