DFT BASED TRANSCEIVERS WITH WINDOWING

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ABSTRACT

In this paper we consider window designs for DFT (discrete Fourier transform) based multicarrier transceivers without using extra cyclic prefix. As in previous works of window designs for DFT based transceiver, a post processing matrix, generally channel dependent, is needed to have a zero-forcing receiver. We show that post processing is channel independent if and only if the window itself has the cyclic-prefixed property. We design optimal windows with minimum spectral leakage subject to the cyclic-prefixed condition. Moreover, we analyze how post processing affects SNR (signal to noise ratio) at the receiver, an aspect that is not considered in most of the earlier works. The resulting SNR can be given in a closed form. Join optimization of spectral leakage and SNR are also considered. Furthermore, examples demonstrate that we can have a significant reduction in spectral leakage at the cost of a small SNR penalty.

1. INTRODUCTION

The DFT based multicarrier systems have found applications in a wide range of transmission systems, e.g., DMT (discrete multitone) for ADSL (asymmetric digital subscriber lines) [2], VDSL (very high speed digital subscriber lines) [3], and OFDM (orthogonal frequency division multiplexing) for wireless LAN (local area network) [4], DVB (digital video broadcasting) [5]. The transmitter and receiver perform respectively M-point IDFT (inverse DFT) and DFT computation, where M is the number of subchannels. At the transmitter side, each block is padded with cyclic prefix of length L. The number L is chosen to be no smaller than the order of the channel, which is usually assumed to be an FIR (finite impulse response) filter. Using redundant cyclic prefix, ISI (inter-symbol interference) is canceled completely. As a result, an FIR channel is converted into M subchannels. The subchannel gains are the M-point DFT of the FIR channel impulse response.

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In the conventional DFT based multicarrier system the pulse shaping filter is a rectangular window. As a rectangular window has large spectral sidelobes, there is a large spectral leakage. This could pose a problem in applications where the PSD (power spectral density) of the transmit signal is required to have a large roll-off in certain frequency bands. For example in some wired transmission application, the PSD of the downstream transmit signal needs to fall below a threshold in the frequency bands of upstream transmission to avoid interference [2, 3]. The PSD should also be attenuated in amateur radio bands to reduce interference or egress emission [3].

Many methods have been proposed to reduce sidelobes by windowing, filtering or using different pulse shaping filters. A number of non-rectangular continuous-time pulse shapes have been proposed to improve the spectral roll-off of the transmit signal, e.g., [6][7][8]. Usually continuous-time pulse shapes are designed based on analog implementation of OFDM transmitters and these pulses usually do not admit a digital implementation [9]. Discrete-time windows that can be easily incorporated in digital transmitters implementation have been considered in [10, 11]. The design of overlapping windows for OFDM with offset QAM (quadrature amplitude modulation) over distortion-less channels are studied fully in [10, 11]. When the channel is distortionless, orthogonality among the subchannels is preserved [10, 11] and a better spectral efficiency is achieved.

However for channels with distortion, i.e., ISI channels, the subchannel outputs contain intra- and inter-subchannel interference; additional processing is required to remove interference in this case. If extra guard time is available, post processing can be avoided at the cost of a reduced transmission rate [12]. When there is no extra cyclic prefix, the use of windowing at the transmitter requires post processing at the receiver. More recently, transmitting windows with the cyclic-prefixed property have been proposed in [13] for egress control. Windows that are the inverse of a raised cosine function are optimized to minimize spectral leakage and hence minimize egress emission. The corresponding zero-forcing receiver also requires post-processing equalization.

In this paper we will consider window designs for DFT

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based multicarrier system without using extra cyclic prefix. We will see that post-processing is in general channel dependent. We will derive the explicit dependency on the channel and show that the post-processing matrix that cancels ISI at the output of the receiver is channel independent if the window itself has the cyclic-prefixed property. In this case, the output of the transmitter has the usual cyclic-prefixed property. Techniques that exploit cyclic prefix for synchronization can still be used. We will design windows that minimize spectral leakage subject to the cyclic-prefixed constraint.

Moreover we show that post processing can affect the SNR at the receiver, an aspect mostly overlooked in earlier window designs for DFT based transceivers. We will see that the resulting SNR can be given in a closed form in terms of the transmit window if the channel noise is AWGN (additive white Gaussian noise). Furthermore, joint optimization of spectral leakage and SNR can be achieved by using an objective function that incorporates both terms. Examples will be given to demonstrate that a good trade-off between spectral roll-off and SNR can be obtained through such an optimization.

The sections are organized as follows: In section 2, we consider windowed transceivers and derive the receiver for a given window. The design method and examples of windowed systems will be given in section 3. A conclusion is given in section 4.

2. DFT BASED TRANSCEIVERS WITH WINDOWS

In this section, we consider windowed DFT based transceivers (Fig. 1). The modulation symbols to be transmitted are first blocked into M by 1 vectors, where M is the number of subchannels. The input symbols s_k are passed through an M by M IDFT matrix, followed by the parallel to serial (P/S) operation and the insertion of cyclic prefix. The length of the cyclic prefix L is chosen to be equal to or larger than the order of the channel $C(z)$. At the receiver, the cyclic prefix is discarded and the samples are again blocked into M by 1 vectors for M-point DFT computation. The scalar multipliers $1/C_k$ are also called frequency domain equalizers, where $C_0, C_1, \cdots, C_{M-1}$ are the M-point DFT of the channel impulse response c_n. The prefix is discarded at the receiver to remove inter-block interference. The transceiver is ISI free and the receiver is a zero-forcing receiver.

2.1. System Model

The transceiver in Fig. 1 can be redrawn as in Fig. 2. The matrices $\mathbf{G}$ and $\mathbf{S}$ shown in Fig. 2 are of dimensions $N \times M$ and $M \times N$, where $N = M + L$. They are given respectively by

$$\mathbf{G} = \begin{pmatrix} \mathbf{0} & \mathbf{I}_L \\ \mathbf{I}_M & \end{pmatrix} \mathbf{W}^\dagger, \quad \text{and} \quad \mathbf{S} = \mathbf{W}\begin{pmatrix} \mathbf{0} & \mathbf{I} \end{pmatrix}. \qquad (1)$$

The matrix Λ indicated in Fig. 2 is diagonal, given by

$$\Lambda = \text{diag}\begin{pmatrix} \frac{1}{C_0} & \frac{1}{C_1} & \cdots & \frac{1}{C_{M-1}} \end{pmatrix}.$$

We can obtain a windowed system by applying a window to each transmitter output block as shown in Fig. 3. The length of the window is the same as the block length N. The window has coefficients $d_0, d_1, \cdots, d_{N-1}$. The conventional system in Fig. 2 can be viewed as having a rectangular window with length N. Due to the non-rectangular window at the transmitter, the receiver needs an additional post processing matrix $\mathbf{P}$ to cancel inter-subchannel interference. As there is no constraint on the matrix $\mathbf{P}$, there is no loss of generality in considering the receiver of the form shown in Fig. 3. The transmitting matrix can be written as $\mathbf{DG}$, where $\mathbf{D}$ is the diagonal matrix $\mathbf{D} = \text{diag}\begin{pmatrix} d_0 & d_1 & \cdots & d_{N-1} \end{pmatrix}$. We partition $\mathbf{D}$ as

$$\mathbf{D} = \begin{pmatrix} \mathbf{D}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_2 \end{pmatrix},$$

where $\mathbf{D}_0$ and $\mathbf{D}_2$ are of dimensions $L \times L$, and $\mathbf{D}_1$ is of dimensions $(M-L) \times (M-L)$. For a given window, we now derive the condition on $\mathbf{P}$ so that the overall system is ISI free.

Lemma 1 *Consider the windowed DFT based transceiver in Fig. 3. The receiver is zero forcing if and only if the post processing matrix $\mathbf{P}$ is given by*

$$\mathbf{P} = \left[\mathbf{W}\begin{pmatrix} \mathbf{D}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_2 \end{pmatrix}\mathbf{W}^\dagger + \Lambda\mathbf{W}\begin{pmatrix} \mathbf{0} & \mathbf{C}_2(\mathbf{D}_0 - \mathbf{D}_2) \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\mathbf{W}^\dagger\right]^{-1}, \qquad (2)$$

where $\mathbf{C}_2$ is an L by L lower triangle Toeplitz matrix with the first column given by $\begin{pmatrix} c_0 & c_1 & \cdots & c_{L-1} \end{pmatrix}^T$.

A proof can be found in [14].

From the above lemma, we see that the solution of the post processing matrix depends on the window $\mathbf{D}$ as well as the channel. This channel dependency means that $\mathbf{P}$ needs to be computed along with other channel dependent parameters. To remove such a dependency, we observe that $\mathbf{P}$ is channel independent if $\mathbf{D}_0 = \mathbf{D}_2$. That is, the window itself has the cyclic-prefixed property. In this case, the post processing matrix is given by $\mathbf{P} = \mathbf{W}\text{diag}(1/d_L \quad 1/d_{L+1} \quad \cdots \quad 1/d_{N-}$
Notice that to have a channel independent $\mathbf{P}$ for any channel, the condition $\mathbf{D}_0 = \mathbf{D}_2$ is not only sufficient but also necessary.

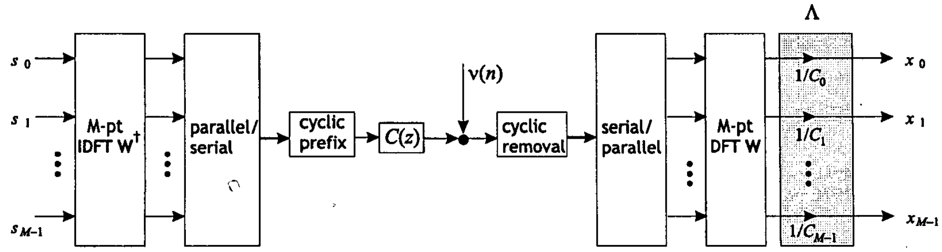

Figure 1: The DFT based multicarrier system over a channel $C(z)$ with additive noise $\nu(n)$.

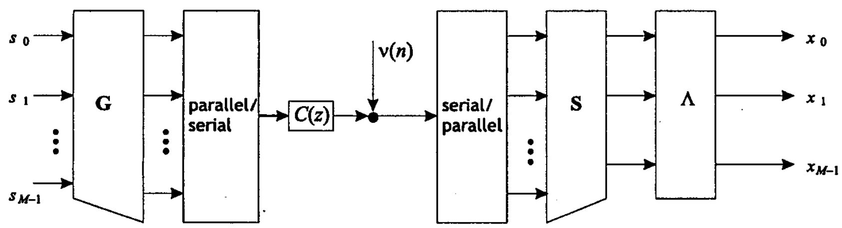

Figure 2: The block based representation of the transceiver in Fig. 1.

Spectral Leakage. Let $D(e^{j\omega})$ denote the Fourier transform of the window function. The stopband energy of the window is

$$\mathcal{S} = \int_{\omega_s}^{2\pi-\omega_s} |D(e^{j\omega})|^2 \frac{d\omega}{2\pi}, \tag{3}$$

where $\omega_s = 2\pi/N$. We define the spectral leakage β as

$$\beta \triangleq \mathcal{S}_{d,cp}/\mathcal{S}_{rec,cp}, \tag{4}$$

where $\mathcal{S}_{rec}$ is the stopband energy of the rectangular window and $\mathcal{S}_{d,cp}$ is the stopband energy of the window $\mathbf{d}$. In section 4, we will see how to design windows that improve the spectral leakage of the window subject to the cyclic prefix condition.

2.2. Output SNR

We assume that the window has the cyclic-prefixed property and the post processing matrix is channel independent. Suppose the channel noise $\nu(n)$ is AWGN with variance $\mathcal{N}_0$. We constrain the transmission power to be the same as the conventional system with a rectangular window. That is, the window satisfies the condition

$$\frac{1}{N}\sum_{k=0}^{N-1} |d_k|^2 = 1. \tag{5}$$

Lemma 2 *[14] Consider the windowed DFT based transceiver in Fig. 3. The channel noise is AWGN with variance $\mathcal{N}_0$. Assume the window has cyclic-prefixed property and the post processing matrix is as given in (2). The total output noise power $\mathcal{E}_{d,cp}$ is given by*

$$\mathcal{E}_{d,cp} = \sigma_q^2 \sum_{k=L}^{N-1} \frac{1}{|d_k|^2} = \frac{\mathcal{N}_0}{M}\left[\sum_{i=0}^{M-1}\frac{1}{|C_i|^2}\right]\left[\sum_{k=0}^{M-1}\frac{1}{|d_k|^2}\right], \tag{6}$$

In the conventional system, the window is rectangular with $d_k = 1$, for $k = 0, 1, \cdots, M-1$. The total output noise power in this case is simply

$$\mathcal{E}_{rec,cp} = \mathcal{N}_0 \sum_{i=0}^{M-1} \frac{1}{|C_i|^2}. \tag{7}$$

To compare the output noise with the conventional system with rectangular window for the same signal variance, we define the quantity SNR loss $\alpha = \frac{\mathcal{E}_{d,cp}}{\mathcal{E}_{rec,cp}}$. Using (6) and (7), we have

$$\alpha = \frac{1}{M}\sum_{k=0}^{M-1}\frac{1}{|d_k|^2}. \tag{8}$$

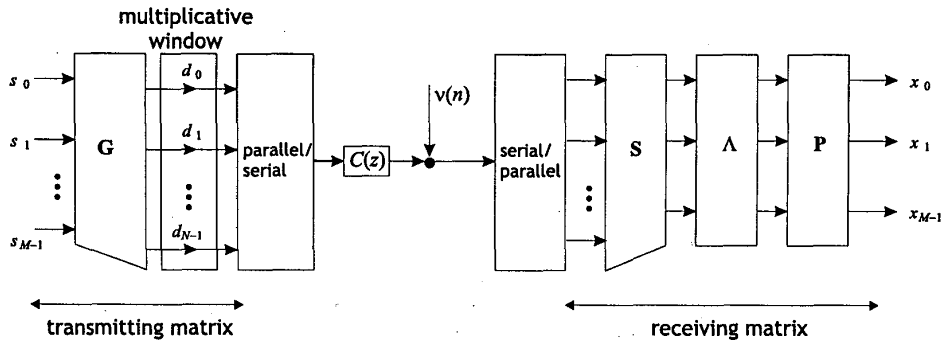

Figure 3: A windowed DFT based transceiver.

3. WINDOW DESIGNS

In this section, we will design optimal windows to minimize spectral leakage. The use of windows improves the spectral roll-off of the transmitter outputs significantly. We will also optimize both spectral leakage and SNR by considering a joint objective function consisting of both terms. The joint optimization yields a trade-off between spectral leakage and SNR.

We have shown in section 2 that a cyclic-prefixed window yields channel independent post processing. We will design windows subject to this constraint. Let $\mathbf{d}$ be the N by 1 window vector and $\widehat{\mathbf{d}} = \begin{pmatrix} d_L & d_{L+1} & \cdots & d_{N-1} \end{pmatrix}^T$, a column vector containing only the last M coefficients of the window. The cyclic-prefixed property means that $\mathbf{d}$ can be written as

$$\mathbf{d} = \mathbf{F}\widehat{\mathbf{d}}, \quad \text{where} \quad \mathbf{F} = \begin{pmatrix} \mathbf{0} & \mathbf{I}_L \\ \mathbf{I}_M & \end{pmatrix}.$$

The Fourier transform of the window can be expressed as

$$D(e^{j\omega}) = \mathbf{e}(e^{j\omega})\mathbf{F}\widehat{\mathbf{d}}, \tag{9}$$

where $\mathbf{e}(e^{j\omega}) = \begin{pmatrix} 1 & e^{-j\omega} & e^{-j2\omega} & \dots & e^{-j(N-1)\omega} \end{pmatrix}$. It follows that the squared magnitude response of the window is

$$|D(e^{j\omega})|^2 = \widehat{\mathbf{d}}^\dagger \mathbf{F}^T \mathbf{e}^\dagger(e^{j\omega})\mathbf{e}(e^{j\omega})\mathbf{F}\widehat{\mathbf{d}} \triangleq \widehat{\mathbf{d}}^\dagger \mathbf{F}^T \mathbf{E}(e^{j\omega})\mathbf{F}\widehat{\mathbf{d}},$$

where $[\mathbf{E}(e^{j\omega})]_{mn} = e^{j\omega(m-n)}$. The stopband energy of the window in (3) can be expressed as

$$\mathcal{S} = \widehat{\mathbf{d}}^\dagger \mathbf{F}^T \int_{\omega_s}^{2\pi-\omega_s} \mathbf{E}(e^{j\omega})\frac{d\omega}{2\pi}\mathbf{F}\widehat{\mathbf{d}} \triangleq \widehat{\mathbf{d}}^\dagger \mathbf{F}^T \mathbf{Q}\mathbf{F}\widehat{\mathbf{d}}. \tag{10}$$

The $N \times N$ matrix $\mathbf{Q}$ is given by

$$[\mathbf{Q}]_{mn} = \begin{cases} 1 - \dfrac{\omega_s}{\pi}, & m = n, \\ -\dfrac{\sin((m-n)\omega_s)}{\pi(m-n)}, & \text{otherwise.} \end{cases}$$

It is real, symmetric and positive semi definite.

Using (10), we can see that the minimization of spectral leakage becomes the minimization of $\widehat{\mathbf{d}}^\dagger \mathbf{F}^T \mathbf{Q}\mathbf{F}\widehat{\mathbf{d}}$. As the product matrix $\mathbf{F}^T\mathbf{Q}\mathbf{F}$ is positive semi definite, the objective function $\widehat{\mathbf{d}}^\dagger \mathbf{F}^T \mathbf{Q}\mathbf{F}\widehat{\mathbf{d}}$ can be minimized by choosing $\widehat{\mathbf{d}}$ to be the eigen vector corresponding to the smallest eigen value of $\mathbf{F}^T\mathbf{Q}\mathbf{F}$. As the matrix $\mathbf{F}^T\mathbf{Q}\mathbf{F}$ is real, the optimal window also has real coefficients. Notice that the resulting window is different from the solution obtained in [13]. In [13], the window is derived subject to the constraint that the window is the inverse of a raised cosine function.

To incorporate SNR in the optimization, we can form the objective function

$$\phi = c\widehat{\mathbf{d}}^T \mathbf{F}^T \mathbf{Q}\mathbf{F}\widehat{\mathbf{d}} + (1-c)\sum_{k=0}^{M-1} \frac{1}{|d_k|^2}, \tag{11}$$

where $0 \le c \le 1$. In this case, the objective function is no longer in quadratic form. Nonlinear optimization packages can be used to design the window, e.g., [15]. The parameter c gives a trade-off between spectral leakage and SNR.

Example 1. Windowed DFT based transceiver. The block size $M = 512$ and prefix length $L = 32$. We form the positive semi definite matrix $\mathbf{F}^T\mathbf{Q}\mathbf{F}$ and compute the eigen vector corresponding to the smallest eigen value to obtain $\widehat{\mathbf{d}}$. The resulting window $\mathbf{d}$ is as shown in Fig. 4(a). The magnitude response of $D(e^{j\omega})$ is shown in Fig. 4(b). Fig. 4(c) shows the spectrum of the transmitter output using the window in Fig. 4(a). The subcarriers used are 38 to 99 and 111 to 255 as in [13]. The subcarriers with indices smaller than 38 are reserved for voice band and upstream transmission, and those with indices between 99 and 111 are for egress control. We see that the spectrum of the windowed output has a much smaller spectral leakage in unused bands.

Example 2. Joint optimization. Using the nonlinear optimization package in [15], we optimize the window to minimize the objective function in (11) that incorporates both spectral leakage and SNR. The resulting windows d for $c = 0, 0.3$ are as shown in Fig. 5(a). Fig. 5(b) shows α and β versus the trade-off factor c. For example, when $c = 0.3$, spectral leakage β is 0.4 and α is 1.1. Different values of c give us different trade-off between spectral leakage and SNR. As c increases, we get a smaller spectral leakage β at the price of a larger SNR loss. Observe that when $c = 0$, the objective function becomes SNR loss only. The resulting window has SNR loss=0.99, slightly less than 1.

Fig. 5(c) shows the transmission rate when the symbol error rate is fixed at 10^{-4}. The input s_k are QAM modulation symbols and subchannel SNRs are used to determined bit allocation. We have used the same set of subcarriers in Fig. 4(c). The windows are designed using $c = 0.3, 0.5$. The channel used in the simulation is Loop 6, a typical channel in carrier serving area [2] and the channel noise is AWGN. Fig. 5(c) shows the number of bits transmitted per block. For example, when SNR=58 dB, 2233 bits and 2204 bits per block are transmitted respectively for $c = 0.3$ and $c = 0.5$. Compared with 2268 bits per block for the rectangular window, the rate losses are only around 1.6% and 2.8% respectively.

4. CONCLUSIONS

In this paper, we considered windowed DFT based multicarrier systems. The spectral leakage of the transmit signal can be reduced significantly by using windows. Like earlier works of windowed transceivers, the use of windows at the transmitter side requires post processing at the receiver side, which is usually channel dependent. We show that, post processing is channel independent if the window itself has cyclic-prefixed property. The optimal window that minimizes spectral leakage of the transmit signal can be given in closed forms. We also show that post processing affects the output SNR. The output SNR can be given in terms of the window and the channel. Furthermore, we can jointly optimize spectral leakage and SNR. The results show that these window designs provide a good trade-off between SNR and spectral leakage.

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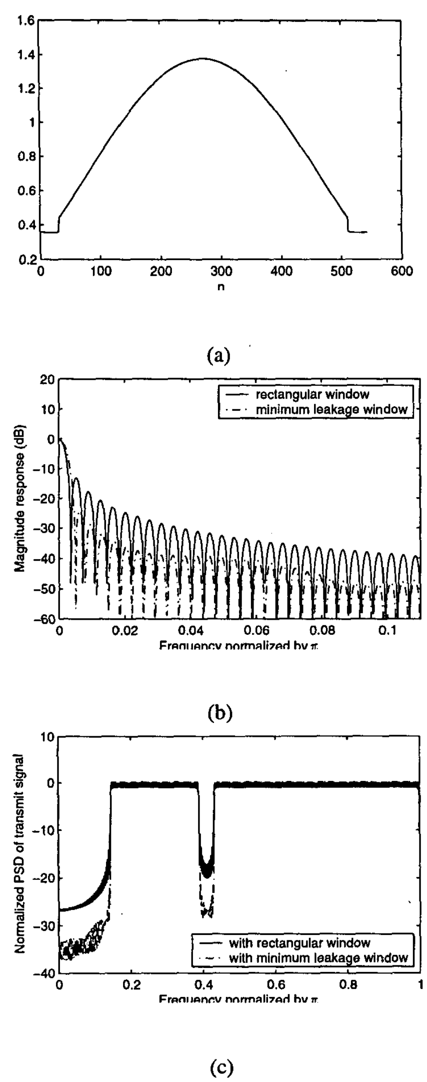

Figure 4: Example 1. Windows for DFT based transceivers; (a) time-domain plot of the window with minimum spectral leakage; (b) magnitude response of the window in (a); (c) power spectral density of the transmit signal.

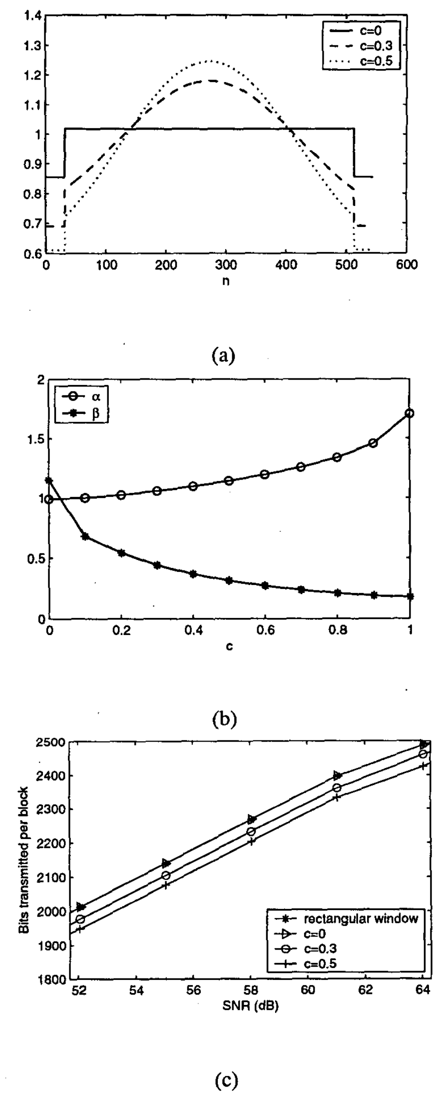

Figure 5: Example 2. Joint optimization; (a) windows obtained by joint optimization of spectral leakage and SNR; (b) spectral leakage β and SNR loss α versus the trade-off factor c; (c) performances of windowed transceivers.