A FREQUENCY-DOMAIN SIR MAXIMIZING TIME-DOMAIN EQUALIZER FOR VDSL SYSTEMS

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ABSTRACT

In this paper we propose a semi-blind time domain equalizer (TEQ) design method that maximize SIR (signal-to-interference ratio) in frequency domain for VDSL systems. The proposed method exploits the training symbols in VDSL initialization using an eigen approach. Unlike earlier eigen based TEQ designs, the proposed method does not require the channel impulse response. The TEQ can be obtained by computing an eigenvector of a positive definite matrix that depends only on the averaged received VDSL symbols. Examples will be given to demonstrate that the proposed TEQ design method can effectively shorten the channel impulse response, and achieve very good bit rates with only a small number of training symbols.

1. INTRODUCTION

The time domain equalizers play an important role in the application of DMT (discrete multitone) to DSL (digital subscriber loop transmission) [1, 2]. For a DMT system with cyclic prefix length L, there is no IBI if the channel order is no larger than L. In DSL applications, the channel impulse response can spread to a duration much larger than cyclic prefix. At the receiver, there is usually a TEQ that shortens the channel to reduce IBI (inter-block-interference).

Many TEQ designs have been proposed in the literature. In many of the existing methods, optimal TEQ in a certain sense can be computed once the channel impulse response and channel noise statistics are given, [3]-[8]. In [3]-[7], the TEQ design problem is formulated as an eigen filter problem. The optimal TEQ is the eigen vector corresponding to the largest eigen value of an appropriately defined channel dependent matrix. In these methods, the TEQ depends directly on the channel impulse response, an estimate of which is needed using training symbols before TEQ optimization can take place. In [7], zero padding is applied in the frequency domain to impose extra null symbols. Using a quadratic objective function based on the null symbols, blind adaptive equalization that does not require the channel impulse response can be achieved. The equalizer designed in [7] is different from the usual TEQ for DMT systems in the sense that the goal is to have ideal equalization so that the equivalent channel has only one tap. In [8], a blind adaptive TEQ (called MERRY) that exploits cyclic prefix is proposed. MERRY is shown in [8] to be a globally convergent algorithm.

In this paper we propose a TEQ design method for VDSL system that maximizes SIR in frequency domain using an eigen filter approach. In the VDSL system frequency division duplex is used to separate upstream and downstream signals. In downstream or upstream application, only half the tones are used and the other unused half are referred to as the null tones in this paper. In VDSL training symbols, around half of the used tones are pilots and the other half carry messages. We will exploit these properties of the training symbols to maximize SIR in frequency domain. Our proposed method is different from existing eigen methods in two aspects. First, the channel impulse response is not needed and the TEQ can be computed directly using an average of the received VDSL symbols without channel estimation. Secondly, the objection function is formulated in frequency domain; the optimization implicitly takes into account the equivalent channel's frequency response, which directly affects subchannel SNR and thus bit rate. We will demonstrate through examples that proposed TEQ can achieve good bit rates with only a small number of training symbols.

2. VDSL SYSTEM TRAINING SYMBOLS

In the VDSL system frequency division duplex is used to separate upstream and downstream signals. In downstream application, only half the tones are used and these tones are referred to as the data tones; for the tones reserved for upstream transmission zeros are used and these tones are

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called the null tones. Similarly, in upstream application, the tones used for transmission are the data tones and the downstream tones are the null tones. In an VDSL training symbol, about half of the data tones are set aside for pilots (even tones and tones that are multiples of 10 plus 9) and the others used for transmitting messages. Constellation of 4-QAM are used for all data tones. The QAM symbols on pilots tones are determined in a pseudo random manner but they are the same for all training symbols. The QAM symbols on message tones vary with training symbols. Therefore, the n-th input vector of the IDFT matrix on the transmitter side is of the form $\begin{pmatrix}\mathbf{s}_p^T & \mathbf{s}_m(n)^T & \mathbf{0}\end{pmatrix}^T$ after proper permutation (as the actual pilot tone indices and message tone indices are interleaved), where $\mathbf{s}_p$ is a constant vector consisting of QAM symbols on pilot tones and $\mathbf{s}_m(n)$ is a time dependent vector consisting of QAM symbols on message tones.

When the channel order is smaller than the length of the cyclic prefix L, we know there is no IBI after removing guard samples (prefix removal). In the absence of channel noise, the outputs of the DFT matrix at the receiving end are the scaled versions of the transmitter inputs. The scalars are the M-point DFT of the channel impulse response. In this case, the null tones will be nothing but channel noise. However, if the channel order is larger than the length of cyclic prefix, there will be IBI even after removing guard samples. The outputs of the null tones now have channel noise plus interference from the data tones of the previous block due to IBI (assuming channel order is smaller than $N = M + L$). Our proposed method will exploit the fact that the symbols sent on pilot tones and the null tones are fixed in each VDSL training symbol to formulate a quadratic objective function of signal to interference ratio in the frequency domain for TEQ optimization.

3. OBJECTION FUNCTION: SIGNAL TO INTERFERENCE RATIO

In this section, we derive the objective function to be used in the proposed TEQ design method. Fig. 1(a) shows the block diagram of a DMT transceiver. Using matrix representation for cyclic prefix insertion and prefix removal, Fig. 1(a) can be redrawn as Fig. 1(b), where the matrices $\mathbf{F}_0$ and $\mathbf{F}_1$ are respectively of dimensions $N \times M$ and $M \times N$,

$$\mathbf{F}_0 = \begin{pmatrix} & \mathbf{I}_L \\ \mathbf{0} & \mathbf{I}_M \end{pmatrix}, \mathbf{F}_1 = \begin{pmatrix}\mathbf{0} & \mathbf{I}_M\end{pmatrix}.$$

In Fig. 1(b), we have lumped the channel $c(n)$ and the equalizer $t(n)$ together as $h(n) = c(n) * t(n)$, and $q(n)$ is the noise after TEQ, $q(n) = \nu(n) * t(n)$.

It is known that the $N \times N$ system from $\mathbf{y}(n)$ to $\mathbf{x}(n)$ is an LTI system [9]. Assume the length of $h(n)$ is $N = M + L$. The transfer matrix $\mathbf{H}(z)$ is pseudo-circulant with the first column given by [9]

$$\begin{pmatrix} h(0) & h(1) & \cdots & h(N-1)\end{pmatrix}^T,$$

and $\mathbf{H}(z)$ is of the form $\mathbf{H}_0 + z^{-1}\mathbf{H}_1$. The transceiver in Fig. 1(b) can be redrawn as Fig. 1(c), where $\mathbf{q}(n)$ is the channel noise vector blocked from $q(n)$. The vector $\mathbf{y}(n)$ as shown in Fig. 1(c) is

$$\mathbf{y}(n) = \mathbf{F}_0\mathbf{W}^\dagger\mathbf{P}\begin{pmatrix}\mathbf{s}_p \\ \mathbf{s}_m(n) \\ \mathbf{0}\end{pmatrix},$$

where a permutation matrix $\mathbf{P}$ is included so that the input vector can be conveniently expressed as the pilot vector followed by message vector and null vector.

The vector $\mathbf{x}(n)$ can be written as

$$\mathbf{x}(n) = \mathbf{H}_0\mathbf{y}(n) + \mathbf{H}_1\mathbf{y}(n-1) + \mathbf{q}(n). \tag{1}$$

Suppose we collect B vectors of $\mathbf{x}(n)$ and let

$$\overline{\mathbf{x}} = \frac{1}{B}\sum_{n=1}^{B}\mathbf{x}(n).$$

Using (1), we have

$$\begin{aligned}\overline{\mathbf{x}} = \quad & \mathbf{H}_0\mathbf{F}_0\mathbf{W}^\dagger\mathbf{P}\begin{pmatrix}\mathbf{s}_p \\ \frac{1}{B}\sum_{n=1}^{B}\mathbf{s}_m(n) \\ \mathbf{0}\end{pmatrix} + \\ & \mathbf{H}_1\mathbf{F}_0\mathbf{W}^\dagger\mathbf{P}\begin{pmatrix}\mathbf{s}_p \\ \frac{1}{B}\sum_{n=1}^{B}\mathbf{s}_m(n-1) \\ \mathbf{0}\end{pmatrix} + \overline{\mathbf{q}}, \end{aligned} \tag{2}$$

where $\overline{\mathbf{q}}$ is the averaged noise vector. Assume, reasonably, that the noise is zero mean that the message is zero mean, then $\overline{\mathbf{q}} \approx \mathbf{0}$, $\frac{1}{B}\sum_{n=1}^{B}\mathbf{s}_m(n) \approx \mathbf{0}$ and we have

$$\overline{\mathbf{x}} \approx (\mathbf{H}_0 + \mathbf{H}_1)\mathbf{F}_0\mathbf{W}^\dagger\mathbf{P}\begin{pmatrix}\mathbf{s}_p \\ \mathbf{0}\end{pmatrix}.$$

The averaged output vector of the DFT matrix, after permutation, is $\overline{\mathbf{u}} = \mathbf{P}^T\mathbf{W}\mathbf{F}_1\overline{\mathbf{x}}$. It is approximately

$$\overline{\mathbf{u}} \approx \mathbf{P}^T\mathbf{W}\mathbf{F}_1(\mathbf{H}_0 + \mathbf{H}_1)\mathbf{F}_0\mathbf{W}^\dagger\mathbf{P}\begin{pmatrix}\mathbf{s}_p \\ \mathbf{0}\end{pmatrix}.$$

Let us express $\mathbf{H}_0 + \mathbf{H}_1$ as $\mathbf{C}_0 + \mathbf{C}_1$, where $\mathbf{C}_0$ depends only on $h(0), h(1), \cdots, h(L)$ and $\mathbf{C}_1$ depends on the rest of the coefficients. Notice that the product $\mathbf{P}^T\mathbf{W}\mathbf{F}_1\mathbf{C}_0\mathbf{F}_0\mathbf{W}\mathbf{P}$ is a diagonal matrix $\mathbf{\Lambda}$. When the equivalent channel $h(n)$ has only $L+1$ coefficients, there is no IBI, $\mathbf{C}_1 = \mathbf{0}$ and $\overline{\mathbf{u}}$ becomes

$$\overline{\mathbf{u}} \approx \mathbf{\Lambda}\begin{pmatrix}\mathbf{s}_p \\ \mathbf{0}\end{pmatrix}.$$

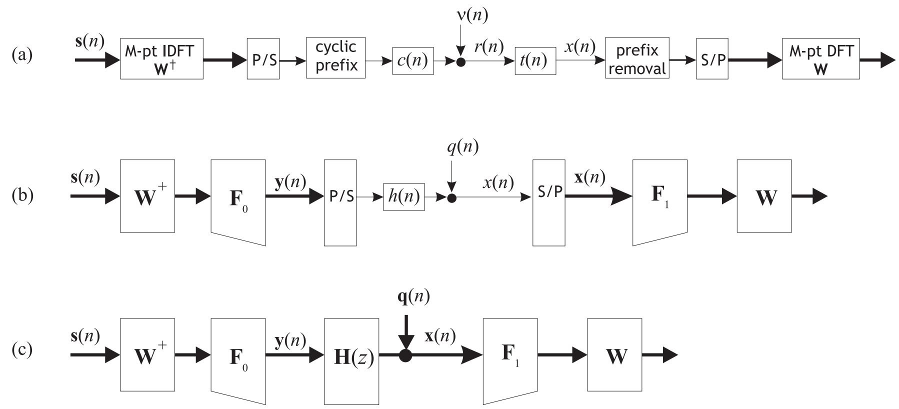

Figure 1: Equivalent block diagrams of the DMT transceiver.

The output of the null tones and message tones are approximately zero. If the equivalent channel has more than $L+1$ coefficients and there is IBI, then

$$\overline{\mathbf{u}} \approx \mathbf{\Lambda} \begin{pmatrix} \mathbf{s}_p \\ \mathbf{0} \end{pmatrix} + \mathbf{C}_1' \begin{pmatrix} \mathbf{s}_p \\ \mathbf{0} \end{pmatrix},$$

where $\mathbf{C}_1' = \mathbf{P}^T \mathbf{W} \mathbf{F}_1 \mathbf{C}_1 \mathbf{F}_0 \mathbf{W}^\dagger \mathbf{P}$. Let us partition $\overline{\mathbf{u}}$ as $\begin{pmatrix} \overline{\mathbf{u}}_p^T & \overline{\mathbf{u}}_m^T & \overline{\mathbf{u}}_n^T \end{pmatrix}^T$, where $\overline{\mathbf{u}}_p$, $\overline{\mathbf{u}}_m$, and $\overline{\mathbf{u}}_n$ correspond to outputs of pilot tones, message tones and null tones, respectively. We see that the average outputs $\overline{\mathbf{u}}_m$, and $\overline{\mathbf{u}}_n$ on the message tones and null tones contain mostly interference from the pilot tones due to IBI. Note that, disregard intereference, the i-th average pilot output $\overline{\mathbf{u}}_{p,i} \approx H_{p,i} \mathbf{s}_{p,i}$, where $H_{p,i}$ denotes the subchannel gain corresponding to the i-th pilot tone. As the symbols on pilot tones are 4-QAM, we have $\overline{\mathbf{u}}_p^\dagger \overline{\mathbf{u}}_p \approx \sum_i |H_{p,i}|^2$ (assuming $|\mathbf{s}_{p,i}| = 1$). Therefore, the term $\overline{\mathbf{u}}_p^\dagger \overline{\mathbf{u}}_p$ represents the signal power in the bands used for transmission. Although only the pilot tones are included in $\overline{\mathbf{u}}_p^\dagger \overline{\mathbf{u}}_p$, rather than all the tones used, it is a good measure. We propose the following objective function of SIR, which is the pilot tone energy over interference in the null tones and message tones,

$$\phi = \frac{\overline{\mathbf{u}}_p^\dagger \overline{\mathbf{u}}_p}{\overline{\mathbf{u}}_m^\dagger \overline{\mathbf{u}}_m + \overline{\mathbf{u}}_n^\dagger \overline{\mathbf{u}}_n}. \tag{3}$$

Although $\overline{\mathbf{u}}_p^\dagger \overline{\mathbf{u}}_p$ contains pilot energy as well as terms due interference, we still use it to reflect pilot energy as interference is usually much smaller by comparison. This is because the pilots are densely distributed–all the even tones are pilot tones. In the next section, we will optimize the TEQ coefficients to maximize ϕ.

4. OPTIMAL TEQ DESIGN

In what follows, we will see that the numerator and denominator of the objective function in (3) can be formulated as quadratic terms of the TEQ coefficients and the problem can be solved elegantly by computing an eigen vector of an appropriately defined positive definite matrix. Suppose the TEQ has order T. The output of the TEQ can be written as $x(n) = \sum_{\ell=0}^{T} t(\ell) r(n-\ell)$. The input vector $\mathbf{x}(n)$ of the matrix $\mathbf{F}_1$ at the receiver can be written in terms of TEQ coefficients as

$$\mathbf{x}(n) = \mathbf{R}_n \mathbf{t}, \tag{4}$$

where $\mathbf{R}_n$ is an $N \times (T+1)$ Toeplitz matrix with $[\mathbf{R}_n]_{mi} = r(nN + m - i)$, for $0 \leq m < N$, $0 \leq i \leq T$, and $\mathbf{t}$ is a column vector, $\mathbf{t} = \begin{pmatrix} t(0) & t(1) & \cdots & t(T) \end{pmatrix}^T$. The averaged vector in (1) is thus,

$$\overline{\mathbf{x}} = \overline{\mathbf{R}} \mathbf{t}, \text{ where } \overline{\mathbf{R}} = \frac{1}{B} \sum_{n=1}^{B} \mathbf{R}_n.$$

Therefore the averaged output vector $\overline{\mathbf{u}}$ is given by $\overline{\mathbf{u}} = \mathbf{P}^T \mathbf{W} \mathbf{F}_1 \overline{\mathbf{R}} \mathbf{t}$. We partition the matrix $\mathbf{P}^T \mathbf{W} \mathbf{F}_1 \overline{\mathbf{R}}$ to obtain

$$\begin{pmatrix} \overline{\mathbf{u}}_p \\ \overline{\mathbf{u}}_m \\ \overline{\mathbf{u}}_n \end{pmatrix} = \begin{pmatrix} \mathbf{A}_p \\ \mathbf{A}_m \\ \mathbf{A}_n \end{pmatrix} \mathbf{t}.$$

The objective function given in (3) become

$$\phi = \frac{\mathbf{t}^\dagger \mathbf{A}_p^\dagger \mathbf{A}_p \mathbf{t}}{\mathbf{t}^\dagger (\mathbf{A}_m^\dagger \mathbf{A}_m + \mathbf{A}_n^\dagger \mathbf{A}_n) \mathbf{t}}.$$

Now the objective function is written as quadratic forms of the TEQ coefficients. To avoid trivial solutions, we constrain the TEQ to have unit energy, i.e., $\mathbf{t}^\dagger \mathbf{t} = 1$.

Optimal solution. We now find the optimal $\mathbf{t}$ that maximizes ϕ. As $\mathbf{A}_m^\dagger \mathbf{A}_m + \mathbf{A}_n^\dagger \mathbf{A}_n$ is positive definite, we can write it as

$$\mathbf{A}_m^\dagger \mathbf{A}_m + \mathbf{A}_n^\dagger \mathbf{A}_n = \mathbf{B}^{-\dagger} \mathbf{B}^{-1}.$$

Then ϕ can be written as the ratio $\phi = \frac{\mathbf{t}^\dagger \mathbf{A}_p^\dagger \mathbf{A}_p \mathbf{t}}{\mathbf{t}^\dagger \mathbf{B}^{-\dagger} \mathbf{B}^{-1} \mathbf{t}}$. Let $\mathbf{v} = \mathbf{B}^{-1} \mathbf{t}$, then $\mathbf{t} = \mathbf{B} \mathbf{v}$ and

$$\phi = \frac{\mathbf{v}^\dagger \mathbf{B}^\dagger \mathbf{A}_p^\dagger \mathbf{A}_p \mathbf{B} \mathbf{v}}{\mathbf{v}^\dagger \mathbf{v}}.$$

Using Rayleigh's principle, ϕ can be maximized by choosing $\mathbf{v}$ to be the eigen vector corresponding to the largest eigen value of $\mathbf{B}^\dagger \mathbf{A}_p^\dagger \mathbf{A}_p \mathbf{B}$. The optimal TEQ is given by $\mathbf{t} = \mathbf{B}\mathbf{v}$.

5. SIMULATION EXAMPLES

In the simulations, the DFT size is 8192, cyclic prefix length is 640, sampling rate is 35.328 MHz and 10 blocks of VDSL symbols are used. We consider downstream transmission so the upstream tones are null tones. The noise is comprised of FEXT, NEXT and AWGN as described in the testing environment of [2]. We use VDSL training symbols given in [2]. The channel used is loop 1 from [2]. The loop has length 4500 ft. The TEQ has 20 taps. Fig. 2 shows the impulse response of the original channel and the equalized channel.

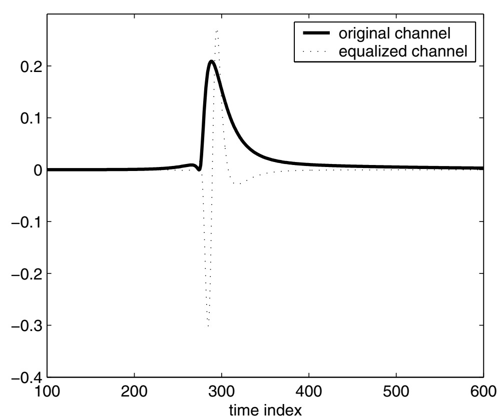

Figure 2: The impulse responses of the original VDSL loop 1 and the equalized channel.

Using the proposed TEQ design method, we list in Table 1 the bit rates of the equalized channels for the test loops listed in [2]. The test loops VDSL 1-4 used are long loops of length 4,500 ft. We have also listed the bit rates when the TEQ is designed using MSSNR (maximum shortening signal-to-noise ratio)[3]. For the case of MSSNR, perfect channel knowledge is assumed to be available. In both cases, the TEQ is of 20 taps. We can see that the proposed TEQ design can achieve bit rate much higher than MSSNR for all test loops. This is because MSSNR maximizes signal to interference ratio in the time domain, disregarding the frequency response of the equivalent channel, and zeros close to the transmission bands are often introduced by TEQ. These zeros often lead to a significant loss in SNR and bit rate. Our proposed method maximizes signal to interference ratio in the frequency domain. As a result, the frequency response of the equalized channel is implicitly taken into account and a better bit rate is achieved.

Table 1: Comparison of bit rates (Mbits/sec) on VDSL loops.

VDSL loop	proposed	MSSNR
VDSL-1L	49.0	41.2
VDSL-2L	43.7	32.0
VDSL-3L	42.7	35.3
VDSL-4L	23.3	19.5
VDSL-5	96.2	81.6
VDSL-6	55.2	46.6
VDSL-7	29.9	24.5

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