A FREQUENCY DOMAIN BASED TEQ DESIGN FOR DSL SYSTEMS

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ABSTRACT

In this paper, we propose a frequency domain based design method for designing TEQ (time domain equalizer) for DSL (digital subscriber loops) applications. In DSL applications, usually frequency division multiplexing is used to separate the upstream and downstream signals. In either direction of transmission, some of the frequency bands are not used for transmission. Interference is formulated and minimized in the frequency domain to take advantage of the unused frequency bands. The frequency domain approach allows us to have a more direct control over the frequency response of the resulting TEQ, which is crucial in the final achievable transmission bit rates. The simulation examples demonstrate that the proposed method shortens the channel effectively and very good transmission bit rates can be achieved.

1. INTRODUCTION

The DFT based discrete multitone (DMT) transceiver has found important applications for high-speed transmission, [1]-[2]. At the transmitter end, each block is padded with a cyclic prefix of length L. When the channel is longer than L, which is usually the case in DSL applications, a time domain equalizer (TEQ) is inserted at the receiver to shorten the channel impulse response to within a window of $L+1$ samples. The samples outside the window, however, will lead to interference and reduces transmission bit rate. The TEQ plays an important role in the application of transmission over DSL. In the literature, many TEQ designs have been proposed [3]-[6]. In [3], Melsa *et al.* design the optimal TEQ that minimizes the out-of-window energy of the equivalent channel to minimizes IBI, also called MSSNR method (maximum shortening signal-to-noise ratio). In [4], Arslan *et al.* consider interference on tones used for transmission and minimize weighted interference on used tones. In [5], zero padding is applied in the frequency domain so that the equivalent channel is approximately an impulse. In [7], Martin *et al.* propose a globally convergent blind adaptive TEQ (called MERRY) that provides a good trade-off between complexity and performance. A comprehensive overview on TEQ and a unified design approach is available in [8]. Most of the TEQ designs do not take into consideration the frequency response of the TEQ. It is noted in [6] that the frequency response of the TEQ greatly affects the achievable transmission bit rates. In particular, zeros of the TEQ in the transmission bands often result in a loss of bit rates and it is desired that the TEQ does not have zeros in the frequency bands used for transmission. A semi-blind TEQ design that utilizes training symbols of VDSL systems and takes the frequency response of the TEQ implicitly into account is given in [9].

In this paper, we propose a frequency domain based method for designing TEQ for DSL applications. In DSL (digital subscriber loops) applications, the upstream and downstream bands usually do not overlap and thus in either direction of transmission some of the frequency bands do not carry signals. The unused frequency bands can be exploited to design TEQ. We will consider interference over a chosen set of tones (target tones) due to another set of tones (source tones). In particular, by placing the source and target tones in the unused frequency bands the TEQ will be free from zeros in the transmission bands. This allows us to have a more direct control over the frequency response of the resulting TEQ. Furthermore we will see that the channel can be successfully shortened with as few as only one source tone, which greatly reduces design complexity. The objective function can be formulated as a quadratic term of TEQ coefficients. The TEQ solution can be obtained by solving an eigen problem. The simulation examples demonstrate that the proposed method shortens the channel effectively and very good transmission bit rates can be achieved.

2. SYSTEM MODEL

The block diagram of a DMT system is shown in Fig. 1(a). The transmitter and receiver perform respectively M-point

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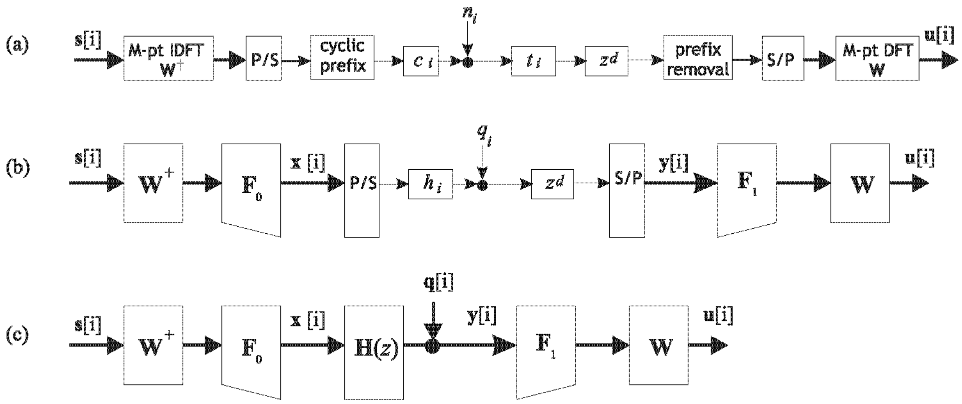

Figure 1: Matrix representation of a DMT transceiver.

IDFT and DFT computations, where M is the number of tones or number of subchannels. The $M \times M$ DFT matrix is denoted by $\mathbf{W}$, with $[\mathbf{W}]_{m,n} = e^{-j2\pi/M}/\sqrt{M}$. A cyclic prefix of length L is added after the parallel to serial (P/S) operation. The receiver includes the block z^d, where d is a parameter of synchronization delay. Fig. 1(a) can be redrawn as Fig. 1(b), where the channel and TEQ are lumped together and

$$h_i = c_i * t_i.$$

We assume that the length of t_i is Q and the length of h_i is shorter than $N = M + L$. The equivalent noise after TEQ is given by $q_i = n_i * t_i$. The prefix insertion and removal operations can be represented as $\mathbf{F}_0$ and $\mathbf{F}_1$ respectively,

$$\mathbf{F}_0 = \begin{bmatrix} \mathbf{0} & \mathbf{I}_L \\ \multicolumn{2}{c}{\mathbf{I}_M} \end{bmatrix}, \quad \mathbf{F}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{I}_M \end{bmatrix}. \tag{1}$$

Note that in Fig. 1(b), the $N \times N$ system from $\mathbf{x}(n)$ to $\mathbf{y}(n)$ is LTI with transfer matrix $\mathbf{H}(z)$ given by,

$$\mathbf{H}(z) = \begin{bmatrix} h_d & \cdots & h_0 & z^{-1}h_{N-1} & \cdots & z^{-1}h_{d+1} \\ \vdots & & & & \ddots & \vdots \\ h_{N-1} & & & \ddots & & z^{-1}h_{N-1} \\ zh_0 & \ddots & & & & h_0 \\ \vdots & \ddots & \ddots & & & \vdots \\ zh_{d-1} & \cdots & zh_0 & & \cdots & h_d \end{bmatrix}. \tag{2}$$

We can write $\mathbf{H}(z)$ as

$$\mathbf{H}(z) = \mathbf{H}_0 + z^{-1}\mathbf{H}_1 + z\mathbf{H}_{-1}, \tag{3}$$

Using such a matrix representation, Fig. 1(b) can be redrawn as Fig. 1(c), where $\mathbf{q}[i]$ is an N by one vector obtained by blocking the noise q_i. The i-th received vector $\mathbf{y}[i]$ is related to $\mathbf{x}[i]$ by

$$\mathbf{y}[i] = \mathbf{H}_0\mathbf{x}[i] + \mathbf{H}_1\mathbf{x}[i-1] + \mathbf{H}_{-1}\mathbf{x}[i+1].$$

There is interference from the previous block $\mathbf{H}_1\mathbf{x}[i-1]$ and possibly interference from the next block $\mathbf{H}_{-1}\mathbf{x}[i+1]$ due to synchronization delay.

We can further split the constant matrix $\mathbf{H}_0$ into two parts,

$$\mathbf{H}_0 = \mathbf{H}_{00} + \mathbf{H}_{01},$$

where $\mathbf{H}_{00}$ consists of only the coefficients in the window $h_d, h_{d+1}, \cdots, h_{d+L}$ while $\mathbf{H}_{01}$ depends on only the coefficients outside the window $h_0, h_1, \cdots, h_{d-1}$, and $h_{d+L+1}, h_{d+L+2} \cdots, h_{N-1}$. The receiver output vector $\mathbf{u}[i]$ is related to the transmitter input vector $\mathbf{s}[i]$ by

$$\begin{aligned} \mathbf{u}[i] &= \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_{00}\mathbf{F}_0\mathbf{W}^\dagger}_{\mathbf{\Lambda}}\mathbf{s}[i] + \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_{01}\mathbf{F}_0\mathbf{W}^\dagger}_{\mathbf{A}}\mathbf{s}[i] \\ &\quad + \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_1\mathbf{F}_0\mathbf{W}^\dagger}_{\mathbf{B}}\mathbf{s}[i-1] \\ &\quad + \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{H}_{-1}\mathbf{F}_0\mathbf{W}^\dagger}_{\mathbf{C}}\mathbf{s}[i+1] + \underbrace{\mathbf{W}\mathbf{F}_1\mathbf{q}[i]}_{\mathbf{e}[i]} \end{aligned} \tag{4}$$

We can express $\mathbf{u}[i]$ as

$$\mathbf{u}[i] = \mathbf{\Lambda}\mathbf{s}[i] + \mathbf{A}\mathbf{s}[i] + \mathbf{B}\mathbf{s}[i-1] + \mathbf{C}\mathbf{s}[i+1] + \mathbf{e}[i].$$

The position of the window is determined by the delay parameter d and thus the elements of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ depend on d.

Notice that in the expression of $\mathbf{\Lambda}$ in (4), $\mathbf{F}_1\mathbf{H}_{00}\mathbf{F}_0$ is an $M \times M$ circulant matrix with the first column given by

$$\begin{pmatrix} h_d & h_{d+1} & \cdots & h_{d+L} & 0 & \cdots & 0 \end{pmatrix}^T.$$

Therefore $\mathbf{\Lambda}$ is a diagonal matrix whose diagonal elements are the M-point DFT of $h_d, h_{d+1}, \cdots h_{d+L}$. On the other hand the off-diaognal elements of $\mathbf{A}$ represent the interference within the same block. In particular the (k,ℓ)-th entry $A_{k,\ell}$, for $k \neq \ell$, represents the interference of the ℓ-th tone to the k-th tone of the same block. (The sum $A_{k,k} + \Lambda_{k,k}$

is the signal gain of the k-th tone.) Also the elements of $\mathbf{B}$ and $\mathbf{C}$ represent the interference from, respectively, the previous and the next block.

3. PROPOSED TEQ DESIGN

We know if the equivalent channel h_i has all the out-of-window coefficients equal to zero, then $\mathbf{A} = \mathbf{B} = \mathbf{C} = \mathbf{0}$, where $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are as given in (4). When the out-of-window coefficients are not zero, the nonzero coefficients lead to intra-block and inter-block interference. In this case, each tone contributes interference to other tones. In particular, each symbol interfere with symbols on other tones of the same block and also with symbols of the previous and the next blocks.

We will consider the interference of a selected set of tones $\mathcal{S}$ (source tones) to a chosen set of tones $\mathcal{T}$ (target tones). We propose the following objective function of the interference on the target tones due to the source tones

$$\phi = \min_{d} \min_{\mathbf{t}} \sum_{\ell \in \mathcal{S}} \sum_{k \in \mathcal{T}} \left(|A_{k,\ell}|^2 + |B_{k,\ell}|^2 + |C_{k,\ell}|^2 \right). \quad (5)$$

Although we are considering only a subset of tones as interference source, we will see that TEQ with good shortening effect can be obtained. In fact we can design good TEQ with as few as one source tone as will be demonstrated by simulation examples in Section 4. The TEQ designed by minimizing (5) usually has zeros in the frequency bands corresponding to the source and target tones. We can exploit the frequency division multiplexing property and place the source and target tones in the unused frequency bands. Then the TEQ will be free from zeros in the transmission bands and a better bit rate can be obtained. In what follows, we will see that the objective function in (5) can be minimized by solving eigen problems.

To minimize (5), we observe that the elements of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ can be expressed in terms of the TEQ coefficients. In particular,

$$A_{k,\ell} = \mathbf{a}_{k,\ell}\mathbf{t}, \; B_{k,\ell} = \mathbf{b}_{k,\ell}\mathbf{t}, \; C_{k,\ell} = \mathbf{c}_{k,\ell}\mathbf{t}, \quad (6)$$

where $\mathbf{t}$ is the $Q \times 1$ vector consisting of the TEQ coefficients. To see this, we note that $A_{k,\ell}$ a linear combination of the coefficients of h_i and it can be expressed as $A_{k,\ell} = \mathbf{a}'_{k,\ell}\mathbf{h}$, where $\mathbf{h}$ is the $N \times 1$ vector consisting of the coefficients of h_i. The equivalent channel is $h_i = c_i * t_i$ and we can write it as $\mathbf{h} = \mathbf{Q}\mathbf{t}$, where $\mathbf{Q}$ is an $N \times Q$ convolution matrix. Therefore, we have

$$A_{k,\ell} = \mathbf{a}'_{k,\ell}\mathbf{h} = \mathbf{a}'_{k,\ell}\mathbf{Q}\mathbf{t}.$$

Defining $\mathbf{a}_{k,\ell} = \mathbf{a}'_{k,\ell}\mathbf{Q}$, we have $A_{k,\ell}$ in the form of (6). Following a similar approach, we can express $B_{k,\ell}$ and $C_{k,\ell}$ as in (6). Using (6), we have $|A_{k,\ell}|^2 = \mathbf{t}^\dagger \mathbf{a}^\dagger_{k,\ell}\mathbf{a}_{k,\ell}\mathbf{t}$, where $\dagger$ denotes conjugation and transposition. Let

$$\mathbf{P} = \sum_{\ell \in \mathcal{S}} \sum_{k \in \mathcal{T}} \left(\mathbf{a}^\dagger_{k,\ell}\mathbf{a}_{k,\ell} + \mathbf{b}^\dagger_{k,\ell}\mathbf{b}_{k,\ell} + \mathbf{c}^\dagger_{k,\ell}\mathbf{c}_{k,\ell} \right).$$

Then the cost function ϕ becomes

$$\phi = \min_{d} \min_{\mathbf{t}} \mathbf{t}^\dagger \mathbf{P}\mathbf{t}, \quad (7)$$

subject to $\mathbf{t}^\dagger\mathbf{t} = 1$. For a given d, $\mathbf{t}^\dagger\mathbf{P}\mathbf{t}$ is a quadratic form of TEQ coefficients. It can be minimized by finding the eigenvector corresponding to the smallest eigenvalue of $\mathbf{P}$ and the minimum is the smallest eigenvalue of $\mathbf{P}$. The optimal TEQ is obtained by finding the d that minimizes ϕ.

Remark. For the convenience of derivations, we have assumed that h_i is shorter than N. The proposed method can be generalized for longer h_i. In this case, $\mathbf{H}(z)$ in (2) will have more coefficient matrices.

4. NUMERICAL SIMULATION

We use the VDSL (Very high speed Digital Subscriber Loop) system as an example in our simulations [2]. The DFT size M is 8192, and the cyclic prefix length L is 640. The sampling rate is $35.328MHz$. The channel noise is additive white Gaussian noise of -170dBm/Hz. We will consider downstream transmission, which has two transmission bands. The tones used for downstream transmission are roughly 33-870 (band 1) and 1206-1970 (band 2) [2]. The TEQ has 40taps. A common measure of TEQ performance is SIR (signal to interference ratio) defined as

$$SIR = \max_{\Delta} 10 \log \frac{\sum_{n=\Delta}^{\Delta+L} |h_i|^2}{\sum_{i \notin \{\Delta, \Delta+1, \cdots, \Delta+L\}} |h_i|^2},$$

We first use VDSL loop 7 as an example. Fig. 2(a) shows the impulse response of the original channel, which has an SIR of 35.8 dB. We design the TEQ using the proposed method with $\mathcal{S} = \{3000\}$ and $\mathcal{T} = \{3001, 3002, \cdots, 4095\}$. Both the source and target tones are in the frequency bands not used for downstream transmission. The equalized channel is given in Fig. 2(a). It has an SIR of 59.5 dB and the channel has been successfully shortened with only one source tone. Fig. 2(b) shows the magnitude response of the TEQ. The two downstream transmission bands are marked by 'D1' and 'D2'. We can see that both transmission bands fall into the passband of the TEQ and the TEQ is free from zeros in the transmission bands. The magnitude response of the equalized channel is shown in Fig. 2(c). Simulations show that the performance of the proposed TEQ is not very sensitive to the choice of the source and target tones so long as they are placed in the unused frequency bands.

The transmission rate performance of the proposed TEQ for 7 VDSL test loops [2] is listed in Table 1. For comparison, we have also listed the transmission rates of the MSSNR [3] and MERRY [7] methods. The transmission rate performance of the proposed method compares favorably with MSSNR and MERRY.

Loop	Proposed method	MSSNR	MERRY
VDSL-1L	73.06	69.34	72.92
VDSL-2L	69.82	62.94	68.05
VDSL-3L	67.74	55.11	65.82
VDSL-4L	42.79	41.63	36.32
VDSL-5	94.18	90.92	94.25
VDSL-6	79.76	73.53	79.38
VDSL-7	54.17	53.01	49.42

Table 1: Bit rates (Mbits/sec) on 7 VDSL test loops.

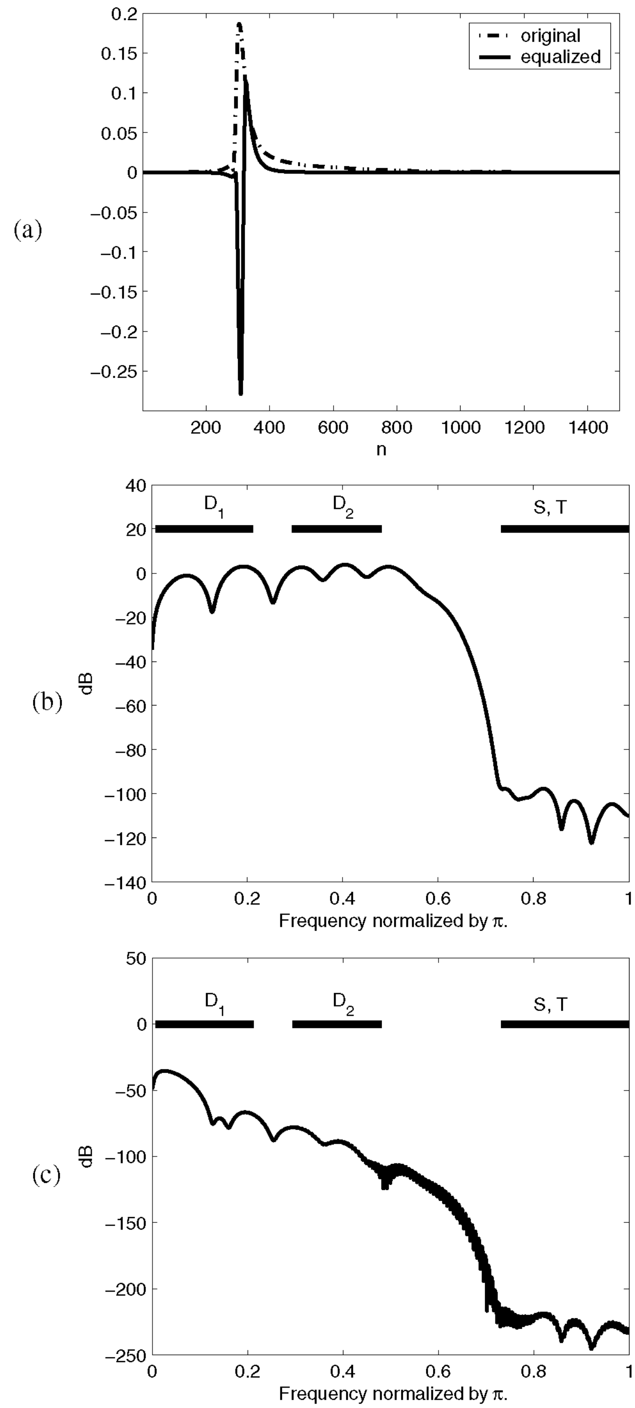

Figure 2: (a) The impulse response of the original VDSL loop 7 and the equalized channel, (b) magnitude response of the TEQ , (c) magnitude response of the equalized channel.

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