

# SUBBAND ADAPTIVE FILTERING USING APPROXIMATELY ALIAS-FREE COSINE MODULATED FILTERBANKS

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## ABSTRACT

In this paper, we use a class of cosine modulated filterbank (CMFB) with approximately reconstruction property for subband adaptive filtering. We show that even after the subband filters are included, the CMFB maintains the approximately alias-free property. In earlier designs, when the filterbank has real coefficients, the alias-free property is usually destroyed after subband filters are inserted. The CMFB has real coefficients and the computational complexity for filtering and subband adaptation is less than that of complex-valued filterbanks. Numerical simulations will be given to demonstrate its approximately alias-free property and fast convergence.

## 1. INTRODUCTION

The subband adaptive filtering technique is very attractive for many applications, such as system identification, adaptive equalization, and acoustic echo cancelation, especially for systems with a very long impulse response. For example, in the application of acoustic echo cancelation, the echo path usually has several thousands of taps. If we apply the conventional fullband adaptive filtering, it is necessary to model the echo path by using a very long adaptive filter with several thousand taps. Computational burden is costly. Besides, for signals with non-flat spectrum, the convergence speed will be slowed down [1].

Subband adaptive filtering has been proposed to reduce complexity and speed up convergence [2] - [11]. It has two major advantages. Subband filtering is done after downsampling, so the working rate is reduced. Furthermore, as a fullband signal process is divided into subband signals, each subband signal has a flatter spectrum and convergence speed can be improved. Maximally decimated filter banks are used in [3] for subband filtering. To remove aliasing, extra cross filters are needed. The DFT filter bank, derived from one or two prototypes, has been applied to subband filters due to its low complexity. An optimal design of filter-

bank that optimizes mean-squared error (MSE) due to the aliasing components, distortion function, and stopband attenuation of the analysis prototype is proposed in [4]. In [5], given the analysis filters, the prototype filter of the synthesis filterbank is designed by minimizing the output error. Near perfect reconstruction NPR oversampled filterbank, using iteratively least-squares algorithm, are designed in [6]. Designs of oversampled uniform DFT filterbanks with constraints such as delay specification, inband aliasing reduction, and group delay specifications, are given in [7] - [9]. The problem of aliasing effect and amplitude distortion are studied and prototype filters are optimized by nonlinear programming technique in [10]. Recently, an almost alias-free subband adaptive filtering structure with critical sampling using a bandwidth-increased analysis filter is proposed in [12].

In this paper, we apply the oversampled cosine modulated filterbanks [13] to subband adaptive filtering. We show that the CMFB has approximately zero aliasing even after subband filters are included if the prototype satisfies some simple conditions. As opposed to complex DFT filterbank, the subband CMFB needs only real computation for the analysis, synthesis band and subband filters. Numerical simulations demonstrate that the subband CMFB achieves a small MSE at a fast convergence rate. This paper is organized as follows. Section 2 presents the basic structure of subband adaptive structure. In section 3, the CMFB with approximate reconstruction property is reviewed. Section 4 presents the subband adaptive CMFB that has the approximate alias-free property. Simulation results are given in section 5. A conclusion is given in section 6.

## 2. SUBBAND ADAPTIVE STRUCTURE

The structure of subband adaptive filtering [15] is shown in Fig. 1. The analysis and synthesis filters are denoted respectively by  $H_k(z)$  and  $F_k(z)$ , for  $0 \leq k \leq K - 1$ . The  $k$ -th subband filter is  $G_k(z)$ . In Fig. 1,  $x(n)$  is the input signal.  $s(n)$  is the system impulse response.  $u(n)$  is the output signal of  $s(n)$ .  $d(n)$  is the fullband desired signal, which is the sum of  $u(n)$  and additive noise  $\nu(n)$ . Given analysis fil-

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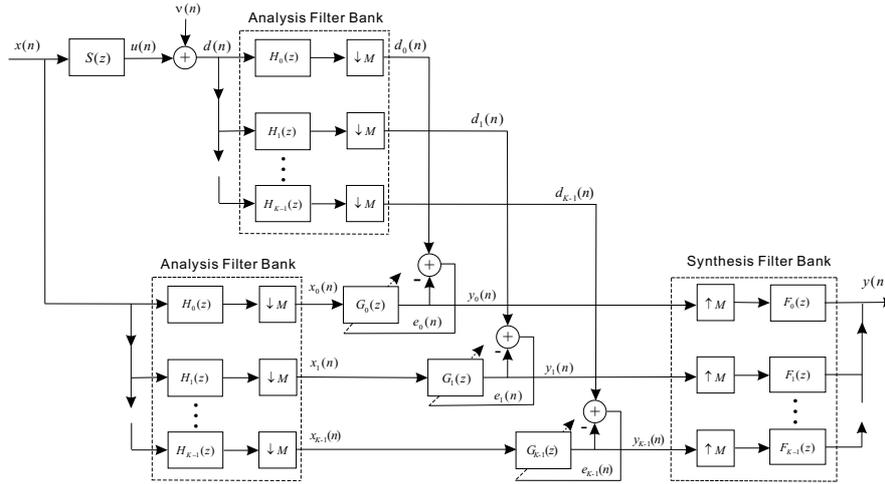


Figure 1: Subband adaptive filtering structure.

terbank and synthesis filterbank,  $G_k(z)$  will be adapted so that  $y(n) \approx d(n)$ . We apply  $d(n)$  to the analysis filterbank and generate the subband signals for training the subband filters. The subband signals  $x_k(n)$ , for  $0 \leq k \leq K-1$ , can be expressed as

$$X_k(z) = \frac{1}{M} \sum_{\ell=0}^{M-1} H_k(z^{1/M} W_M^\ell) X(z^{1/M} W_M^\ell), \quad (1)$$

where  $W_M$  is defined as  $e^{-j2\pi/M}$ . Upon subband filtering and the operation of synthesis bank, the reconstructed output signal  $y(n)$  has z-transform given by

$$Y(z) = \sum_{\ell=0}^{M-1} T_\ell(z) X(z W_M^\ell) \quad (2)$$

where

$$T_\ell(z) = \frac{1}{M} \sum_{k=0}^{K-1} H_k(z W_M^\ell) F_k(z) G_k(z^M). \quad (3)$$

We call  $T_0(z)$  the distortion function, and  $T_\ell(z)$ ,  $1 \leq \ell \leq M-1$  the  $\ell$ -th aliasing transfer function. The aliasing terms are approximately zero if

$$T_\ell(z) \approx 0, \quad \forall 1 \leq \ell \leq M-1 \quad (4)$$

In this case, the output is related to the input by

$$Y(z) \approx T_0(z) X(z). \quad (5)$$

When there is no aliasing, the output  $y(n)$  will be a good approximation of the desired signal  $d(n)$  if  $t_0(n)$  approximates the desired impulse response well.

### 3. CMFB WITH APPROXIMATE RECONSTRUCTION

In the DFT filterbank, the analysis and synthesis filters have complex coefficients. The cosine modulated filterbank (CMFB) comes into play if real coefficients are desired. In this section, we introduce the oversampled cosine modulated filterbank [13]. Consider an oversampled CMFB with  $2M$  subbands and decimation ratio  $M$ . Let the prototype filter of the analysis filterbank be a real coefficient  $P(z)$ . Let  $\tilde{P}(z)$  be z-transform of the time-reversal version  $p(-n)$ . Let  $P_k(z) = P(z W_{2M}^{k+0.5})$ ,  $0 \leq k \leq 2M-1$ . The analysis filters  $H_k(z)$  and the synthesis filters  $F_k(z)$  are given by

$$\begin{aligned} H_k(z) &= \begin{cases} P_k(z) + P_{2M-1-k}(z), & 0 \leq k \leq M-1 \\ -jP_k(z) + jP_{2M-1-k}(z), & M \leq k \leq 2M-1 \end{cases} \\ F_k(z) &= \tilde{H}_k(z), \quad 0 \leq k \leq 2M-1 \end{aligned} \quad (6)$$

All the analysis and synthesis filters  $H_k(z)$  and  $F_k(z)$  have real coefficients. It is shown in [13] that if  $|P(e^{j\omega})|^2$  is a Nyquist ( $2M$ ) filter with  $P(e^{j\omega}) \approx 0$ ,  $|\omega| > \pi/M$ , then the CMFB has the approximate reconstruction property.

### 4. PROPOSED SUBBAND ADAPTIVE CMFB

In the following, we consider the CMFB system. With the choice of analysis and synthesis filters in 6, the  $\ell$ -th aliasing

transfer function becomes

$$\begin{aligned}
T_\ell(z) = & \frac{1}{M} \sum_{k=0}^{M-1} (P_{k+2\ell} \tilde{P}_k + P_{k+2\ell} \tilde{P}_{2M-1-k} \\
& + P_{2M-1-k+2\ell} \tilde{P}_k + P_{2M-1-k+2\ell} \tilde{P}_{2M-1-k}) G_k(z^M) \\
& + \frac{1}{M} \sum_{k=0}^{M-1} (P_{2M-1-k+2\ell} \tilde{P}_{2M-1-k} - P_{2M-1-k+2\ell} \tilde{P}_k \\
& - P_{k+2\ell} \tilde{P}_{2M-1-k} + P_{k+2\ell} \tilde{P}_k) \\
& \cdot G_{2M-1-k}(z^M), \tag{7}
\end{aligned}$$

where we have used  $P_k$  as a shorthand for  $P_k(z)$ . Notice that we choose the subband filters by

$$G_k(z) = G_{2M-1-k}(z), \quad 0 \leq k \leq M-1, \tag{8}$$

then the  $\ell$ -th aliasing function simplifies to

$$T_\ell(z) = \frac{2}{M} \sum_{k=0}^{M-1} (P_{k+2\ell} \tilde{P}_k + P_{2M-1-k+2\ell} \tilde{P}_{2M-1-k}) G_k(z^M). \tag{9}$$

If  $P(e^{j\omega})$  satisfies

$$P(e^{j\omega} W_{2M}^{2\ell}) \tilde{P}(e^{j\omega}) \approx 0, \quad \text{for } 1 \leq \ell \leq M-1 \tag{10}$$

then all the aliasing transfer functions are approximately zero. Furthermore, letting  $\ell = 0$ , we can get the distortion function  $T_0(e^{j\omega})$  as

$$T_0(e^{j\omega}) = \frac{2}{M} \sum_{k=0}^{2M-1} |P_k(e^{j\omega})|^2 G_k(e^{jM\omega}). \tag{11}$$

Therefore, if the condition stated in (10) holds, the inclusion of subband filters does not destroy the approximately alias-free property of CMFB.

### Prototype filter design

From the discussion above, we can know that the condition for approximate reconstruction can be stated in terms of  $P(e^{j\omega})$  as follows

$$P(e^{j\omega}) \approx 0, \quad \text{for } |\omega| > \pi/M \tag{12}$$

$$|T(e^{j\omega})| \approx 1 \tag{13}$$

where

$$T(e^{j\omega}) = \sum_{k=0}^{2M-1} |P(e^{j(\omega-k\pi/M)})|^2. \tag{14}$$

Here we apply Kaiser window design method to design the prototype filter [16]. Given  $A_s$  and  $\Delta\omega$ , the only free parameter for design of the prototype filter is the cutoff frequency  $\omega_c$ . We can design the prototype filter to find the extreme value of the performance criterion  $\phi$  by adjusting  $\omega_c$  [14].

## 5. NUMERICAL SIMULATIONS

Here we will apply the proposed subband adaptive CMFB to echo cancellation. The echo path from ITU-T Recommendation G.168 [17] and a room impulse response from [18] will be used in our simulations. The echo path has 1024 taps. The oversampled filterbank has  $2M$  subbands, and subband adaptive filters are of length 256. In this section, we fix  $M = 4$ . The prototype has stopband attenuation 100dB and length 56. The input signal  $x(n)$  is a first-order autoregressive (AR) signal. The signal-to-noise ratio (SNR) is 30dB, where SNR is defined as the ratio between the power of  $u(n)$  and  $v(n)$ . A measurement of aliasing error given in [15] is

$$\sqrt{\sum_{\ell=1}^{M-1} |T_\ell(e^{j\omega})|^2} \tag{15}$$

where  $T_\ell(z)$  is the aliasing transfer function as defined in (3). The aliasing error of the subband adaptive CMFB is shown in Fig. 2. We can see that aliasing error falls below -90dB. The proposed subband adaptive CMFB is approximately alias-free in the presence of subband filters.

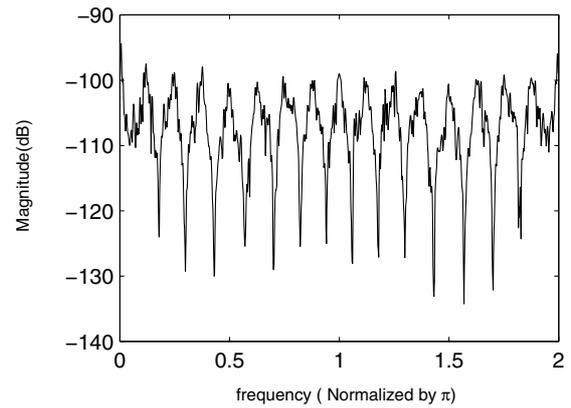


Figure 2: Aliasing error of the proposed subband adaptive CMFB.

Fig. 3 (a) shows the learning curves for SNR = 30dB. We compare the simulation result of CMFB with those of Block DFT [11], near perfect reconstruction filterbank (NPRFB) [6] and fullband adaptive filtering. We can see that the CMFB has very fast convergence. Fig. 3 (b) shows the learning curves for SNR = 30dB when the echo path is the room impulse response. We can also see the very fast convergence behavior of CMFB.

## 6. CONCLUSION

In this paper, we propose an oversampled cosine modulated filterbank (CMFB) for subband adaptive filtering. The

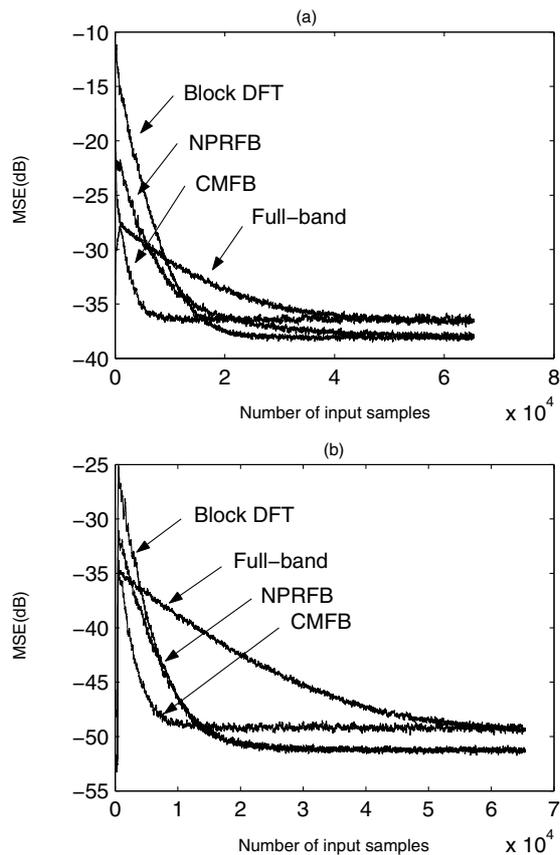


Figure 3: Learning curves for SNR = 30dB when the echo path is (a) G.168 (b) room impulse response.

filter bank has the approximately alias-free property even in the presence of subband filters. With real-coefficient analysis, synthesis and subband filters, only real additions and multiplications are needed. Furthermore, only half the subband filters need adaptation and the second half can be directly obtained from the first half of the subband filters. Simulation results corroborate that the aliasing error is very close to zero. The learning curves demonstrate that the proposed subband CMFB achieves a small MSE with a fast convergence speed.

## 7. REFERENCES

- [1] S. Haykin, Adaptive Filter Theory, 3rd ed., Prentice-Hall, Englewood Cliffs, New Jersey, 1996.
- [2] W. Kellermann, "Analysis and design of multirate systems for cancellation of acoustical echoes," ICASSP, 1988
- [3] Gilloire, and M. Vetterli, "Adaptive filtering in subbands with critical sampling: analysis, experiments and application to acoustic echo cancellation," IEEE Trans. Signal Processing, 1992.
- [4] M. R. Petraglia and S. K. Mitra, "Performance analysis of adaptive filter structures based on subband decomposition," ISCAS, 1993.
- [5] M. R. Petraglia and P. R. V. Piber, "Prototype filter design for oversampled subband adaptive filtering structures," ISCAS, 1999.
- [6] M. Harteneck, S. Weiss and R. W. Stewart, "Design of near perfect reconstruction oversampled filter banks for subband adaptive filters," IEEE Trans. Circuits and Systems, 1999.
- [7] J. M. de Haan, N. Grbic, I. Claesson, and S. Nordholm, "Design of oversampled uniform DFT filter banks with delay specification using quadratic optimization," ICASSP, 2001.
- [8] N. Grbic, J.M. de Haan, I. Claesson, S. Nordholm, "Design of oversampled uniform DFT filter banks with reduced inband aliasing and delay constraints," Sixth International Symposium on Signal Processing and its Applications, 2001.
- [9] Hai Huyen Dam, Sven Nordholm, and Antonio Candoni, "Uniform FIR Filterbank Optimization With Group Delay Specifications," IEEE Tran. Signal Processing, 2005.
- [10] K. F. C. Yiu, N. Grbic, S. Nordholm, Kok Lay Teo, "Multicriteria design of oversampled uniform DFT filter banks," IEEE Signal Processing Letters, 2004.
- [11] R. Merched and A. H. Sayed, "An embedding approach to frequency-domain and subband adaptive filtering," IEEE Trans. Signal Processing, 2000.
- [12] Gyun Kim, C. D. Yoo, and T. Q. Nguyen, "Alias-free subband adaptive filtering with critical sampling," IEEE Trans. Signal Processing, 2008.
- [13] Y.-P. Lin and P. P. Vaidyanathan, "Application of DFT filter banks and cosine modulated filter banks in filtering," IEEE APCCAS, 1994.
- [14] Y.-P. Lin and P. P. Vaidyanathan, "A Kaiser Window Approach to the Design of Prototype Filters of Cosine Modulated Filter Banks," IEEE Signal Processing Letters, 1998.
- [15] P. P. Vaidyanathan, Multirate Systems and Filter Banks. Englewood Cliffs, NJ: Prentice-Hall, 1993.
- [16] A. V. Oppenheim and R. W. Schaffer, Discrete-Time Signal Processing, Prentice-Hall, Englewood Cliffs, New Jersey, 1989.
- [17] ITU-T Recommendations G. 168, "Digital network echo cancellers", International Telecommunication Union, April 2000.
- [18] <http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do?objectId=51116&objectType=file>