Design of Time Domain Equalizers Incorporating Radio Frequency Interference Suppression

Ya-Wen Wu, Yuan-Pei Lin, Chien-Chang Li
Dept. Elec. and Control Engr.
National Chiao Tung Univ.
Hsinchu, Taiwan
wu826.ece93g@nctu.edu.tw
ypl@mail.nctu.edu.tw
halliday.ece92g@nctu.edu.tw

See-May Phoong
Dept. of EE & Grad. Inst. of Comm Engr.
National Taiwan Univ.
Taipei, Taiwan
smp@cc.ee.ntu.edu.tw

***Abstract*— It is known that radio frequency interference (RFI) degrades the performance of DMT systems for digital subscriber loops. The RFI signal, though narrowband in nature, may be spread to subchannels around the RFI frequencies due to the large sidelobes of the receiving rectangular window. In this paper, we propose a joint consideration of channel shortening and RFI suppression in the design of time domain equalizers (TEQ). We will design the TEQ by minimizing ISI, channel noise and RFI interference. Simulation results are given to show that the proposed method can considerably reduce interference around the RFI frequencies and increase the transmission rate.**

***Keywords*—time-domain equalizer, radio frequency interference, RFI suppression**

I. Introduction

Discrete multitone (DMT) modulation is a very useful method for high-speed data transmission, e.g. asymmetric digital subscriber lines and very high speed digital subscriber lines (VDSL). The transmitter and receiver perform M-point IDFT and DFT computation, respectively, where M is the number of subchannels. For every block of M data samples, the transmitter adds a cyclic prefix (CP) of length ν. If the number ν is chosen to be no smaller than the order of the channel, then interblock interference (IBI) can be easily removed. The DFT outputs are multiplied with a set of scalars, known as the frequency domain equalizers (FEQ).

Usually the channel is longer than ν and the receiver includes a TEQ to shorten the channel impulse response. Many TEQ designs have been proposed [3]-[7]. In [3], Melsa *et al*. design the optimal TEQ that minimizes the out-of-window energy of the equivalent channel to minimizes IBI. In [4], Arslan *et al*. minimize ISI and channel noise on the tones used for transmission to design the TEQ. Per-tone equalization for bit rate maximization is proposed in [5]. A filterbank approach to the design of TEQ for maximizing the bit rate is given in [6]. A comprehensive overview on TEQ and a unified design approach is available in [7].

In DMT applications such as ADSL and VDSL, some of the frequency bands are also used by radio transmission systems, e.g., amplitude-modulation stations and amateur radio. These radio signals can be coupled into wires and interfere; they are known as RFI ingress [8]. The large sidelobe of the rectangular window in conventional multicarrier systems lead to spectral leakage. As a result, many neighboring tones can be affected. The signal to interference noise ratio of these tones are reduced and the total transmission rate is decreased. To improve RFI suppression, receiver windowing have been proposed in [9]-[10]. A combination of a raised-cosine window and per tone equalization are proposed to suppress RFI interference in [11]. In [13], it provides a general framework where a combined window and TEQ can be designed to maximize the bit rate for a given number of TEQ and window taps. A channel independent window that minimizes output interference is given in [12].

In this paper, we propose a filterbank framework to the design of TEQ. We consider a joint optimization of channel shortening and RFI suppression. We will design the TEQ by minimizing ISI, channel noise and RFI interference. We will see that the proposed TEQ can significantly alleviate the effect of RFI for the tones around RFI frequencies. Simulation results are given to show that the proposed method can considerably reduce interference and achieve a higher transmission rate. The sections are organized as follows: In section II, we derive the filterbank representation of the DMT receiver. In section III, we review the TEQ design for minimizing ISI and channel noise [4], [6]. The TEQ design by taking RFI suppression into consideration will be given in section IV. Simulation results are given in section V. A conclusion is given in section VI.

II. Filterbank Representation

Fig. 1 shows the DMT system with TEQ $t(n)$. The size of the IDFT matrix is M and the cyclic prefix length is ν. The channel is modeled as an LTI filter with additive noise $v(n)$ and radio interference $u(n)$. The shortened channel becomes $c(n) * t(n)$. Let $\lambda(n)$ be the part of $c(n) * t(n)$ within a window of $\nu + 1$ samples where the energy is most concentrated. We can write $\lambda(n) = g(n)(c(n) * t(n))$. The

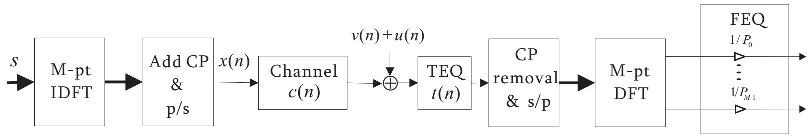

Fig. 1. The DMT transceiver.

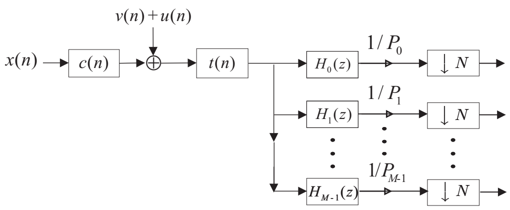

Fig. 2. Filterbank representation of DMT receiver.

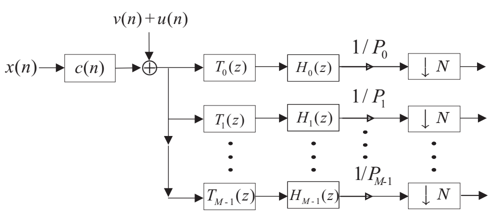

Fig. 3. DMT receiver with multiple TEQs.

window function $g(n)$ is given by

$$g(n) = \begin{cases} 1, & d \le n \le d+\nu, \\ 0, & \text{otherwise}, \end{cases}$$

where d is the synchronization delay. The scalar multiplier $1/P_k$ are known as the frequency domain equalizers, where P_k is equal to $\Lambda(z)\mid_{z=e^{j2\pi k/M}}$.

It is known that the receiver in Fig. 1 can be redrawn as Fig. 2 [6], where $N = M+\nu$. The M receiving filters are frequency shifted versions of the first receiving filter $H_0(z)$. In particular, $H_i(z) = W^{-i\nu}H_0(zW^i)$, for $i = 0, 1, \ldots, M-1$, where $H_0(z) = \sum_{k=\nu}^{N-1} z^k$, and $W = e^{-j\frac{2\pi}{M}}$. If multiple TEQs are used, one for each subchannel, the receiver becomes the one shown in Fig. 3 [6]. Now the effective channel for the kth subchannel is $c(n) * t_k(n)$, and $\lambda_k(n) = g(n)(c(n) * t_k(n))$. Let $\lambda_k(n)$ be the part of $c(n) * t_k(n)$ inside the window, i.e., $\lambda_k(n) = g(n)(c(n) * t_k(n))$. The k-th FEQ coefficient P_k is given by $P_k = \Lambda_k(z)\mid_{z=e^{j2\pi k/M}}$, where $\Lambda_k(z)$ is the z-transform of $\lambda_k(n)$. We note that if the order of the shortened channel $C(z)T_k(z)$ is not larger than cyclic prefix length, the DMT system still has ISI free property. Also, by setting $T_k(z) = T(z)$ for $k = 0, 1, \ldots, M-1$, then Fig. 3 reduces to the case in Fig. 2. In the design of $T_k(z)$, we can exploit the extra freedom of the proposed receiver so that the ISI, noise and RFI can be minimized.

III. TEQ DESIGN FOR MINIMUM ISI AND CHANNEL NOISE

In this section, we review the TEQ design method for minimizing ISI and channel noise [4], [6]. Suppose the length of the original channel $c(n)$ is L and the lengths of the TEQs are T. Define $f_k(n) = (1-g(n))(c(n) * t_k(n))$. From Fig. 3, we see that the output error caused by ISI and noise at the kth tone is $e_k(n) = [e_{isi,k}(n) + e_{noise,k}(n)]_{\downarrow N}$, where

$$e_{isi,k}(n) = x(n) * f_k(n) * h_k(n)/P_k, \tag{1}$$

$$e_{noise,k}(n) = v(n) * t_k(n) * h_k(n)/P_k. \tag{2}$$

Owing to the fact that the decimator does not change the signal variance, we have $\sigma_k^2 = \sigma_{isi,k}^2 + \sigma_{noise,k}^2$, where $\sigma_{isi,k}^2$ and $\sigma_{noise,k}^2$ are respectively the variances of $e_{isi,k}(n)$ and $e_{noise,k}(n)$. The signal and noise are assumed to be uncorrelated, and $x(n)$ is assumed to be a white WSS process for simplicity.

Let $\mathbf{t}_k$ be the $T \times 1$ column vector consisting of the k-th TEQ coefficients. The coefficients of $\lambda_k(n)$ can be written in a matrix form as $\mathbf{GCt}_k$, where $\mathbf{C}$ is an $(L+T-1)\times T$ lower triangular Toeplitz matrix with first columns given by

$$(\ c(0)\ c(1)\ \cdots\ c(L-1)\ 0\ \cdots\ 0\ )^T,$$

and $\mathbf{G}$ is diagonal with $[\mathbf{G}]_{ii} = g(i), i = 0, 1, \cdots, L+T-2$. Let $\mathbf{w}_k$ be the k-th row vector of the M-point DFT matrix. Then P_k can be written as $P_k = \mathbf{w}_k\mathbf{GCt}_k$ and thus $|P_k|^2 = \mathbf{t}_k^\dagger \mathbf{B}_k \mathbf{t}_k$, where

$$\mathbf{B}_k = \mathbf{C}^\dagger \mathbf{G}^\dagger \mathbf{w}_k^\dagger \mathbf{w}_k \mathbf{GC}. \tag{3}$$

Using matrix representation for convolution, we can write the coefficients of $f_k(n) * h_k(n)$ as $\mathbf{H}_k\mathbf{DCt}_k$, where $\mathbf{H}_k$ is an $(M+L+T-2)\times(L+T-1)$ lower triangular Toeplitz matrix with the first column given by

$$(\ e^{-j\frac{2\pi}{M}k(M-1)}\ e^{-j\frac{2\pi}{M}k(M-2)}\ \ldots\ 1\ 0\ \ldots\ 0\ )^T, \tag{4}$$

and $\mathbf{D}$ is diagonal, $\mathbf{D} = \mathbf{I} - \mathbf{G}$. Using this matrix representation, $\sigma_{isi,k}^2$ can be expressed as

$$\sigma_{isi,k}^2 = \frac{\mathbf{t}_k^\dagger \mathbf{Q}_{isi,k} \mathbf{t}_k}{\mathbf{t}_k^\dagger \mathbf{B}_k \mathbf{t}_k}, \tag{5}$$

where $\mathbf{Q}_{isi,k} = \sigma_x^2 \mathbf{C}^\dagger \mathbf{D}^\dagger \mathbf{H}_k^\dagger \mathbf{H}_k \mathbf{D}\mathbf{C}$ Similarly, we have $\sigma^2_{noise,k}$ in the following matrix form

$$\sigma^2_{noise,k} = \frac{\mathbf{t}_k^\dagger \mathbf{Q}_{noise,k} \mathbf{t}_k}{\mathbf{t}_k^\dagger \mathbf{B}_k \mathbf{t}_k}, \tag{6}$$

where $\mathbf{Q}_{noise,k} = \tilde{\mathbf{H}}_k^\dagger \mathbf{R}_v \tilde{\mathbf{H}}_k$. The matrix $\mathbf{R}_v$ is the $(M+T-1)\times(M+T-1)$ autocorrelation matrix of $v(n)$ and $\tilde{\mathbf{H}}_k$ is an $(M+T-1)\times T$ lower triangular Toeplitz matrix whose first column is the same as in (4) except that the last $L-1$ zeros are discarded.

IV. TEQ DESIGN INCORPORATING RFI SUPPRESSION

We assume that the statistics of interference sources are available. The radio interference is known as a narrow band signal and for the duration of one DMT symbol, it can be considered as a sum of sinusoids like as $u(n) = \sum_{l=0}^{J-1} \alpha_l cos(\omega_l n + \theta_l)$, where ω_l is the frequency of the l-th interference source, and α_l and θ_l are the corresponding amplitude and phase. To analyze the effect of interference, we apply an interference-only signal $u(n)$ to the receiver in Fig. 3. The output of the kth tone is $e_k(n) = [e_{rfi,k}(n)]_{\downarrow N}$, where

$$e_{rfi,k}(n) = \frac{1}{P_k}\sum_{l=0}^{J-1}\frac{\alpha_l}{2}[r_{1,l}(n) + r_{2,l}(n)],$$

$$r_{1,l}(n) = H_k(e^{j\omega_l})T_k(e^{j\omega_l})e^{j(\omega_l n+\theta_l)},$$

$$r_{2,l}(n) = H_k(e^{-j\omega_l})T_k(e^{-j\omega_l})e^{-j(\omega_l n+\theta_l)}.$$

Note that the decimators do not change the amplitude. To suppress interference, we can we minimize

$$\phi_{rfi,k} = \frac{1}{|P_k|^2}\sum_{l=0}^{J-1}\frac{\alpha_l^2}{4}[|\rho_{k,l}|^2 + |\eta_{k,l}|^2], \tag{7}$$

where

$$\rho_{k,l}(n) = H_k(e^{j\omega_l})T_k(e^{j\omega_l}), \eta_{k,l}(n) = H_k(e^{-j\omega_l})T_k(e^{-j\omega_l}). \tag{8}$$

Notice that the equations $\rho_{k,l}$ and $\eta_{k,l}$ can be expressed as $\rho_{k,l} = \boldsymbol{\tau}_{k,l}\mathbf{t}_k$, $\eta_{k,l} = \boldsymbol{\zeta}_{k,l}\mathbf{t}_k$, where

$$\boldsymbol{\tau}_{k,l} = H_k(e^{j\omega_l})[1 \; e^{-j\omega_l} \; \ldots \; e^{-j\omega_l(T-1)}],$$

$$\boldsymbol{\zeta}_{k,l} = H_k(e^{-j\omega_l})[1 \; e^{j\omega_l} \; \ldots \; e^{j\omega_l(T-1)}].$$

Using the above derivations, $\phi_{rfi,k}$ in (7) can be rewritten as:

$$\phi_{rfi,k} = \frac{\mathbf{t}_k^\dagger \mathbf{Q}_{rfi,k}\mathbf{t}_k}{\mathbf{t}_k^\dagger \mathbf{B}_k \mathbf{t}_k}, \tag{9}$$

where $\mathbf{B}_k$ is as in (3) and $\mathbf{Q}_{rfi,k}$ is

$$\mathbf{Q}_{rfi,k} = \frac{1}{4}\sum_{l=0}^{J-1}\alpha_l^2[\boldsymbol{\tau}_{k,l}^\dagger\boldsymbol{\tau}_{k,l} + \boldsymbol{\zeta}_{k,l}^\dagger\boldsymbol{\zeta}_{k,l}]. \tag{10}$$

An objective function that minimizes ISI, noise variance and RFI interference is

$$\phi_k = \sigma^2_{isi,k} + \sigma^2_{noise,k} + \phi_{rfi,k} = \frac{\mathbf{t}_k^\dagger \mathbf{Q}_k \mathbf{t}_k}{\mathbf{t}_k^\dagger \mathbf{B}_k \mathbf{t}_k}, \tag{11}$$

where $\mathbf{Q}_k = \mathbf{Q}_{isi,k} + \mathbf{Q}_{noise,k} + \mathbf{Q}_{rfi,k}$. The optimal TEQ $\mathbf{t}_k$ minimizes the above ratio or maximizes its inverse. We can obtain the optimal TEQs $\mathbf{t}_k$ by finding the eigenvector corresponding to the maximum eigenvalue of $\mathbf{Q}_k^{-1}\mathbf{B}_k$. Notice that the ratio in (11) is proportional to the inverse of the signal to interference ratio (SINR) of the kth subchannel. Therefore the optimal TEQ $\mathbf{t}_k$ will maximize the subchannel SINR$_k$.

V. SIMULATION RESULTS

We evaluate the performance of the proposed TEQ design algorithm using VDSL loops 1-7 [2]. For the first 4 loops, long lengths (4500 ft) are used. Downstream transmission is considered. The DFT size $M = 1024$ and the cyclic prefix length $\nu = 80$. The channel noise consists additive white Gaussian noise of -140 dBm/Hz, and also NEXT and FEXT crosstalk. The RFI interference is of -90 dBm differential mode [2]. We assume the RFI interference are of frequencies 1.05MHz, 1.9MHz and 2.0MHz. The TEQ for each subchannel has length 30.

Table. I shows the transmission rate of the proposed method. For comparison, we have shown the performance of windowing methods with a single TEQ. We have shown the cases of rectangular, Hanning, and Blackman windows as well as the window design in [12] and the joint window and TEQ design in [13]. The lengths of the window and the TEQ are 10 and 20, respectively. The transmission rate is computed using

$$b_i = \left\lfloor \log_2(1 + \frac{SINR_i}{\Gamma}) \right\rfloor,$$

where the parameter Γ corresponds to a bit error probability of 10^{-5}, and SINR$_i$ are obtained from simulation measurements. To gain further insight, we show in Fig. 4

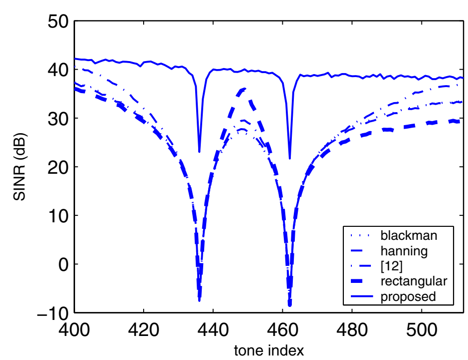

Fig. 4. SINRs of individual tones.

VDSL loop	Proposed	[12]	Blackman	Hanning	Rectangular	[13]
1	28.7	26.9	26.9	26.9	26.9	28.0
2	28.7	25.9	25.9	25.9	25.9	28.0
3	28.8	25.5	25.5	25.5	25.5	27.5
4	28.7	19.7	19.7	19.7	19.7	20.7
5	28.8	28.3	28.3	28.3	28.3	28.8
6	28.8	28.5	28.5	28.5	28.5	28.7
7	28.7	27.2	27.2	27.2	27.2	27.2

TABLE I

COMPARISON OF TRANSMISSION RATE (MBITS/SEC) ON VDSL LOOPS.

the subchannel SINRs around the RFI source frequencies for loop 1. We see that the RFI on the subchannels near the RFI source frequencies can be significantly reduced. Thus, we can transmit more bits in those subchannels and a higher transmission rate is achieved.

VI. CONCLUSION

In this paper, we use a filter bank formulation for designing TEQ incorporating RFI suppression. The proposed TEQ can significantly alleviate the effect of RFI for the tone around RFI frequencies. Simulation results demonstrate that larger SINR can be obtained and higher transmission rates can be achieved.

ACKNOWLEDGMENT

The work was supported in parts by NSC 94-2213-E-009-038, NSC 94-2752-E-002-006 PAE, and NSC 94-2213-E-002-075, Taiwan, R.O.C.

REFERENCES

[1] *Asymmetric Digital Subscriber Lines (ADSL)-Metallic Interface*, ANSI T1.413, 1998.

[2] *Very-high Speed Digital Subscriber Lines (VDSL)-Metallic Interface*, ANSI T1.424, 2002.

[3] P. J. W. Melsa, R. C. Younce, C. E. Rohrs, *Impulse response shortening for discrete multitone transceiver*, *IEEE Trans. Commun.*, vol. 44, no.12, pp. 1662-1672, Dec. 1996.

[4] G. Arslan, B. L. Evans, S. Kiaei, *Equalization for discrete multitone transceivers to maximize bit rate*", *IEEE Transactions on Signal Processing*, vol. 49, no. 12, pp. 3123-3135, Dec. 2001.

[5] K. Van Acker, G. Leus, M. Moonen, O. van deWiel, and T. Pollet, *Pertone equalization for DMT-based systems*, *IEEE Trans. Commun.*, vol. 49, no. 1, pp. 109V119, Jan. 2001.

[6] C.-Y. Chen, S.-M. Phoong, *Bit rate optimized time-domain equalizers for DMT systems*, IEEE International Symposium on Circuits and Systems, May 2003.

[7] R. K. Martin, K. Vanbleu, M. Ding, G. Ysebaert, M. Milosevic, B. L. Evans, M. Moonen, and C. R. Johnson Jr., *Unification and Evaluation of Equalization Structures and Design Algorithms for Discrete Multitone Modulation Systems*, *IEEE Trans. Signal Processing*, to appear.

[8] L. De Clercg, M. Peeters, S. Schelstraete, and T. Pollet, *Mitigation of radio interference in xDSL Transmission*, *IEEE Communications Magazine*, vol. 38, no. 3, March 2000.

[9] A. J. Redfern, *Receiver window design for multicarrier communication systems*, *IEEE Trans. Commun.*, vol. 20, no. 5, pp. 1029-1036, June 2002.

[10] S. Kapoor and S. Nedic, *Interference suppression in DMT receiver using windowing*, International Conf. on Commun., 2000.

[11] G. Cuypers, K. Vanbleu, G. Ysebaert, M. Moonen, *Combining raised cosine windowing and per tone equalization for RFI mitigation in DMT receivers*, International Conf. on Commun., MAY 2003.

[12] Chien-Chang Li, and Yuan-Pei Lin, *Optimal Receiver Window Design For Radio Frequency Interference Suppression*, European Signal Processing Conference, 2006.

[13] G. Ysebaert, K. Vanbleu, G. Cuypers, and M. Moonen, *Joint Window and Time Domain Equalizer Design for Bit Rate Maximization in DMT-Receivers*, *IEEE Journal on Selected Areas in Communication,* vol. 20, no. 5, pp. 1029-1036, June 2002.