Joint Estimation of CFO and Receiver I/Q Imbalance Using Virtual Subcarriers for OFDM Systems

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***Abstract*—In this paper, we study the estimation of carrier frequency offset (CFO) in the presence of the receiver in-phase and quadrature-phase (I/Q) imbalance for orthogonal frequency division multiplexing (OFDM) systems. By minimizing the energy of the samples on the virtual subcarriers, our proposed algorithm can jointly estimate the CFO and I/Q parameters using one OFDM block. When the CFO is small, a closed-form solution can be obtained. Simulation results show that our proposed method can provide a good performance for both of the I/Q and CFO estimation and it compares favorably with a recent method.**

***Index Terms*—OFDM, CFO, I/Q Imbalance, Virtual Subcarrier.**

I. INTRODUCTION

The topic of carrier frequency offset (CFO) estimation for orthogonal frequency division multiplexing (OFDM) systems has been studied by many researchers in the last two decades. Many methods have been proposed to solve the problem of CFO estimation [1]-[5]. The authors in [1] showed that by sending repeated OFDM blocks, a correlation method can be used to obtain a CFO estimate. The authors in [2] estimated CFO by minimizing the energy of the signals at the unused subcarriers in OFDM systems. These unused subcarriers are called virtual subcarriers. The identifiability of the estimator in [2] was studied in [3][4][5]. In [3], the authors showed that the location of the virtual subcarriers shall be distinct to guarantee the identifiability. It was shown [4][5] that the uniformly-spaced placement is optimal in the sense of either minimizing the Cramer-Rao bound [4] or maximizing the signal-to-interference ratio [5]. The above references do not consider the effect of the in-phase and quadrature (I/Q) phase imbalance. The I/Q imbalance is due to the mismatch of the amplitude and phase components between the I-branch and Q-branch [6]. It is known that the I/Q imbalance can severely degrade the performance of OFDM systems. It also affects the accuracy of the CFO estimation [7][10][12]. The CFO estimation in the presence of I/Q imbalance was studied in [7]-[12]. In [8], the authors estimated CFO in the presence of I/Q imbalance by sending 3 training OFDM blocks and a numerical search was needed. In [9], assuming sending at least 3 repeated OFDM blocks, a closed-form solution for CFO and I/Q estimation was derived. In [10], the authors studied the I/Q and CFO estimation by using a differential filter to suppress the direct current offset in direct conversion receivers. In [11], a subspace method was proposed to solve the joint estimation problem. All of the above methods needed more than one OFDM block to do the estimation. The authors in [12] estimated CFO in the presence of I/Q imbalance by using the virtual subcarriers in OFDM systems. This method [12] needed one OFDM block for finding an I/Q and CFO estimate.

In this paper, we propose a new method for the CFO estimation in the presence of the receiver I/Q imbalance using the virtual subcarriers. By minimizing the energy of the samples on the virtual subcarriers, we can jointly estimate the I/Q and CFO parameters. The proposed method needs only one OFDM block. When the CFO is small, we can obtain a closed-form solution for the proposed estimator. Simulation results show that our proposed method can achieve a good performance for both of the I/Q and CFO estimation and it compares favorably with [12]. The rest of the paper is organized as below. The proposed joint estimation method for I/Q and CFO estimation is given in Sec. 2. The issue of the virtual subcarrier location is discussed in Sec. 3. The case of small CFO is investigated in Sec. 4. We extend the proposed method to more than one OFDM block in Sec. 5. Simulations and conclusions are given in Sec. 6 and Sec. 7 respectively.

Notation: The transpose, conjugate and conjugate-transpose of the matrix $\mathbf{A}$ are defined by $\mathbf{A}^T$, $\mathbf{A}^*$ and $\mathbf{A}^\dagger$ respectively. $\mathbf{W}$ is the $M \times M$ normalized DFT matrix with $[\mathbf{W}]_{kl} = \frac{1}{\sqrt{M}} e^{-j\frac{2\pi}{M}kl}$. $< b >$ denotes b modulo M and the delta function $\delta(k) = 1$ for $k = 0$ and 0 otherwise.

II. SYSTEM DESCRIPTION

Let M be the DFT size in the OFDM system and L be the cyclic prefix length. The channel is modeled as a finite impulse response (FIR) filter with order L. Let $s(k)$ be the modulation symbols sent on the kth subcarrier. In this paper, we assume that there are K virtual subcarriers located at subcarriers $l_0, l_1, \ldots, l_{K-1}$. That is, $s(l_k) = 0$. Suppose that the OFDM system suffers from CFO and I/Q imbalance. Let θ and α denote respectively the CFO and I/Q parameters, where the CFO is normalized by the subcarrier spacing. It was shown

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[7][12] that the received signal at the receiver is given by

$$\mathbf{z} = \mathbf{E}(\theta)\mathbf{W}^{\dagger}\mathbf{\Lambda s} + \alpha(\mathbf{E}(\theta)\mathbf{W}^{\dagger}\mathbf{\Lambda s})^{*} + \mathbf{q}, \quad (1)$$

where $\mathbf{q}$ is the noise vector,

$$\mathbf{E}(\theta) = e^{j\frac{2\pi}{M}\theta L}\text{diag}\begin{bmatrix} 1 & e^{j\frac{2\pi}{M}\theta} & \cdots & e^{j\frac{2\pi}{M}(M-1)\theta} \end{bmatrix}, \quad (2)$$

$\mathbf{\Lambda}$ is a diagonal matrix whose diagonal entries are the channel gain $H(k)$ and $\mathbf{s} = [\ s(0)\ \ s(1)\ \ \cdots\ \ s(M-1)\ ]^T$. The aim of this paper is to jointly estimate θ and α from the received vector $\mathbf{z}$. In what follows, we review a method [12] for the CFO estimation using the virtual subcarriers.

A review of [12]: Suppose we have the CFO value θ. We first multiply the vector $\mathbf{z}$ with the matrix $\mathbf{E}(-\theta)$. After taking the $M-$point DFT, we obtain

$$\mathbf{WE}(-\theta)\mathbf{z} = \mathbf{\Lambda s} + \alpha\mathbf{WE}(-2\theta)\mathbf{W}(\mathbf{\Lambda s})^{*} + \mathbf{WE}(-\theta)\mathbf{q}. \quad (3)$$

Define

$$\mathbf{P} \triangleq \begin{bmatrix} p_{l_0} & p_{l_1} & \cdots & p_{l_{K-1}} \end{bmatrix}^T, \quad (4)$$

where $\mathbf{p}_{l_k}^T$ is the l_kth row of the $M \times M$ identity matrix. It can be easily shown that $\mathbf{P\Lambda s} = \mathbf{0}$ because of the insertion of the virtual subcarriers. So multiplying the vector $\mathbf{WE}(-\theta)\mathbf{z}$ with $\mathbf{P}$, we get

$$\mathbf{PWE}(-\theta)\mathbf{z} \ = \ \alpha\mathbf{PWE}(-2\theta)\mathbf{W}(\mathbf{\Lambda s})^{*} + \mathbf{PWE}(-\theta)\mathbf{q}. \quad (5)$$

On the other hand, we can also obtain the other vector $\mathbf{PWE}(-\theta)\mathbf{z}^*$

$$\mathbf{PWE}(-\theta)\mathbf{z}^{*} \ = \ \mathbf{PWE}(-2\theta)\mathbf{W}(\mathbf{\Lambda s})^{*} + \mathbf{PWE}(-\theta)\mathbf{q}^{*}. \quad (6)$$

It can be observed from (5)(6) that in the absence of channel noise, the two vectors $\mathbf{PWE}(-\theta)\mathbf{z}$ and $\mathbf{PWE}(-\theta)\mathbf{z}^*$ differ by a scaling factor α. This property is exploited for the CFO estimation in [12]. Let $\widehat{\theta}$ denote an estimate of θ and define a vector $\mathbf{y}$ with the entries given by $[\mathbf{y}(\widehat{\theta})]_k = \frac{[\mathbf{PWE}(-\widehat{\theta})\mathbf{z}]_k}{[\mathbf{PWE}(-\widehat{\theta})\mathbf{z}^*]_k}$. The authors in [12] estimated the CFO value as

$$\theta_{opt} = \arg\min_{\widehat{\theta}} \left\{ ||M\mathbf{y}(\widehat{\theta}) - \mathbf{1}\mathbf{1}^T\mathbf{y}(\widehat{\theta})||^2 \right\}, \quad (7)$$

where the $1 \times K$ vector $\mathbf{1}^T = [\ 1\ \ 1\ \ \cdots\ \ 1\ ]$. The CFO estimate is obtained by searching $\widehat{\theta}$ that minimizes the above cost function.

III. PROPOSED METHOD FOR THE I/Q AND CFO ESTIMATION

In this section, we jointly estimate the I/Q and CFO parameters by minimizing the energy of the received signal at the virtual subcarriers. To do this, let us rearrange the terms in (1) so that the frequency-domain signal at the receiver is expressed as

$$\mathbf{\Lambda s} + \mathbf{q}' = \mathbf{WE}(-\theta)\frac{\mathbf{z} - \alpha\mathbf{z}^*}{1 - |\alpha|^2}, \quad (8)$$

where $\mathbf{q}'$ is a vector related to the channel noise $\mathbf{q}$. Let $\widehat{\theta}$ and $\widehat{\alpha}$ be an estimate of θ and α respectively. Using $\mathbf{P}$ and (8), we can write the energy of the frequency-domain signal at the virtual subcarriers as

$$J(\widehat{\alpha}, \widehat{\theta}) = \left\| \mathbf{PWE}(-\widehat{\theta})\frac{\mathbf{z} - \widehat{\alpha}\mathbf{z}^*}{1 - |\widehat{\alpha}|^2} \right\|^2. \quad (9)$$

From (8), it is clear that when the CFO and I/Q parameters are estimated perfectly, we have

$$J(\widehat{\alpha}, \widehat{\theta})|_{\widehat{\theta}=\theta, \widehat{\alpha}=\alpha} = \left\| \mathbf{P\Lambda s} + \mathbf{Pq}' \right\|^2 = ||\mathbf{Pq}'||^2, \quad (10)$$

where the second equality holds because $\mathbf{P\Lambda s}$ contains only signals at the virtual subcarriers, which are zero. It shows that the cost function $J(\widehat{\alpha}, \widehat{\theta})$ has a minimum at $\widehat{\theta} = \theta$ and $\widehat{\alpha} = \alpha$ when there is no channel noise. So the optimal $\widehat{\theta}$ and $\widehat{\alpha}$ can be obtained by minimizing $J(\widehat{\alpha}, \widehat{\theta})$. The above optimization problem is a two-dimensional search problem. Below we will reduce the optimization problem to a one-dimensional search problem. Since α is small in practice, we can make the approximation $1 - |\alpha|^2 \approx 1$ and the cost function in (9) is given by

$$J(\widehat{\alpha}, \widehat{\theta}) \approx \left\| \mathbf{PWE}(-\widehat{\theta})(\mathbf{z} - \widehat{\alpha}\mathbf{z}^*) \right\|^2. \quad (11)$$

Define

$$\mathbf{B} = \mathbf{PWE}(-\widehat{\theta}). \quad (12)$$

We can rewrite the cost function as

$$J(\widehat{\alpha}, \widehat{\theta}) \approx \left\| \mathbf{Bz} - \widehat{\alpha}\mathbf{Bz}^* \right\|^2. \quad (13)$$

From linear algebra, we know that for a fixed $\widehat{\theta}$, the optimal $\widehat{\alpha}$ that minimizes $J(\widehat{\alpha}, \widehat{\theta})$ is given by

$$\alpha_{opt}(\widehat{\theta}) = \frac{(\mathbf{Bz}^*)^{\dagger}(\mathbf{Bz})}{||\mathbf{Bz}^*||^2}. \quad (14)$$

Substituting (14) into (13), we can write

$$J(\widehat{\theta}) = ||\mathbf{Bz}||^2 - \frac{|(\mathbf{Bz})^{\dagger}(\mathbf{Bz}^*)|^2}{||\mathbf{Bz}^*||^2}. \quad (15)$$

The optimal $\widehat{\theta}$ is given by

$$\theta_{opt} = \arg\min_{\widehat{\theta}} J(\widehat{\theta}). \quad (16)$$

The optimal $\widehat{\alpha}$ can be obtained by substituting $\widehat{\theta} = \theta_{opt}$ into (14). Note that when there is no I/Q mismatch $\alpha = 0$, the cost in (13) reduces to that of the MUSIC-like method [2]. That means our method can be seen as an extension of the MUSIC-like method for the joint estimation of I/Q and CFO.

IV. PLACEMENT OF VIRTUAL SUBCARRIERS

It is known [4][5] that the optimal location of the virtual subcarriers is uniformly-spaced for CFO estimation. In the following, we will study impact of the location of the virtual subcarriers on the I/Q and CFO estimators. For this, we assume that there is no channel noise $\mathbf{q} = \mathbf{0}$. We first study the case with only the I/Q imbalance. The case with both the I/Q and CFO mismatches will be investigated later.

I/Q imbalance estimation: Substituting $\mathbf{E}(\theta) = \mathbf{I}$ and $\mathbf{q} = \mathbf{0}$ into (1), we can rewrite the two vectors $\mathbf{Bz}$ and $\mathbf{Bz}^*$ as

$$\begin{aligned}\mathbf{Bz} &= \mathbf{PWW}^\dagger\mathbf{\Lambda s} + \alpha\mathbf{PWW\Lambda}^*\mathbf{s}^* \\ \mathbf{Bz}^* &= \mathbf{PWW\Lambda}^*\mathbf{s}^* + \alpha^*\mathbf{PWW}^\dagger\mathbf{\Lambda s}.\end{aligned} \quad (17)$$

Using $\mathbf{WW}^\dagger = \mathbf{I}$ and the matrix $\mathbf{P}$ in (4), it can be observed that $\mathbf{PWW}^\dagger\mathbf{\Lambda s} = \mathbf{P\Lambda s} = \mathbf{0}$. So we have

$$\mathbf{Bz} = \alpha\mathbf{PWW\Lambda}^*\mathbf{s}^*, \quad \mathbf{Bz}^* = \mathbf{PWW\Lambda}^*\mathbf{s}^*. \quad (18)$$

We see that if $\mathbf{PWW\Lambda}^*\mathbf{s}^* = \mathbf{0}$, then $\mathbf{Bz} = \mathbf{Bz}^* = \mathbf{0}$. In this case α can not be estimated by using (14). In other words, α is not identifiable. From [14], we know that $[\mathbf{WW}]_{k,l} = \delta(< k+l >)$, where $< b >$ denotes b modulo M. Also notice that the kth entry of $\mathbf{P}(\mathbf{WW\Lambda}^*\mathbf{s}^*)$ is the l_kth entry of $\mathbf{WW\Lambda}^*\mathbf{s}^*$. Using these two results, one can show that the kth entry of the vector $\mathbf{PWW\Lambda}^*\mathbf{s}^*$ is given by $[\mathbf{PWW\Lambda}^*\mathbf{s}^*]_k = H^*(< M - l_k >)s^*(< M - l_k >)$. Thus $[\mathbf{PWW\Lambda}^*\mathbf{s}^*]_k = 0$ for all k if and only if the product $H(< M - l_k >)s(< M - l_k >) = 0$ for all k. Summarizing the results, we have proved:

Theorem 1. *α is not identifiable if $H(< M - l_k >)s(< M - l_k >) = 0$ for all $k = 0, \ldots, K-1$.*

In particular, the theorem implies that α is not identifiable when $s(< M - l_k >) = 0$ for all $k = 0, \ldots, K-1$. This case can happen if the $< M - l_k >$th subcarriers are also virtual subcarriers. That means both of the l_kth and $< M - l_k >$th subcarriers are virtual subcarriers. In other words, the location of all the virtual subcarriers satisfies $< l_k + l_j >= 0$. We call the virtual subcarriers satisfying this property as conjugate-symmetric. One example of conjugate-symmetric virtual subcarriers is when $M = 64$ and $l_k = 8k$. It can be observed that $< l_k + l_{8-k} >= 0$ for $k = 1, \ldots, 7$ and $< l_0 + l_0 >= 0$. In many practical applications, the virtual subcarriers are placed symmetrically two sides of the DC subcarrier. One can verify that in this case, the virtual subcarriers satisfy the conjugate-symmetric property.

CFO and I/Q Estimation: Suppose there are I/Q imbalance and CFO. We will show that if the locations of the virtual subcarriers are conjugate-symmetric as described in Theorem 1, then $J(\theta) = J(-\theta) = 0$. That means the two estimates $\widehat{\theta} = \theta$ and $\widehat{\theta} = -\theta$ both minimize the cost function $J(\widehat{\theta})$. To show $J(\theta) = 0$, we first substitute $\widehat{\theta} = \theta$ into the matrix $\mathbf{B}$ in (12). Then we write the two vectors $\mathbf{Bz}$ and $\mathbf{Bz}^*$ from (1) as

$$\begin{aligned}\mathbf{Bz} &= \mathbf{P\Lambda s} + \alpha\mathbf{PWE}(-2\theta)\mathbf{W\Lambda}^*\mathbf{s}^* \\ \mathbf{Bz}^* &= \mathbf{PWE}(-2\theta)\mathbf{W\Lambda}^*\mathbf{s}^* + \alpha^*\mathbf{P\Lambda s}.\end{aligned} \quad (19)$$

Since $\mathbf{P\Lambda s} = \mathbf{0}$, we have

$$\begin{aligned}\mathbf{Bz} &= \alpha\mathbf{PWE}(-2\theta)\mathbf{W\Lambda}^*\mathbf{s}^* \\ \mathbf{Bz}^* &= \mathbf{PWE}(-2\theta)\mathbf{W\Lambda}^*\mathbf{s}^*.\end{aligned} \quad (20)$$

Substituting the above equations into (15), we obtain $J(\theta) = 0$. Similarly, substituting $\widehat{\theta} = -\theta$ into $\mathbf{B}$ in (12), we can write

$$\begin{aligned}\mathbf{Bz} &= \mathbf{PWE}(2\theta)\mathbf{W}^\dagger\mathbf{\Lambda s} + \alpha\mathbf{PWW\Lambda}^*\mathbf{s}^* \\ \mathbf{Bz}^* &= \mathbf{PWW\Lambda}^*\mathbf{s}^* + \alpha^*\mathbf{PWE}(2\theta)\mathbf{W}^\dagger\mathbf{\Lambda s}.\end{aligned} \quad (21)$$

From the above discussion, we see that once the virtual subcarriers are conjugate-symmetric, $H(< M - l_k >)s(< M - l_k >) = 0$ for all $k = 0, \ldots, K-1$. In this case, we have $\mathbf{PWW\Lambda}^*\mathbf{s}^* = \mathbf{0}$. Using this, we can simplify (21) as

$$\mathbf{Bz} = \mathbf{PWE}(2\theta)\mathbf{W}^\dagger\mathbf{\Lambda s}, \quad \mathbf{Bz}^* = \alpha^*\mathbf{PWE}(2\theta)\mathbf{W}^\dagger\mathbf{\Lambda s}. \quad (22)$$

Substituting the two vectors into (14), we see $J(-\theta) = 0$. Since $J(\theta) = J(-\theta) = 0$, both $\widehat{\theta} = \theta$ and $\widehat{\theta} = -\theta$ minimize $J(\widehat{\theta})$.

To eliminate the false solution $\widehat{\theta} = -\theta$, we look at $\widehat{\alpha}(\widehat{\theta})$. Using (22) in (14), we obtain

$$\widehat{\alpha}(-\theta) = \frac{1}{\alpha^*}. \quad (23)$$

In practice, α is small. So if $|\widehat{\alpha}(\widehat{\theta})| > 1$, we know $\widehat{\theta}$ is a false solution.

For CFO estimation, the virtual subcarriers are optimal if they are uniformly-spaced. For I/Q estimation, α becomes unidentifiable if the virtual subcarriers are conjugate-symmetric. When there are both I/Q imbalance and CFO, the I/Q parameter can be identifiable even if the virtual subcarriers are conjugate-symmetric. The more general case of joint I/Q and CFO estimation will be explored experimentally in Sec. VII.

V. JOINT ESTIMATION FOR SMALL CFO

In many practical applications, it is often true that a coarse CFO estimation is carried out at the initial stage. After the coarse estimation, the CFO is small ($|\theta| \ll 1$). When $|\theta| \ll 1$, we can make the approximation $e^{jk\theta} \approx 1 + jk\theta$. Thus $\mathbf{E}(-\widehat{\theta})$ can be approximated as

$$\mathbf{E}(-\widehat{\theta}) \approx e^{-j\frac{2\pi}{M}\widehat{\theta}(L+\frac{M}{2})}\left(\mathbf{I} - j\frac{2\pi}{M}\widehat{\theta}\mathbf{\Gamma}\right), \quad (24)$$

where $\mathbf{\Gamma} \triangleq \mathrm{diag}\begin{bmatrix} -\frac{M}{2} & -\frac{M}{2}+1 & \cdots & \frac{M}{2}-1 \end{bmatrix}$. Applying (24) and $1 - |\alpha|^2 \approx 1$ to (9), we get

$$J(\widehat{\theta}, \widehat{\alpha}) \approx ||\mathbf{v}_0 + \widehat{\theta}\mathbf{v}_1 - \widehat{\alpha}\mathbf{v}_2||^2, \quad (25)$$

where the $K \times 1$ vectors $\mathbf{v}_0 \triangleq \mathbf{PWz}, \mathbf{v}_1 \triangleq j\frac{2\pi}{M}\mathbf{PW\Gamma z}, \mathbf{v}_2 \triangleq \mathbf{PWz}^*$. The problem of finding $\widehat{\theta}$ and $\widehat{\alpha}$ that minimize $J(\widehat{\theta}, \widehat{\alpha})$ in (25) becomes a least-squares problem and the solution is given in closed-form. Define $\widehat{\alpha}_R$, $\widehat{\alpha}_I$, $\mathbf{v}_{i,R}$ and $\mathbf{v}_{i,I}$ as the real and imaginary parts of $\widehat{\alpha}$ and $\mathbf{v}_i$ respectively. The optimal solution of α and θ that minimize $J(\widehat{\theta}, \widehat{\alpha})$ is given by

$$\begin{bmatrix} \theta_{opt} & \alpha_{R,opt} & \alpha_{I,opt} \end{bmatrix}^T = (\mathbf{V}^\dagger\mathbf{V})^{-1}\mathbf{V}^\dagger\mathbf{v}, \quad (26)$$

where the $2K \times 3$ matrix $\mathbf{V}$ and the $2K \times 1$ vector $\mathbf{v}$ are respectively given by

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}_{1,R} & -\mathbf{v}_{2,R} & \mathbf{v}_{2,I} \\ \mathbf{v}_{1,I} & -\mathbf{v}_{2,I} & -\mathbf{v}_{2,R} \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -\mathbf{v}_{0,R} \\ -\mathbf{v}_{0,I} \end{bmatrix}. \quad (27)$$

VI. EXTENSION TO THE CASE OF MORE THAN ONE OFDM BLOCK

Suppose the receiver receives N OFDM blocks, and each received block contains K_i virtual subcarriers. The number and the placement of the virtual subcarriers can be different in each block. Let $\mathbf{z}_i$ denote the ith received block. Assume that the CFO and I/Q imbalance, θ and α, do not change during the transmission of N blocks. Our goal is to jointly estimate θ and α that minimize the energy of the signals on the virtual subcarriers in each block. Define a $NK \times NM$ block diagonal matrix

$$\mathbf{B}^{'} \triangleq \text{diag}\left[\begin{array}{cccc} \mathbf{B}_0 & \mathbf{B}_1 & \cdots & \mathbf{B}_{N-1} \end{array}\right], \quad (28)$$

where

$$\mathbf{B}_i = \mathbf{P}_i \mathbf{W} \mathbf{E}_i(-\widehat{\theta}) \quad (29)$$

and

$$\mathbf{E}_i(\widehat{\theta}) = e^{j\frac{2\pi}{M} i\widehat{\theta}(L+M)} \text{diag}\left[\begin{array}{cccc} 1 & e^{j\frac{2\pi}{M}\widehat{\theta}} & \cdots & e^{j\frac{2\pi}{M}(M-1)\widehat{\theta}} \end{array}\right] \quad (30)$$

and an $NM \times 1$ vector $\mathbf{z}^{'} \triangleq \left[\begin{array}{cccc} \mathbf{z}_0^T & \mathbf{z}_1^T & \cdots & \mathbf{z}_{N-1}^T \end{array}\right]^T$. Using the above definitions, the optimal θ_{opt} can be obtained by replacing $\mathbf{B}$ and $\mathbf{z}$ in (15) with $\mathbf{B}^{'}$ and $\mathbf{z}^{'}$ respectively. The corresponding α_{opt} is obtained using (14). Note that the channel and the data symbols can be different in each block. The case when the CFO is small can also extended to more than one OFDM block by using similar procedures described above.

VII. NUMERICAL RESULTS

We carry out Monte-Carlo experiments to verify the performance. The I/Q parameter is given by $\alpha = -0.0244 - 0.0436j$ (This corresponds to a phase mismatch of $-5°$ and a amplitude mismatch of 1.05). The channel is an FIR filter of length 5. The channel taps are i.i.d. complex Gaussian random variables with variance normalized to 1. The size of the DFT matrix is $M = 64$ and the CP length is $L = 4$. The transmission symbols are QPSK. The signal-to-noise ratio (SNR) is defined as the ratio of the modulation symbol power over the channel noise power. Assume there are 8 virtual subcarriers in each block. We first investigate the impact of the location of the virtual subcarriers on the proposed estimators. CFO is assumed to be $\theta = -0.078$. Four different placements of the virtual subcarriers are considered with different combinations of conjugate-symmetric and uniformly-spaced:

(A) Non-conjugate-symmetric and uniformly-spaced: $l_k \in \{1, 9, 17, 25, 33, 41, 49, 57\}$
(B) Conjugate-symmetric and uniformly-spaced: $l_k \in \{0, 8, 16, 24, 32, 40, 48, 56\}$
(C) Non-conjugate-symmetric and nonuniformly-spaced: $l_k \in \{24, 26, 28, 30, 33, 35, 37, 39\}$
(D) Conjugate-symmetric and nonuniformly-spaced: $l_k \in \{24, 26, 28, 30, 34, 36, 38, 40\}$

Fig 1(a) and Fig. 1(b) show the MSE of the CFO and I/Q estimators respectively. From Fig. 1(a), we see that the CFO estimator using uniformly-spaced virtual subcarriers performs better than the nonuniformly-spaced virtual subcarriers. On the other hand, Fig. 1(b) shows that the performance of the I/Q estimator can be affected significantly once the virtual subcarriers are conjugate-symmetric. But the CFO estimator is not as sensitive to the virtual subcarrier placements as the I/Q estimator, as shown in Fig. 1(a). Hence we conclude that to get a good performance for the joint estimation, the virtual subcarriers should be uniformly-spaced and non-conjugate-symmetric.

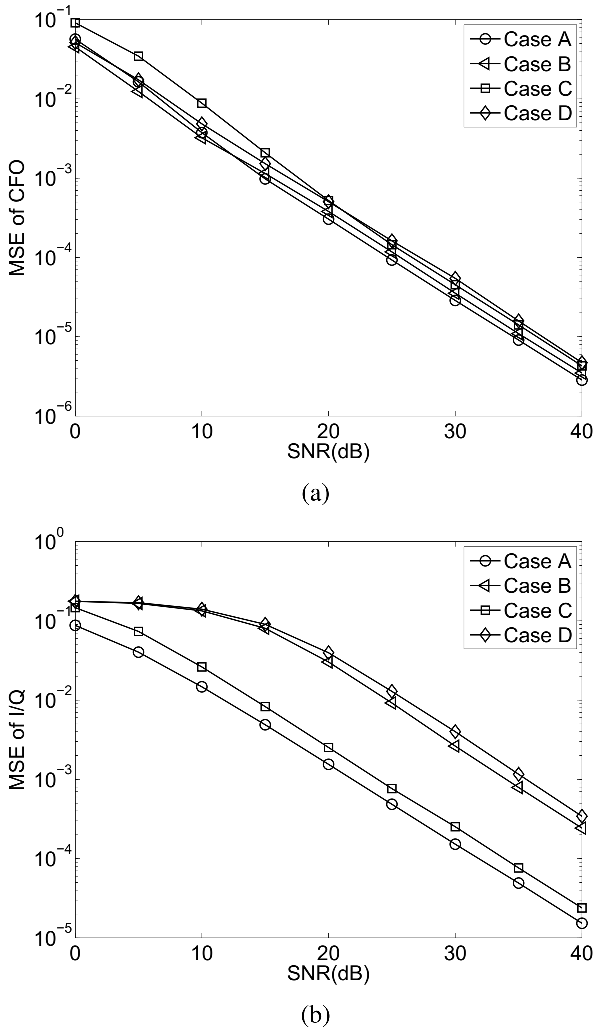

Fig. 1. MSE of CFO and I/Q estimators for different subcarrier locations: (a) θ and (b) α.

Next we show the MSE of the CFO estimator over different values of θ. We assume that the virtual subcarriers are uniformly-spaced and non-conjugate-symmetric. Both the solutions using the numerical search and the closed-form solution for small CFO are considered. We also draw the MSE of the method in [12]. The algorithm in [12] also involves a one-dimensional search problem. Fig. 2 shows the MSE of CFO versus the actual CFO θ for $SNR = 20dB$. It can

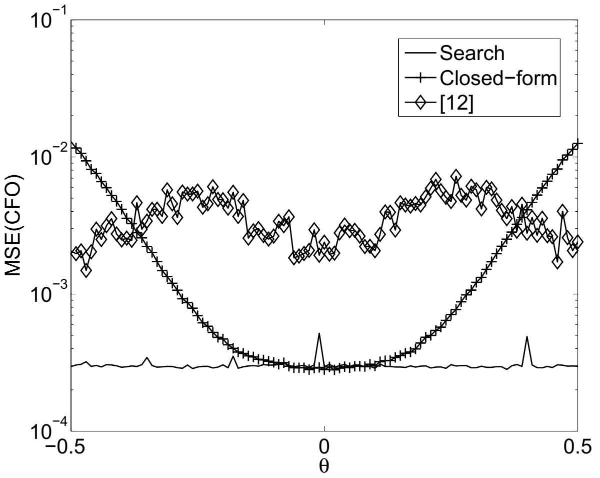

Fig. 2. MSE of CFO estimator over different θ at $SNR = 20dB$.

be seen that our proposed estimator is robust in the range of $-0.5 \le \theta \le 0.5$. When using the closed-form solution, our method can provide a good performance in the range of $-0.2 \le \theta \le 0.2$. Fig. 3(a) and Fig. 3(b) show the MSE for the I/Q and CFO estimation over different SNR. The CFO is $\theta = -0.078$. Both the cases of using 1 OFDM and 5 OFDM blocks with the uniformly-spaced and non-conjugate-symmetric virtual subcarrier assignment are considered. We also draw the MSE of the MUSIC-like method in [2], which only considers the CFO estimation. It can be observed from Fig. 3 that the suboptimal solution using the closed-form solution performs as good as the optimal solution using the numerical search. The proposed method outperforms the method in [12] for both of the I/Q and CFO estimation. The MUSIC-like method [2] suffers an error-flooring for the CFO estimation due to the I/Q imbalances when SNR $\ge 20dB$.

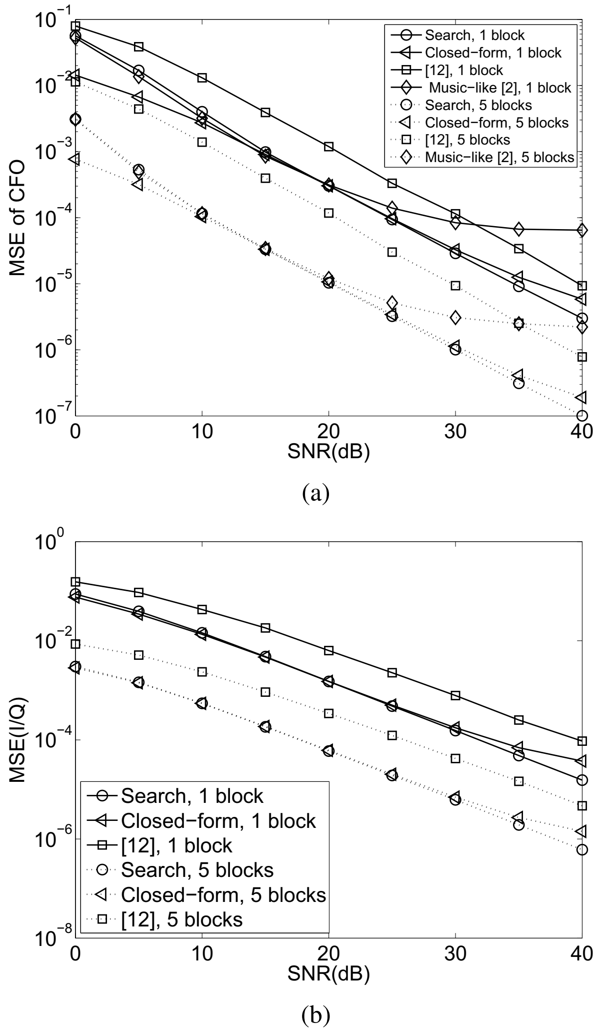

Fig. 3. MSE of the CFO and I/Q estimator for the proposed method over different SNR: (a) θ and (b) α.

VIII. Concluding Remarks

In this paper, we study the joint estimation of the CFO and I/Q parameters using the virtual subcarriers. The optimal I/Q and CFO parameters can be obtained by minimizing the energy of the samples on the virtual subcarriers. The proposed algorithm needs only one OFDM block. Simulation results show that the proposed method can provide a good performance for both CFO and I/Q estimation.

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