ON THE DESIGN OF CMFB TRANSCEIVERS FOR UNKNOWN CHANNELS

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ABSTRACT

In this paper, we consider the cosine modulated filter bank (CMFB) transceiver for unknown channels. Although it is a CMFB transceiver, the only channel dependent part of the transceiver is a set of scalar multipliers at the receiver, like the OFDM. We show how to optimize the transmit and receive prototype filters so that the signal to interference ratio (SIR) is maximized. Moreover a new objective function that can incorporate the frequency response criteria is proposed. Simulation results show that we can obtain CMFB transceivers with high SIR values and good frequency responses.

1. INTRODUCTION

The discrete multitone (DMT) and orthogonal frequency division multiplexing (OFDM) systems have found many applications in broadband communications. In these systems, IDFT and DFT operations are performed at the transmitter and receiver respectively, and redundancy known as cyclic prefix (CP) is added at the transmitter to aid the channel equalization. If the CP length is not smaller than the channel order, then any FIR LTI channel is converted to a set of parallel single-tap subchannels. Recently a new transmission technique based on discrete cosine transform (DCT) was proposed [1]. In the DCT based system, IDCT and DCT are performed at the transmitter and receiver respectively. By judiciously adding suffix and prefix at the transmitter, it was shown that any linear-phase channels with order 2ρ (where 2ρ is the total length of prefix and suffix) is converted to a set of parallel single-tap subchannels. Like the OFDM and DMT systems, simple equalization can be employed at the receiver for symbol recovery. Though these DFT and DCT based transceivers can effectively combat intersymbol interference (ISI), their transmit and receive filters do have not good frequency response. This is because both DFT and DCT filters have poor stopband attenuation. These transceivers have a large out of band energy at the transmitter and their receivers are vulnerable to narrowband interference.

It is the purpose of this paper to extend the DCT based transceiver in [1] to the more general case of CMFB transceiver. In the past, many transceiver systems based on filter bank (FB) design have been proposed [2]-[6] for their good frequency response. However FB transceivers in general suffer from severe ISI when the channel is frequency selective. Post processing is therefore needed at the receiver to reduce the effect of ISI. Recently a new method was proposed for the design of DFT FB transceivers for unknown channels [7]. For a fixed receiver, the transmitter is optimized for SIR maximization. In this paper, we extend the results to the case of CMFB transceivers. In addition, a new iterative algorithm that can incorporate the frequency response criteria is proposed for designing CMFB transceivers with high SIR and good frequency responses.

The proposed CMFB transceivers differ from other existing CMFB transceivers in two aspects. Firstly, the proposed SIR-maximized transceiver is channel independent and thus there is no need to design a new CMFB when the channel changes. Secondly, except for scalar multiplications no post processing is needed at the receiver and hence the frequency response of receive filters will not be affected by post processing.

Notation: The conjugate transpose of a matrix $\mathbf{A}$ is denoted by $\mathbf{A}^\dagger$. For an LTI system with impulse response $f(n)$, the notation $[F(z)]_{\downarrow N}$ represents an LTI system with impulse response $f(Nn)$. The notation $\mathcal{E}_x\{\bullet\}$ represents the statistical expectation taken with respect to the random variable x.

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2. DCT BASED TRANSCEIVERS [1]

The block diagram of a DCT based transceiver is shown in Fig. 1. At the transmitter, an M-point operation is applied to the input vector $\mathbf{s}$. The number M is even and the matrix $\mathbf{C}^T$ is a Type-II IDCT matrix[1]. The output vector of the IDCT matrix $\mathbf{x} = [x_0\ x_1\ \ldots\ x_{M-1}]^T$ is first passed through a parallel-to-serial converter. Then a prefix of length ρ and a suffix of length ρ are added to form a transmission block of length $(M+2\rho)$:

$$(\underbrace{x_{\rho-1}\ \ldots\ x_0}_{\text{prefix}}\ x_0\ x_1 \ldots\ x_{M-1}\ \underbrace{x_{M-1}\ \ldots\ x_{M-\rho}}_{\text{suffix}}).$$

At the receiver, for each block of $(M+2\rho)$ received samples, the first and last ρ samples are first removed and then an M-point Type-IIE DCT operation is applied. Suppose that the channel $P(z)$ is linear-phase with order 2ρ: $P(z) = p_0 + \sum_{l=1}^{\rho} p_l \left(z^l + z^{-l}\right)$ (For example, in DSL applications, many design technique for time domain equalizer can be modified so that the shortened channel has linear phase[1]). For any 2ρ-th order linear phase $P(z)$, it was shown [1] that the transfer function from the input vector $\mathbf{s}$ to the output of the DCT matrix is an M by M diagonal matrix $diag[P_0\ \ldots\ P_{M-1}]$. The k-th subchannel gain P_k is given by $P_k = p_0 + \sum_{l=1}^{\rho} 2p_l \cos(\pi kl/M)$. By multiplying the DCT output by a set of scalars $1/P_k$ as shown in Fig. 1, we get a zero-forcing receiver.

3. FB TRANSCEIVERS FOR UNKNOWN CHANNELS[7]

The block diagram of a FB transceiver is shown in Fig 2. There are M subchannels and the decimation ratio is N. We assume that $N > M$ and $N - M$ redundant samples are added. The

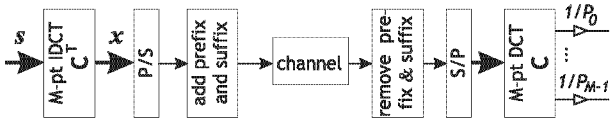

Figure 1: A DCT based transceiver.

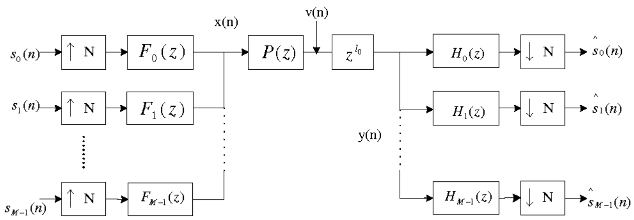
Figure 2: A filter bank transceiver.

advance operator z^{l_0} in Fig. 2 is introduced to compensate for the channel delay. The system from $s_i(n)$ to $\hat{s}_j(n)$ is LTI with transfer function

$$T_{ji}(z) = \left[H_j(z)z^{l_0}P(z)F_i(z)\right]_{\downarrow N}.$$

Assuming that $P(z)$ is an FIR LTI channel with $P(z) = \sum_{l=0}^{L} p_l z^{-l}$, then we have

$$T_{ji}(z) = \sum_{l=0}^{L} p_l \left[H_j(z)F_i(z)z^{l_0-l}\right]_{\downarrow N}. \quad (1)$$

For convenience of discussion, we define

$$\sum_n b_{i,j,l}(n)z^{-n} = \left[H_j(z)F_i(z)z^{l_0-l}\right]_{\downarrow N} \quad (2)$$

for $0 \le i,j \le M-1$ and $0 \le l \le L$. From (1), one can see that the FB transceiver is free from ISI for any p_l if and only if the parameter $b_{i,j,l}(n)$ satisfies

$$b_{i,j,l}(n) = \lambda_{i,l}\delta(i-j)\delta(n). \quad (3)$$

When the transmit filters $F_i(z)$ and receive filters $H_j(z)$ are such that the above condition is satisfied, any $P(z)$ of order L is converted to a set of M parallel subchannels with the kth subchannel gain given by $\sum_{l=0}^{L} p_l\lambda_{k,l}$. Unfortunately except for the special case of block transmission where the orders of $F_i(z)$ and $H_i(z)$ are smaller than N, the ISI-free solution is still unknown.

4. SIR OPTIMIZED CMFB TRANSCEIVERS

The cost of an FB transceiver is very high because we have to implement M filters at the transmitter or receiver. To reduce the cost, we consider CMFB derived from the Type-IIE DCT matrix. For notational convenience, we assume that the filters have the same order K and $(N-M)$ is even. The transmit and receive filters are respectively given by

$$F_i(z) = c_i \sum_{n=0}^{K} f(n)\cos\left(\frac{\pi i(0.5+n-\rho-\theta)}{M}\right) z^{-n}$$
$$H_i(z) = c_i \sum_{n=0}^{K} h(n)\cos\left(\frac{\pi i(0.5+n-\rho-\theta)}{M}\right) z^{n},$$

where $c_0 = \sqrt{1/M}$ and $c_i = \sqrt{2/M}$ for $i \neq 0$. The quantities θ and ρ are respectively given by

$$\theta = \frac{K-N+1}{2},\ \rho = \frac{N-M}{2}.$$

The filter orders K can be larger than N. To implement the transmitter or receiver, we need to implement only one prototype filter and an M by M IDCT or DCT matrix. Note that by setting $K = N$, $f(n) = 1$ for $0 \le n < N$ and $h(n) = 1$ for $\rho \le n < N-\rho$, it can be verified that the CMFB transceiver reduces to DCT based transceiver in Sec. 2.

To get an ISI-free CMFB transceiver, the prototype filters $F(z)$ and $H(z)$ should be such that (3) is satisfied. Except for the special case of DCT based transceiver in Sec. 2, the solution is still unknown. In the following, we will show how to design the prototype filters so that SIR is maximized. From the definition of $b_{i,j,l}(n)$ in (2), the j-th output signal of the CMFB transceiver can be expressed as

$$\begin{aligned}\hat{s}_j(n) &= \left[\sum_{l=0}^{L} b_{j,j,l}(0)p(l)\right] s_j(n) \\ &+ \left[\sum_{l=0}^{L} p(l)(b_{j,j,l}(n) - b_{j,j,l}(0)\delta(n))\right] * s_j(n) \\ &+ \sum_{i=0,i\neq j}^{M-1}\sum_{l=0}^{L} p(l)b_{i,j,l}(n) * s_i(n),\end{aligned}$$

where $*$ denotes convolution. Looking at the right hand side of the above equation, we notice that the first term is the desired signal whereas the second and third terms are ISI. In what follows, we assume that the signals $s_i(n)$ satisfy

$$\mathcal{E}_s\{s_i(n)s_j^*(m)\} = \sigma_s^2\delta(i-j)\delta(m-n).$$

The channel $P(z)$ is a linear phase filter with $p(l) = p(L-l)$ and L is even. The channel taps are zero-mean random variables satisfying

$$\mathcal{E}_p\{p_ip_j^*\} = \sigma_p^2(i)\delta(i-j), \quad (4)$$

for $0 \le i,j \le L/2$. For the case of unknown linear phase channels, one can assume that all the taps have the same variance, $\sigma_p^2(i) = \sigma_p^2$ for all i. Define the $(L+1)$ by 1 vectors

$$\mathbf{b}_{i,j}(n) = \begin{bmatrix} b_{i,j,0}(n) \\ b_{i,j,1}(n) \\ \vdots \\ b_{i,j,L}(n)\end{bmatrix},\ \mathbf{p} = \begin{bmatrix} p_0 \\ p_1 \\ \vdots \\ p_L\end{bmatrix}.$$

Then one can verify that under the above signal and channel model, the signal power at the j-th output is:

$$P_{sig}(j) = \sigma_s^2\mathcal{E}_p\left|\sum_{l=0}^{L} b_{j,j,l}(0)p_l\right|^2 = \sigma_s^2\mathbf{b}_{j,j}^\dagger(0)\underbrace{\mathcal{E}_p\{\mathbf{p}\mathbf{p}^\dagger\}}_{\mathbf{\Lambda}}\mathbf{b}_{j,j}(0).$$

Note that most of the entries of the $(L+1)$ by $(L+1)$ matrix $\mathbf{\Lambda}$ are zero, except the diagonal and anti-diagonal entries. Similarly, it can be found that the ISI power at the j-th output is

$$P_{isi}(j) = \sigma_s^2\left(\sum_{n\neq 0}\mathbf{b}_{j,j}^\dagger(n)\mathbf{\Lambda}\mathbf{b}_{j,j}(n) + \sum_{n,i\neq j}\mathbf{b}_{i,j}^\dagger(n)\mathbf{\Lambda}\mathbf{b}_{i,j}(n)\right).$$

The SIR of the transceiver is therefore given by

$$SIR = \frac{\sum_{j=0}^{M-1} P_{sig}(j)}{\sum_{j=0}^{M-1} P_{isi}(j)}.$$

Next we will show that the SIR can be written as a Rayleigh-Ritz quotient of $f(n)$ or $h(n)$. To do this, let us define

$$\mathbf{f} = [f(0)\ \ f(1)\ \ \ldots\ \ f(K)]^T$$
$$\mathbf{h} = [h(0)\ \ h(1)\ \ \ldots\ \ h(K)]^T$$

For $0 \le i < M$, define the $(K+1)$ by $(K+1)$ diagonal matrices $\mathbf{D}_i$ whose kk-th entry is given by

$$[\mathbf{D}_i]_{kk} = \cos\left(\frac{i\pi(k+0.5-\rho-\theta)}{M}\right).$$

From (2), one can see that $b_{i,j,l}(n)$ is one of the coefficient of the convolution $f_i(n) * h_j(n)$. Hence we can write

$$\mathbf{b}_{i,j}(n) = \mathbf{A}_j(n)\mathbf{D}_i\mathbf{f} = \mathbf{B}_i(n)\mathbf{D}_j\mathbf{h},$$

for some $(L+1)$ by $(K+1)$ matrices $\mathbf{A}_j(n)$ and $\mathbf{B}_i(n)$. The entries of $\mathbf{A}_j(n)$ and $\mathbf{B}_i(n)$ consist of the impulse responses of the j-th receive filter and the i-th transmit filter respectively. Substituting $\mathbf{b}_{i,j}(n) = \mathbf{A}_j(n)\mathbf{D}_i\mathbf{f}$ into the expressions for $P_{sig}(j)$ and $P_{isi}(j)$, we can write the SIR as

$$SIR = \frac{\mathbf{f}^\dagger\mathbf{Q}_0\mathbf{f}}{\mathbf{f}^\dagger\mathbf{Q}_1\mathbf{f}}, \tag{5}$$

where $\mathbf{Q}_0$ and $\mathbf{Q}_1$ are positive semi definite matrices given by

$$\mathbf{Q}_0 = \sum_{j=0}^{M-1} \mathbf{D}_j^\dagger\mathbf{A}_j^\dagger(0)\mathbf{\Lambda}\mathbf{A}_j(0)\mathbf{D}_j \tag{6}$$
$$\mathbf{Q}_1 = \sum_{j,n\neq 0} \mathbf{D}_j^\dagger\mathbf{A}_j^\dagger(n)\mathbf{\Lambda}\mathbf{A}_j(n)\mathbf{D}_j + \sum_{j,n,i\neq j} \mathbf{D}_i^\dagger\mathbf{A}_j^\dagger(n)\mathbf{\Lambda}\mathbf{A}_j(n)\mathbf{D}_i.$$

Similarly we can also express the SIR in terms of $\mathbf{h}$ as

$$SIR = \frac{\mathbf{h}^\dagger\widetilde{\mathbf{Q}}_0\mathbf{h}}{\mathbf{h}^\dagger\widetilde{\mathbf{Q}}_1\mathbf{h}}, \tag{7}$$

where $\widetilde{\mathbf{Q}}_0$ and $\widetilde{\mathbf{Q}}_1$ can be obtained from (6) by replacing $\mathbf{A}_j(n)$ with $\mathbf{B}_j(n)$. Given one of the two prototype filters, we can optimize the other prototype filter so the the SIR is maximized. The optimal $\mathbf{f}$ and $\mathbf{h}$ can be obtained by solving the eigen problem involved the Rayleigh-Ritz ratios of (5) and (7) respectively. By iteratively solving for the optimal prototype filters, the SIR will increase. But as we will see later, the frequency responses of the filters will degrade as the iteration number increases. It is observed in the experiment that the optimal prototype filters will eventually approach the DCT filters and the CMFB transceiver becomes the DCT based transceiver in Sec. 2.

Incorporation of Frequency Constraints: In many applications, it is desirable to have transceivers with good transmit and receive filters. In transceiver design, the passband criteria is not as important as the stopband criteria because of two reasons: (i) the number of subchannels M is usually large and the filters have narrowband passbands; (ii) the reconstruction property of the transceiver is ensured by high SIR values. To get filters with a large stopband attenuation, we can consider the weighted stopband energies

$$E_h = \int_{\omega_s}^{\pi} \omega^\gamma \left|H(e^{j\omega})\right|^2 \frac{d\omega}{\pi},\ E_f = \int_{\omega_s}^{\pi} \omega^\gamma \left|F(e^{j\omega})\right|^2 \frac{d\omega}{\pi},$$

where ω_s is the stopband edge and $\gamma \ge 0$ is a parameter that controls the decay of stopband responses. Using the eigenfilter approach[8], we can express these energies as $E_h = \mathbf{h}^\dagger\mathbf{R}_h\mathbf{h}$ and $E_f = \mathbf{f}^\dagger\mathbf{R}_f\mathbf{f}$ for some positive definite matrices $\mathbf{R}_h$ and $\mathbf{R}_f$. The new objective functions which incorporate the stopband energy as well as SIR are

$$J_f = \frac{\mathbf{f}^\dagger\mathbf{Q}_0\mathbf{f}}{\mathbf{f}^\dagger\left((1-\alpha)\mathbf{Q}_1 + \alpha\mathbf{R}_f\right)\mathbf{f}}$$
$$J_h = \frac{\mathbf{h}^\dagger\widetilde{\mathbf{Q}}_0\mathbf{h}}{\mathbf{h}^\dagger\left((1-\alpha)\widetilde{\mathbf{Q}}_1 + \alpha\mathbf{R}_h\right)\mathbf{h}},$$

where $0 \leqslant \alpha \leqslant 1$. The numerators of these two new cost functions are the signal power whereas their denominators are a weighted sum of the ISI power and the stopband energy. With an appropriate α, we are able to get good SIR value and small stopband energy simultaneously. By alternatively solving for the best $\mathbf{f}$ and $\mathbf{h}$ that maximize J_f and J_h, we can iteratively improve the SIR of the CMFB transceiver without destroying the stopband attenuation.

5. A DESIGN EXAMPLE

In this example, the number of subbands is $M = 128$ and the interpolation ratio is $N = 160$. The orders of the prototype filters are $K = 320$. The stopband edge is $\omega_s = 2\pi/M$. The two prototype filters are initialized as good lowpass filter designed by the eigenfilter approach[8] (labeled as 'no iter' in Fig. 4 and 5). The transmission channel $P(z)$ is a linear phase channel with order $L = 16$. The channel taps are random variables that satisfy (4) with $\sigma_p^2(i) = 1$ for $0 \le i \le 8$. This is the commonly adopted channel model when no channel statistics is available.

We alternatively optimize $\mathbf{f}$ and $\mathbf{h}$. Both cases of optimization with and without frequency constraint are considered. For the optimization without frequency constraint, the objective functions are the two SIR expressions in (5) and (7). For the optimization with frequency constraint, the two objective functions are J_f and J_h. We consider two different weighting functions for the stopband energy, ω^0 (no weighting) with $\alpha = 0.9899$ and ω^1 with $\alpha = 0.9956$. We compare our results with the prefilter method[9] (the values of n_{ha} and n_{fa} defined in [9] are chosen as 80).

The plot of SIR versus the number of iterations is shown in Fig. 3. From the figure, we see that as the number of iterations increases, SIR increases for all methods. However the SIR for the prefilter approach saturates at around 31 dB whereas for the proposed eigenfilter approach, the SIR values saturate at around 37 dB (no weighting ω^0) and 38 dB (weighting function ω^1). On the other hand, without any frequency constraint, the SIR does not saturate and it is observed in the experiment that the filters approach the DCT filters (which has an infinite SIR value as DCT based transceiver in Sec. 2 is ISI-free). In Fig. 4 and 5, we show the magnitude responses of the transmit and receive prototype filters after 30 iterations respectively. As we can see, if no frequency constraint is applied in the optimization, the prototype filters are no longer good filters though they are initialized as good lowpass filters. The stopband attenuation of the proposed eigenfilter approach is much better than that of the prefilter approach, especially when the weighting of ω^1 is applied.

6. CONCLUSIONS

We proposed a method for designing CMFB transceivers for unknown channels. The SIR is formulated as a Rayleigh-Ritz ratio of the prototype filters. Frequency constraint can be incorporated into the cost function. The prototype filters are alternatively optimized for SIR maximization while preserving the good frequency response. Examples show that CMFB transceivers with high SIR and good frequency response can be obtained by the proposed approach.

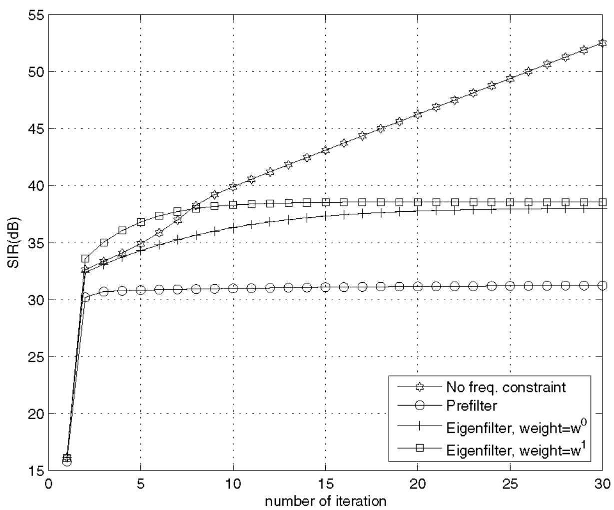

Figure 3: SIR of CMFB transceivers.

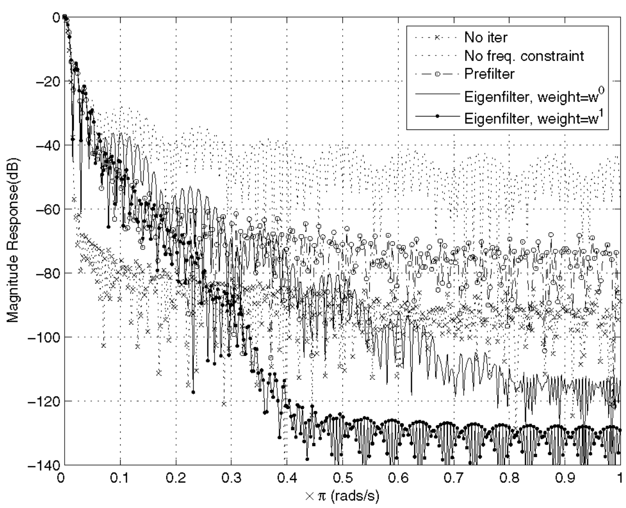

Figure 4: Magnitude response of the transmit prototype filter

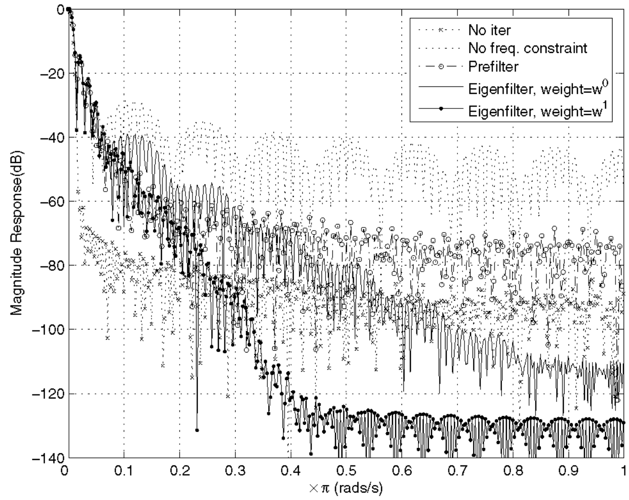

Figure 5: Magnitude response of the receive prototype filter

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