

Code Priority of Multiuser OFDM Systems in Frequency Asynchronous Environments

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Abstract—An approximately MAI-free multiaccess OFDM transceiver, called the precoded multiuser (PMU-OFDM) OFDM system here, was proposed in [5]. In this work, we extend results in [5], and propose a code priority scheme based on the even/odd Hadamard-Walsh code selection to enhance the performance of the PMU-OFDM system in a CFO environment. Based on the observation that different codewords have different contributions to multiaccess interference (MAI) that are correlated with the number of crossings of a particular Hadamard-Walsh codeword, we propose a code priority scheme by ranking codewords according to their contributions to MAI. Simulation results are given to demonstrate the advantages of the proposed priority.

I. INTRODUCTION

Multiuser OFDM systems [3], [4] inherit the advantage of single user OFDM systems [2] in combating the inter-symbol-interference (ISI) and enable high data rate transmission with simple implementation. However, when users transmit their signals simultaneously, several types of multiaccess interference (MAI) take place. Among these MAI effects, the one caused by the carrier frequency offset (CFO) plays a critical role since it limits the mobility of OFDM-based systems.

An approximately MAI-free multiaccess OFDM system, which is called the precoded multiuser (PMU-) OFDM system here, was proposed in [5]. It was shown in [5] that the PMU-OFDM system has several attractive characteristics. First, when the number of parallel transmit symbols, N , is sufficiently large, PMU-OFDM is approximately MAI-free. Second, PMU-OFDM is robust to time asynchronism. Hence, low complexity Hadamard-Walsh codes can be used in uplink transmission. Third, PMU-OFDM can greatly mitigate the CFO effect using even or odd Hadamard-Walsh codewords. This particular code selection scheme can reduce the dominating MAI due to CFO to a negligible amount. In this case, the system performance is determined by the residual MAI.

In this work, we extend the results in [5], and propose a code priority scheme based on the even/odd Hadamard-Walsh code selection to improve the performance of the PMU-OFDM system in a CFO environment. More specifically, We found that different codewords have different contributions to the residual MAI, which correlate with the number of crossings of a particular Hadamard-Walsh codeword [1], and propose a code priority scheme by ranking codewords according to their contribution to the residual MAI in a CFO environment.

The code priority scheme can be useful in several scenarios as described below. First, in some login/logout applications

where not all users are connected simultaneously, we can assign codewords of higher priority to the first connected users. Second, in a serious CFO environment, we may want to support fewer premium users based on the code priority scheme to maintain the nearly MAI-free property in such a hostile environment. Third, individual users may have different CFOs in some occasions. Then, we may assign codewords of higher (or lower) priority to users with a larger (or smaller) CFO value so that the performance degradation of an individual user would not become too severe. This can be done by codeword reservation as we will mention later.

II. SYSTEM MODEL AND REVIEW

For the system block diagram of the PMU-OFDM system, we refer to [5]. The same notations in [5] are adopted here. The system parameters are specified below. The number of parallel transmit symbols is N . Each symbol is spread in the frequency domain by an orthogonal codeword of length M . Thus, the DFT/IDFT size is equal to NM . The m th element of user i 's orthogonal codeword is denoted by $w_i[m]$. The maximum delay spread under consideration is L . It was demonstrated in [5] that the CFO-induced MAI consists of two terms, *i.e.* the dominating MAI and the residual MAI, where the dominating MAI is much larger than the residual MAI. If the even or odd Hadamard-Walsh codewords are used, the dominating MAI is greatly reduced so that it is negligible as compared with the residual MAI. Under this situation, the residual MAI will dominate the system performance. It is observed that different Hadamard-Walsh codewords have different levels of residual MAI in a CFO environment so that we can enhance the system performance using a proper code priority scheme. In the next section, we will analyze the residual MAI and then propose a code priority scheme for PMU-OFDM to reduce the residual MAI in a CFO environment.

III. RESIDUAL MAI ANALYSIS AND CODE PRIORITY

A. Analysis of Residual MAI

Let the CFO of user i be ϵ_i . It was shown in [5] that if the even or odd Hadamard-Walsh codewords are used, the averaged power of residual MAI at the k th symbol from user

i to user j , *i.e.* $E \left\{ \left| B_{j \leftarrow i}^{(1)}[k] \right|^2 \right\}$, can be approximated by

$$\frac{|\beta_i|^2}{M^2} \sigma_{\lambda_i}^2 \sigma_{x_i}^2 \sum_{l=1}^{N-1} \left| \sum_{p=1}^{M-1} [f(p, l) + f(-p, l)] r_{ij}[p] \right|^2, \quad (1)$$

where

$$f(p, l) = e^{-j\pi \frac{p}{NM}} / \left(NM \sin \frac{\pi(p + lM + \epsilon_i)}{NM} \right),$$

$r_{ij}[p] = \sum_{q=0}^{M-1-p} w_i[q] w_j[p+q]$, $\beta_i = \sin(\pi \epsilon_i) e^{j\pi \epsilon_i \frac{NM-1}{NM}}$, $\sigma_{\lambda_i}^2$ and $\sigma_{x_i}^2$ are the averaged channel gain and the averaged symbol power of user i , respectively. For notational convenience, we define the following two parameters

$$\mu_{j \leftarrow i} = E \left\{ \left| B_{j \leftarrow i}^{(1)}[k] \right|^2 \right\} \text{ and } r'_{ij}[p] = \sum_{q=0}^{M-1-p} w_i[p+q] w_j[q].$$

Lemma: If the even or odd Hadamard-Walsh codewords are used, we have the following property:

$$\sum_{p=1}^{M-1} r_{ij}[p] = \sum_{p=1}^{M-1} r'_{ij}[p] = 0, \quad i \neq j. \quad (2)$$

Proof. As proved in [5], if the even or odd Hadamard-Walsh codewords are used, we have the following property

$$r_{ij}[p] = r'_{ij}[p], \quad i \neq j. \quad (3)$$

Using the following two equalities

$$\sum_{v=0}^{M-1} \sum_{u=0, u \neq v}^{M-1} w_i[u] w_j[v] = \sum_{p=1}^{M-1} (r_{ij}[p] + r'_{ij}[p]),$$

and

$$r_{ij}[0] + r'_{ij}[0] = \sum_{v=0}^{M-1} w_i[v] w_j[v] = 0,$$

we have

$$\sum_{v=0}^{M-1} \sum_{u=0}^{M-1} w_i[u] w_j[v] = \sum_{p=1}^{M-1} (r_{ij}[p] + r'_{ij}[p]). \quad (4)$$

From (4), since

$$\sum_{v=0}^{M-1} \sum_{u=0}^{M-1} w_i[u] w_j[v] = \sum_{v=0}^{M-1} w_j[v] \sum_{u=0}^{M-1} w_i[u] = 0,$$

we know that

$$\sum_{p=1}^{M-1} (r_{ij}[p] + r'_{ij}[p]) = 0. \quad (5)$$

From (3) and (5), we can reach (2) ■

From (2), if the even or odd Hadamard-Walsh codewords are used, the codeword cross correlation $r_{ij}[p]$ for two given codewords must have positive and negative values so that the summation over all p is zero. Since the maximum value of p is $M-1$, if N is sufficiently large, $e^{-j\pi \frac{p}{NM}} \approx 1$. In this case, we have $f(p, l) + f(-p, l) \approx \frac{1}{NM \sin \frac{\pi(p+lM+\epsilon_i)}{NM}} +$

$\frac{1}{NM \sin \frac{\pi(-p+lM+\epsilon_i)}{NM}}$, which is a monotonically decreasing or increasing function of p for a given l . Thus, referring to (1), a proper code priority scheme can improve the performance of PMU-OFDM in a CFO environment when not all $M/2$ users are active. Intuitively, a good code priority scheme should let $r_{ij}[p]$ have the largest number of zero values or have the most sign exchanges. Then, the absolute-and-squared term in (1) can be efficiently cancelled out after summation of all p .

To illustrate this point, we may consider a simple example where $M = 8$ and only even codewords, *i.e.* a half-loaded system, are used. One code assignment scheme is to assign the first two users codewords (1 1 1 1 1 1 1) and (1 1 -1 -1 -1 -1 1). Then, $r_{ij}[p]$ is (-1 -2 -1 0 1 2 1) for $1 \leq p \leq M-1$. Another one is to assign them codewords (1 -1 -1 1 1 -1 1) and (1 -1 1 -1 -1 1 -1 1). Then, $r_{ij}[p]$ is (-1 2 -1 0 1 -2 1). Obviously, the latter scheme can cancel out the residual MAI more effectively than the former one.

It is known [1] that each Hadamard Walsh codeword can be identified by an unique crossing number. Moreover, even codewords always have an even number of crossing while odd codewords always have an odd number of crossings. Generally speaking, we have the following conjecture.

Conjecture. The maximum residual MAI, $\mu_{j \leftarrow i}$, occurs when the two codewords having zero and two (or one and three) crossings are used for even (or odd) Hadamard-Walsh codewords.

In the next subsection, we will present numerical examples to shed lights on the above conjecture. A more rigorous proof for this conjecture is under our current investigation.

B. Verification of Residual MAI Property

We study the behavior of Hadamard Walsh codewords of a different crossing number via numerical simulation. Let $\sigma_{h_i}^2 = \sigma_{x_i}^2 = 1$, multipath length $L = 4$, codeword number $M = 16$ and the normalized CFO level $\epsilon_j = 0.2$. The values of $\mu_{j \leftarrow i}$ (dB) are tabulated in Table I for codewords with an even number of crossings. It is clear that $\mu_{j \leftarrow i}$ is symmetric, *i.e.* $\mu_{j \leftarrow i} = \mu_{i \leftarrow j}$. Furthermore, we see that the maximum residual MAI occurs between users with codewords having 0 and 2 crossings. This confirms the conjecture made at the end of Sec. III-A. Moreover, we see that different pairs of codewords will lead to different amount of residual MAI.

Define $\phi_j(T) = \sum_{i=1, i \neq j}^T \mu_{j \leftarrow i}$, where T is the number of active users who are in connection. We show values of $\phi_j(8)$ and $\phi_j(16)$ for $M = 16$ and $M = 32$, respectively, in the third and the fifth columns of Tables II (a) and (b). Note that, by summing up the whole row or column of Table I, we get the value of $\phi_j(8)$ at the 3rd column in Table II (a). When M increases from 16 to 32, we see that the performance of codewords with the same crossing number does not degrade significantly. This can be explained by the fact that the newly added codewords have more crossings so as to result in a smaller residual MAI amount for other codewords. Take the even codeword with 10 crossings as an example. This codeword degrades the most, *i.e.*, 2.9 dB, as M increased from 16 to 32. However, since the increase of

	0 crossing	2 crossings	4 crossings	6 crossings	8 crossings	10 crossings	12 crossings	14 crossings
0 crossing		-18.5	-24.5	-22.4	-31.5	-33.2	-28.9	-27.1
2 crossings	-18.5		-27.3	-26.1	-33.2	-34.0	-31.2	-30.2
4 crossings	-24.5	-27.3		-29.6	-35.2	-35.7	-33.8	-33.2
6 crossings	-22.4	-26.1	-29.6		-34.8	-35.4	-33.2	-32.4
8 crossings	-31.5	-33.2	-35.2	-34.8		-39.2	-38.0	-37.7
10 crossings	-33.2	-34.0	-35.7	-35.4	-39.2		-38.3	-38.1
12 crossings	-28.9	-31.2	-33.8	-33.2	-38.0	-38.3		-36.3
14 crossings	-27.1	-30.2	-33.2	-32.4	-37.7	-38.1	-36.3	

TABLE I

$\mu_{j \leftarrow i}$ AS A FUNCTION OF CODEWORDS IN TERM OF THEIR CROSSING NUMBERS.

M will increase the number of codewords with more crossings that will have much less residual MAI, the overall performance is actually improved.

# of zero crossing	Code index (M=16)	$\phi_j(8)$ (dB)	Code index (M=32)	$\phi_j(16)$ (dB)	# of zero crossing	Code index (M=16)	$\phi_j(8)$ (dB)	Code index (M=32)	$\phi_j(16)$ (dB)
0	1	-15.6	1	-14.6	1	9	-15.8	17	-14.9
2	13	-16.8	25	-15.8	3	5	-16.5	9	-15.5
4	7	-21	13	-19.4	5	15	-21.7	29	-19.9
6	11	-19.6	21	-18.3	7	3	-19.2	5	-17.9
8	4	-26.4	7	-23.8	9	12	-26.7	23	-24.0
10	16	-27.3	31	-24.4	11	8	-27.2	15	-24.3
12	6	-24.6	11	-22.4	13	14	-25.1	27	-22.7
14	10	-23.4	19	-21.5	15	2	-23.0	3	-21.2
16			4	-29.3	17			20	-29.4
18			28	-29.6	19			12	-29.6
20			16	-30.2	21			32	-30.2
22			24	-30.1	23			8	-30.1
24			6	-27.5	25			22	-27.6
26			30	-27.9	27			14	-27.9
28			10	-26.3	29			26	-26.6
30			18	-25.7	31			2	25.4

(a) Even codewords

(b) Odd codewords

TABLE II

THE MAI OF ASSIGNMENT OF (A) EVEN CODEWORDS, (B) ODD CODEWORDS FOR $M = 16$ (COLUMN 3) AND $M = 32$ (COLUMN 5).

C. Proposed Code Priority Scheme

We see from Table II that, as M increases, the newly added codewords, *i.e.*, with 8-14 crossings for even codewords and 9-15 crossings for odd codewords, result in less residual MAI than original even codewords with 0-6 crossings and original odd codewords with 1-7 crossings. Hence, for an $M/2$ -user system with $M \geq 8$, the set of $M/4$ codewords with more crossings should be assigned to the first $M/4$ active users. Afterwards, the remaining $M/4$ codewords with fewer crossings can be further divided into two subsets with $M/8$ codewords each. The $M/8$ codewords with more crossings have higher priority to be assigned to users when the number of active users exceeds $M/4$. We continue this procedure until the divided subset contains only one user.

For example, the set of even or odd codewords with $M = 32$ should be divided into the following five subsets.

- 1) the 1st codeword subset with 0-1 crossings (*i.e.*, 0 for even codeword, and 1 for odd codeword);
- 2) the 2nd codeword subset with 2-3 crossings;
- 3) the 3rd codeword subset with 4-7 crossings;
- 4) the 4th codeword subset with 8-15 crossings; and
- 5) the 5th codeword subset with 16-31 crossings.

The higher the index of a codeword subset, the smaller the residual MAI. Thus, we should assign codewords from a higher indexed subset to a lower indexed subset. Consequently, the overall performance of this codeword assignment is better than that of a random codeword assignment.

Note that the above scheme provides only a coarse-scale code priority. That is, in the same codeword subset, codewords with more crossings may not necessarily have smaller MAI than those with fewer crossings. Take the 5th codeword subset as an example. The codeword with 20 crossings actually has less MAI than that with 30 crossings. A fine-scale codeword priority can be obtained off-line using the close form of $\mu_{j \leftarrow i}$ in (1) to construct Tables I and II. Based on these tables, we can determine the fine-scale code priority.

For instance, consider the case $M = 16$. The fine-scale code priority for the even codewords is (with the Kronecker ordering [1])

$$(\mathbf{w}_{16}, \mathbf{w}_4, \mathbf{w}_6, \mathbf{w}_{10}, \mathbf{w}_7, \mathbf{w}_{11}, \mathbf{w}_{13}, \mathbf{w}_1). \quad (6)$$

Fig. 1 shows $\sum_{i=1, i \neq j}^T \mu_{j \leftarrow i}$, $1 \leq j \leq 8$, as a function of CFO for the set of even codewords. We see that the rank of the performance of different codewords remains the same across the whole range of CFO in the test. In other words, the fine-scale code priority scheme will not be changed by different CFO values. Hence, we only need to determine the fine-scale code priority once for a specific CFO value and apply it in different CFO environments.

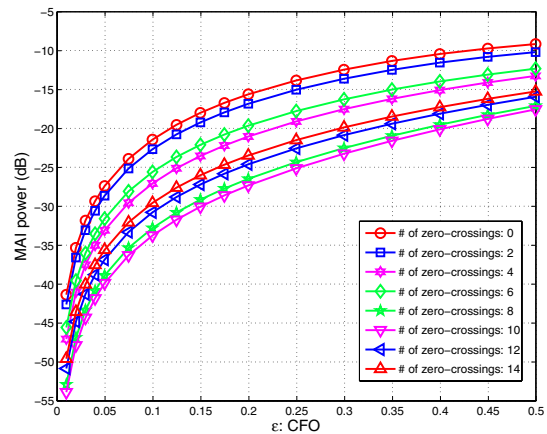


Fig. 1. $\sum_{i=1, i \neq j}^T \mu_{j \leftarrow i}$ as a function of CFO for even codewords.

IV. SIMULATION RESULTS

In this section, we compare the performance of the PMU-OFDM and the OFDMA systems. It was shown in [5] that these two systems have similar performance in a fully-loaded CFO environment. Thus, we will focus on the half-loaded or the lightly-loaded cases here. It will be demonstrated that significant performance improvement can be achieved using the proposed code priority scheme in PMU-OFDM. We consider the uplink direction so that every user has different CFO and channel fading. The channel and CFO are assumed to be quasi-invariant in the sense that it remains unchanged within one OFDM-block duration.

Simulations are conducted with the following parameter setting throughout this section: $N = 64$, $M = 16$, the multipath length $L = 4$, the channel coefficients are i.i.d. complex Gaussian random variables with an unit variance, and the BPSK modulation is used. The Monte Carlo method is used to run more than 500,000 symbols for every individual user. We consider the worst CFO scenario, where the CFO value of each user is randomly assigned to $+\varepsilon$ or $-\varepsilon$.

Example 1. Lightly-loaded systems with serious CFO

In this example, we compare the performance of the PMU-OFDM and the OFDMA systems. Every user in these two systems will transmit N symbols and the DFT/IDFT size is the same, *i.e.* $NM = 1024$, which is a reasonable value for broadband wireless access solution [4]. Since both systems transmit N symbols per block and add the CP of the same length $L-1$, their actual data rates are the same. We consider two loading cases. One is half-loaded and the other is quarterly-loaded. In the half-loaded case, the set of $M/2$ even codewords is used in the PMU-OFDM system. For OFDMA, each user occupies N subchannels which are maximally separated [5]. In a quarterly-loaded case, four codewords with 14, 12, 10, 8 crossings are used in the PMU-OFDM system. For OFDMA, the u th user is assigned subchannels indexed by $4(u-1)+kM$, $1 \leq u \leq M/4$ and $0 \leq k \leq N-1$.

Let us first evaluate the MAI effect in the detection stage, *i.e.* MAI after frequency equalization [5]. In the absence of MAI and channel noise, the received symbols for detection are still BPSK symbols with either $+1$ or -1 . Fig. 2 shows the averaged total MAI after equalization as a function of CFO for the two systems in the half-loaded and the quarterly-loaded situations. (For the averaged total MAI calculation, we refer to [5].) We see that the half-loaded PMU-OFDM even outperforms the quarterly-loaded OFDMA when $\varepsilon < 0.375$. The quarterly-loaded PMU-OFDM outperforms quarterly-loaded OFDMA by around 8-20 dB. We also observe that, in a serious CFO level, *e.g.* $\varepsilon = 0.15$, quarterly-load PMU-OFDM outperforms the half-loaded PMU-OFDM by 14 dB. Thus, in the presence of serious CFO, we may want to support fewer users with the proposed code priority so that PMU-OFDM can still be approximately MAI-free in such a hostile case.

Next, we consider the bit error rate (BER) performance of both systems. Let the CFO value be fixed at 0.2. Fig. 3 plots the BER as a function of the signal-to-noise ratio (SNR).

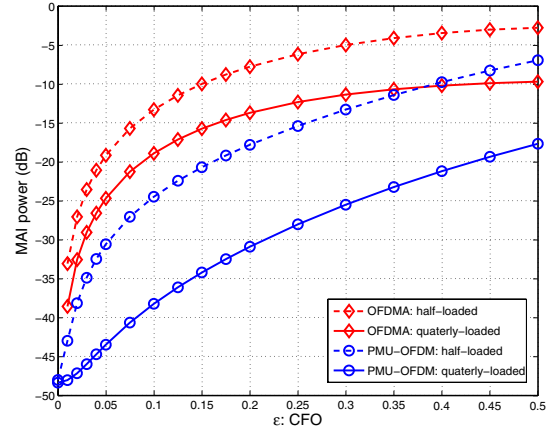


Fig. 2. MAI comparison between PMU-OFDM and OFDMA in half-loaded and quarterly-loaded cases.

It is apparent that the use of even codewords enables half-loaded PMU-OFDM to have lower BER than quarterly-loaded OFDMA. By comparing this figure with Fig. 2, we see that the BER performance in Fig. 3 could be roughly evaluated by the MAI curves in Fig. 2. Let us take half-loaded PMU-OFDM as an example. The performance floor occurs when SNR is around 15 dB. This can be explained as follow. As shown in Fig. 2, the MAI power is around -18 dB at CFO=0.2. For BPSK symbols with 0 dB signal power and SNR=18 dB, it implies that the noise power is around -18 dB. Hence, if we include the MAI power of -18 dB, the total noise-plus-interference power is equal to -15 dB.

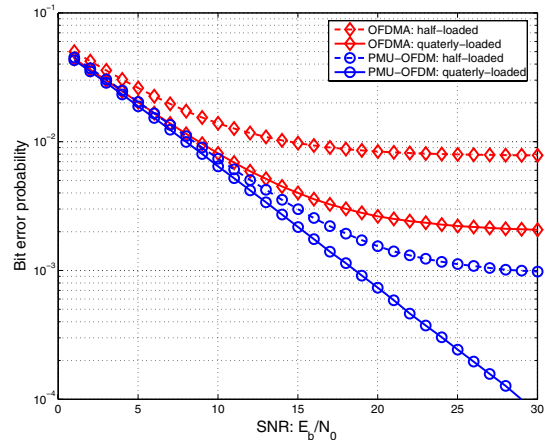


Fig. 3. BER comparison between PMU-OFDM and OFDMA in half-loaded and quarterly-loaded cases with fixed CFO ($|\varepsilon_j| = 0.2$).

We see from Fig. 2 that the BER of half-loaded PMU-OFDM at SNR=18 dB performs roughly the same as quarterly-loaded PMU-OFDM at SNR=15 dB, where its MAI power is around -31 dB and is much smaller than the noise power

-15 dB. As SNR increases, noise will be less than -18 dB. However, the MAI remains -18 dB for half-loaded PMU-OFDM if CFO is 0.2. In this case, the performance floor will occur at SNR=18 dB. The same argument applies to the OFDMA system. The quarterly-loaded PMU-OFDM system with code priority makes the system perform better in this serious CFO environment.

Example 2. Systems with a varying number of users

In this example, we evaluate the performance of different code priority schemes. The proposed fine-scale code priority is given in (6). Another code priority that serves as a performance benchmark is given by (in Kronecker ordering [1])

$$(\mathbf{w}_1, \mathbf{w}_{13}, \mathbf{w}_7, \mathbf{w}_{11}, \mathbf{w}_4, \mathbf{w}_{16}, \mathbf{w}_6, \mathbf{w}_{10}), \quad (7)$$

which assigns codewords with the number of crossings from the smallest to the largest, i.e. from 0 to 14. Fig. 4 shows the averaged total MAI as a function of the number of active users T with these two schemes. The dashed curves are for CFO=0.05 while the solid curves are for CFO=0.2. We see that the proposed code priority scheme outperforms the benchmark code priority significantly. The benchmark scheme assigns the first two users with codewords of 0 and 2 crossings, which result in the most serious MAI in a CFO environment according to the Conjecture claimed before. As T increases, the performance of the benchmark becomes slightly better. The benchmark with CFO=0.05 still has worse performance than the proposed priority scheme with CFO=0.2 when $T < 5$. Thus, a good code priority design can enhance the system performance while a poor design will make the performance nearly unchanged even if the number of users is as low as two.

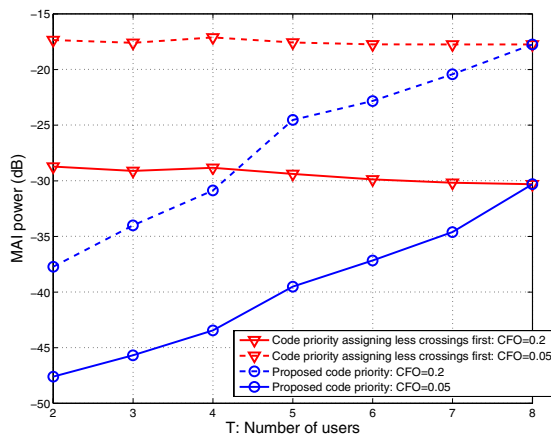


Fig. 4. The MAI power as a function of the number of active users with two code priority schemes.

Example 3. Users with different CFO levels

In some environments, different users may have different CFO levels. For instance, some users are travelling at a high speed so that they have large CFOs. Other users move at a low speed so that their CFO is small. Under this situation, some

codewords with higher priorities should be reserved for users with larger CFOs. Otherwise, if such users are connected in but lower priority codewords are assigned, the overall performance will degrade significantly. Hence, we may assign codewords by determining several CFO thresholds. A user with specific CFO level exceeding a certain threshold should be assigned a codeword with specific priority. This codeword assignment scheme is intuitive since codewords of higher priority do not cause significant residual MAI to other users even in a serious CFO environment as shown in Table I. Here, we assume the codeword reservation is used so that we assign codewords of higher priority to users with larger CFOs, and codewords of lower priority to users with lower CFOs in PMU-OFDM.

We consider a half-loaded system with four different CFO levels; namely, 0.02, 0.03, 0.05 and 0.1, in this example. For PMU-OFDM, \mathbf{w}_1 and \mathbf{w}_{13} are for users with CFO=0.02, \mathbf{w}_{11} and \mathbf{w}_7 are for users with CFO=0.03, \mathbf{w}_{10} and \mathbf{w}_6 are for users with CFO=0.05, and \mathbf{w}_4 and \mathbf{w}_{16} are for users with CFO=0.1. For OFDMA, the 1st and 2nd users i.e. $u = 1$ and 2 are with CFO=0.02, the 3rd and the 4th users are with CFO=0.03, the 5th and the 6th users are with CFO=0.05, and the 7th and the 8th users are with CFO=0.1. Fig. 5 shows the BER curves of these two systems. We see that PMU-OFDM with the proposed code priority outperforms OFDMA greatly.

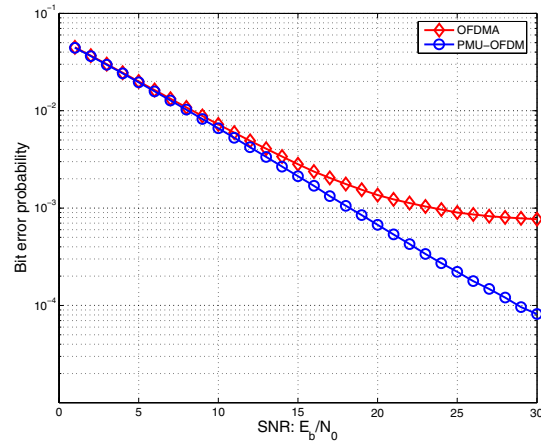


Fig. 5. BER comparison between PMU-OFDM and OFDMA in an environment with different CFOs.

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