FEEDBACK RATE ALLOCATION OF PRECODER AND BIT LOADING FOR MIMO SYSTEMS WITH LIMITED FEEDBACK

Hung-Chun Chen and Yuan-Pei Lin

Dept. Electrical Engr., National Chiao Tung Univ., Hsinchu, Taiwan

ABSTRACT

In this paper we consider feedback rate allocation for MIMO systems with limited feedback of both bit loading and precoder. Allocation of feedback rate is an essential problem when there are two or more kinds of feedback information. We address this issue by analyzing the performance loss due to quantization of precoder and bit loading. We first derive the increase in transmission power when precoder is quantized, and then the additional penalty when bit loading is also quantized. The analysis allows us to find the optimal feedback rate allocation that minimizes the power penalty. Simulations show that the system with proper feedback rate allocation can achieve a very good performance compared with systems that do not consider the feedback of precoder and bit loading jointly.

1. INTRODUCTION

Recently, there has been considerable interest in multi-input multi-output (MIMO) systems with limited feedback [1]. It has been demonstrated that the system performance can be improved significantly with limited amount of feedback. Commonly adopted types of feedback information are precoder, bit loading, power loading or a combination of these three. The feedback of precoder information has been studied extensively [1]-[5]. The precoder is chosen from a codebook using an appropriate selection criterion and the index is fed back to transmitter. Codebooks designs for unitary precoders using Grassmannian subspace packing are developed in [2] for a number of criteria. A randomly generated codebook is proposed in [3] and the required feedback rate can be computed for a given target spectral efficiency. In [4], the precoder is selected from the codebook to minimize bit error rate (BER) and the generalized Lloyd algorithm is used to design codebooks. In the multimode scheme [5], the number of substreams transmitted can vary with the channel and bits are loaded uniformly. The feedback of bit loading and power loading have also been considered in the literature [6]-[7]. An efficient algorithm for per antenna power and rate control is developed in [6]. In [7], the decision feedback receiver feeds back the detection ordering for a fixed bit loading, which is equivalent to using all permutations of the same bit loading vectors. The design of codebooks for an MIMO system that feeds back only bit loading has been considered in [8]. There has also been research on the feedback of both bit loading and precoder [9]-[10]. A number of optimal MIMO transceivers with decision feedback and bit loading are given in [9]. Bit loading is incorporated in the multimode scheme to further improve the performance in [10], and both precoder and bit loading are fed back. The feedback of precoder and power loading is considered in [11]. Two efficient methods are developed in [11] for parameterizing unitary precoders. It is shown therein that the feedback of power loading provides only slight improvement. In [12], the information of power loading, bit loading and precoder are fed back to transmitter to maximize the transmission rate.

In previous works, when there is more than one type of feedback information, the feedback rate is often determined in an ad hoc manner. Feedback rate allocation is an important but often not adequately addressed problem. In this paper we consider feedback rate allocation for MIMO systems with limited feedback of both bit loading and precoder. We allocate the feedback rate by analyzing the power penalty when quantization is applied on precoder and bit loading. We first derive the increase in transmission power when precoder is quantized, and then the additional penalty when bit loading is also quantized. Thus the effect of quantization on precoder and bit loading can be brought together and the optimal feedback rate allocation for minimizing the power penalty can be determined in a systematic manner. Simulation examples are given to demonstrate that judicious allocation of feedback rate leads to a very good performance.

2. SYSTEM MODEL

Consider the MIMO communication system with M_t transmit antennas and M_r receive antennas in Fig. 1. The channel is modeled by an $M_r \times M_t$ matrix $\mathbf{H}$ whose entries are independent and identically distributed circularly symmetric complex Gaussian random variables with zero mean and unit variance. The $M_r \times 1$ channel noise vector $\mathbf{n}$ is additive white Gaussian with zero mean and variance N_0. The precoder $\mathbf{F}$ is an $M_t \times M$ matrix with orthonormal columns, where $M = \min(M_r, M_t)$. The input vector $\mathbf{s}$ consists of symbols

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$s_0,\ s_1, \ldots,\ s_{M-1}$ that are uncorrelated, and zero mean. Let the number of bits loaded on s_k be b_k, then the number of bits transmitted per channel use is $R_b = \sum_{k=0}^{M-1} b_k$. The total transmission power $E[\mathbf{x}^\dagger \mathbf{x}]$ is P_t, where $\mathbf{x}$ is the transmitter output vector as indicated in Fig. 1. The channel output $\mathbf{r}$ is

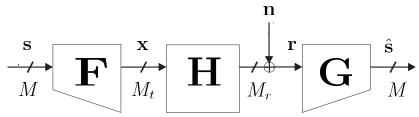

Fig. 1. The MIMO communication system.

given by $\mathbf{r} = \mathbf{HFs} + \mathbf{n}$. The error vector at the output of the $M \times M_r$ receive matrix $\mathbf{G}$ is $\mathbf{e} = \hat{\mathbf{s}} - \mathbf{s} = \mathbf{Gr} - \mathbf{s}$, where $\mathbf{G}$ is zero-forcing, given by $\mathbf{G} = (\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{HF})^{-1}\mathbf{F}^\dagger\mathbf{H}^\dagger$ [15]. The autocorrelation matrix of the error vector $\mathbf{R}_e = E[\mathbf{ee}^\dagger]$ is [15]

$$\mathbf{R}_e = N_0(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{HF})^{-1}. \tag{1}$$

Let the eigen decomposition of $\mathbf{H}^\dagger\mathbf{H}$ be $\mathbf{V}\begin{bmatrix}\mathbf{\Lambda} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\mathbf{V}^\dagger$, where the $M \times M$ diagonal matrix $\mathbf{\Lambda}$ contains the eigenvalues of $\mathbf{H}^\dagger\mathbf{H}$ in nonincreasing order, i.e., $\lambda_0 \geq \lambda_1 \geq \ldots \geq \lambda_{M-1}$, and $\mathbf{V}$ is an $M_t \times M_t$ unitary matrix. For a number of design criteria, e.g. minimization of transmission power [2][9][11][14], the optimal unitary precoder has been found to be $\mathbf{F} = \mathbf{V}_M$, where $\mathbf{V}_M$ is the $M_t \times M$ matrix obtained by keeping the first M columns of $\mathbf{V}$. With the above precoder, the kth error variance is given by

$$\sigma_{e_k}^2 = [\mathbf{R}_e]_{kk} = N_0\lambda_k^{-1}. \tag{2}$$

As $\{\lambda_k\}$ is in nonincreasing order, $\{\sigma_{e_k}^2\}$ is in nondecreasing order. Thus the optimal bit loading $\{b_k\}$ that minimizes the transmission power is in nonincreasing order [9].

In this paper, we consider the limited feedback of precoder and bit loading. At the receiver, the bit loading vector $\mathbf{b} = \begin{bmatrix} b_0 & b_1 & \ldots & b_{M-1}\end{bmatrix}$ and precoder matrix $\mathbf{F}$ are chosen from their respective codebooks and the indexes are sent back to the transmitter. Suppose B_b and B_f bits are used to represent $\mathbf{b}$ and $\mathbf{F}$, respectively, the total feedback rate is $B = B_b + B_f$. In the next section, we derive feedback rate allocation between bit loading and precoder.

3. ALLOCATION OF FEEDBACK RATE

In this section, we allocate the feedback rate between precoder and bit loading. We consider the increase in transmission power due to the quantization of precoder and bit loading with a high feedback rate assumption. First we analyze the power penalty when only the precoder is quantized. Then we derive the additional penalty when bit loading is also quantized. The results arw used to determine feedback rate allocation between precoder and bit loading.

3.1. Performance loss due to precoder quantization

When the precoder is a quantized version of $\mathbf{V}_M$, $\mathbf{F} = \widehat{\mathbf{V}}_M$, the error autocorrelation matrix in (1) is

$$\widehat{\mathbf{R}}_e = N_0(\mathbf{V}_M^\dagger\widehat{\mathbf{V}}_M)^{-1}\mathbf{\Lambda}^{-1}(\widehat{\mathbf{V}}_M^\dagger\mathbf{V}_M)^{-1},$$

where we have assumed that $\mathbf{H}$ has full rank. Let $\mathbf{v}_k$ and $\hat{\mathbf{v}}_k$ be respectively the kth column of $\mathbf{V}_M$ and $\widehat{\mathbf{V}}_M$. Express $\mathbf{V}_M^\dagger\widehat{\mathbf{V}}_M$ as $\mathbf{V}_M^\dagger\widehat{\mathbf{V}}_M = \mathbf{D}(\mathbf{I}_M + \mathbf{E})$, where $\mathbf{D}$ is a diagonal matrix with $[\mathbf{D}]_{kk} = \mathbf{v}_k^\dagger\hat{\mathbf{v}}_k$, $k = 0,\ 1, \ldots,\ M-1$, $\mathbf{I}_M$ is the $M \times M$ identity matrix, and the matrix $\mathbf{E}$ is given by $[\mathbf{E}]_{kj} = \mathbf{v}_k^\dagger\hat{\mathbf{v}}_j/\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k$ when $k \neq j$ and $[\mathbf{E}]_{kk} = 0$. When B_f is large and quantization error is small, $\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k \approx 1$ and $\mathbf{v}_k^\dagger\hat{\mathbf{v}}_j \approx 0$, and thus $[\mathbf{E}]_{kj} \approx 0$. It is known that [17] we can write $(\mathbf{I}_M + \mathbf{E})^{-1}$ as a power series in $\mathbf{E}$, i.e., $(\mathbf{I}_M + \mathbf{E})^{-1} = \sum_{j=0}^{\infty}(-1)^j\mathbf{E}^j$ when $\|\mathbf{E}\|_F < 1$, where $\|\mathbf{E}\|_F$ denotes the Frobenius norm of $\mathbf{E}$. As the elements of $\mathbf{E}$ are small, we have the approximation $(\mathbf{I}_M + \mathbf{E})^{-1} \approx \mathbf{I}_M - \mathbf{E}$. It follows that $(\mathbf{V}_M^\dagger\widehat{\mathbf{V}}_M)^{-1} = (\mathbf{I}_M + \mathbf{E})^{-1}\mathbf{D}^{-1} \approx (\mathbf{I}_M - \mathbf{E})\mathbf{D}^{-1}$. Thus, we have

$$\widehat{\mathbf{R}}_e \approx N_0(\mathbf{I}_M - \mathbf{E})\mathbf{D}^{-1}\mathbf{\Lambda}^{-1}\mathbf{D}^{-\dagger}(\mathbf{I}_M - \mathbf{E}^\dagger). \tag{3}$$

Notice that $\mathbf{D}^{-1}\mathbf{\Lambda}^{-1}\mathbf{D}^{-\dagger}$ is a diagonal matrix, and the diagonal elements of $\mathbf{E}$ are equal to zero, so $[\mathbf{ED}^{-1}\mathbf{\Lambda}^{-1}\mathbf{D}^{-\dagger}]_{kk} = [\mathbf{D}^{-1}\mathbf{\Lambda}^{-1}\mathbf{D}^{-\dagger}\mathbf{E}^\dagger]_{kk} = 0$. Therefore the kth subchannel error variance $\hat{\sigma}_{e_k}^2 = [\widehat{\mathbf{R}}_e]_{kk}$ in (3) can be written as

$$\hat{\sigma}_{e_k}^2 \approx N_0\left(|[\mathbf{D}]_{kk}|^{-2}\lambda_k^{-1} + \|\mathbf{1}_k^\dagger\mathbf{ED}^{-1}\mathbf{\Lambda}^{-1/2}\|^2\right), \tag{4}$$

where $\mathbf{1}_k$ is the kth standard vector with $[\mathbf{1}_k]_k = 1$ and $[\mathbf{1}_k]_j = 0$ when $j \neq k$ and $\|\mathbf{x}\|$ denotes the 2-norm of a vector $\mathbf{x}$. The jth element of $\mathbf{1}_k^\dagger\mathbf{ED}^{-1}\mathbf{\Lambda}^{-1/2}$ is equal to zero when $j = k$, and equal to $\lambda_j^{-1/2}\mathbf{v}_k^\dagger\hat{\mathbf{v}}_j/(\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k\mathbf{v}_j^\dagger\hat{\mathbf{v}}_j)$ when $j \neq k$, so $\|\mathbf{1}_k^\dagger\mathbf{ED}^{-1}\mathbf{\Lambda}^{-1/2}\|^2$ can be expressed as

$$\|\mathbf{1}_k^\dagger\mathbf{ED}^{-1}\mathbf{\Lambda}^{-1/2}\|^2 = \sum_{j=0, j\neq k}^{M-1}\left|\frac{\mathbf{v}_k^\dagger\hat{\mathbf{v}}_j}{\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k}\right|^2\frac{\lambda_j^{-1}}{|\mathbf{v}_j^\dagger\hat{\mathbf{v}}_j|^2}. \tag{5}$$

Substituting (5) to (4), we have

$$\hat{\sigma}_{e_k}^2 \approx N_0\lambda_k^{-1}|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k|^{-2}\left(1 + \sum_{j=0, j\neq k}^{M-1}|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_j|^2\frac{\lambda_j^{-1}\lambda_k}{|\mathbf{v}_j^\dagger\hat{\mathbf{v}}_j|^2}\right). \tag{6}$$

As $|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_j| \approx 0$, we have the approximation

$$\hat{\sigma}_{e_k}^2 \approx N_0\lambda_k^{-1}|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k|^{-2}. \tag{7}$$

To measure the increase in transmission power, note that the total transmission power for a given symbol error rate (SER) can be expressed as [9]

$$P_t = \Gamma\sum_{k=0}^{M-1}(2^{b_k} - 1)\sigma_{e_k}^2, \tag{8}$$

where $\Gamma = \frac{1}{3}Q^{-1}(\frac{SER}{4})^2$ and $Q(y) = \frac{1}{\sqrt{2\pi}}\int_y^{\infty} e^{-t^2/2}dt$, $y \geq 0$. When the transmission rate is large, $2^{b_k} - 1 \approx 2^{b_k}$, the total transmission power can be approximated as $P_t \approx \Gamma\sum_{k=0}^{M-1} 2^{b_k}\sigma_{e_k}^2$. In this case, it is shown in [9] that the optimal bit loading $\{b_k^*\}$ that minimizes the transmission power is given by

$$2^{b_k^*} = P_t^*/(M\Gamma\sigma_{e_k}^2), \tag{9}$$

where $P_t^* = M\Gamma 2^{R_b/M}\prod_{j=0}^{M-1}\sigma_{e_j}^{2/M}$ is the minimized transmission power. When the precoder is not quantized, $\mathbf{F} = \mathbf{V}_M$ and $\sigma_{e_k}^2 = N_0\lambda_k^{-1}$ is as given in (2), b_k^* in (9) satisfies $2^{b_k^*} = \lambda_k P_t^*/(N_0 M\Gamma)$. When the precoder is quantized, the required transmission power becomes $\widehat{P}_t \approx \Gamma\sum_{k=0}^{M-1} 2^{b_k^*}\hat{\sigma}_{e_k}^2$. Using (7), we have

$$\widehat{P}_t \approx \Gamma\sum_{k=0}^{M-1} 2^{b_k^*}N_0\lambda_k^{-1}|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k|^{-2} = \frac{P_t^*}{M}\sum_{k=0}^{M-1}|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k|^{-2}.$$

Therefore the transmission power is increased by a factor of $\widehat{P}_t/P_t^* \approx \frac{1}{M}\sum_{k=0}^{M-1}|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k|^{-2}$. We define the power penalty due to precoder quantization as

$$D_f = 10\log_{10} E\left[\frac{1}{M}\sum_{k=0}^{M-1}|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k|^{-2}\right]. \tag{10}$$

When the entries of the channel $\mathbf{H}$ are independent Gaussian with zero mean and unit variance, it is known that each $\mathbf{v}_k$ is uniformly distributed over the M_t-dimensional space $\{\mathbf{u} \in \mathbb{C}^{M_t} : \|\mathbf{u}\| = 1\}$, where $\mathbb{C}^{M_t}$ is the set of all complex vectors with M_t elements [18].

Lemma 1. *When $\mathbf{v}_k$ is quantized to $\hat{\mathbf{v}}_k$ using B_{v_k} bits, D_f is given by*

$$D_f \approx 10\log_{10}\left(\frac{1}{M}\sum_{k=0}^{M-1}\left(2^{B_{v_k}}(M_t-1)\left[\sum_{j=2}^{M_t-1}\left(\frac{2^{\frac{-(M_t-j)}{M_t-1}B_{v_k}}}{j-M_t}\right) - \ln\left(1-2^{\frac{-B_{v_k}}{M_t-1}}\right)\right]\right)\right). \tag{11}$$

Proof. When B_{v_k} bits is used to quantize $\mathbf{v}_k$ to $\hat{\mathbf{v}}_k$, the probability density function of $|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k|^2$ can be approximated as [13] $f_{|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k|^2}(x) \approx 2^{B_{v_k}}(M_t-1)(1-x)^{M_t-2}1_{[1-\epsilon_k,1)}(x)$, where $0 < x < 1$, $\epsilon_k = 2^{-B_{v_k}/(M_t-1)}$ and $1_{\mathcal{I}}(x)$ is the indicator function, which is equal to 1 if x in the interval $\mathcal{I}$ and zero otherwise. Using the above pdf approximation, it can be verified that $E[|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k|^{-2}] \approx 2^{B_{v_k}}(M_t-1)\int_{1-\epsilon_k}^{1} x^{-1}(1-x)^{M_t-2}dx$. Letting $y = 1-x$ and applying long division, we get $E[|\mathbf{v}_k^\dagger\hat{\mathbf{v}}_k|^{-2}] \approx 2^{B_{v_k}}(M_t-1)\int_{\epsilon_k}^{0}(y^{M_t-3}+y^{M_t-4}+\ldots+y+1)+\frac{1}{y-1}dy$. Evaluating the integrals, we get (11). ☐

3.2. Performance loss due to bit loading quantization

For a given quantized precoder, we can compute the subchannel error variances $\hat{\sigma}_{e_k}^2$, the optimal bit loading $\tilde{b}_k^*$ corresponding to $\hat{\sigma}_{e_k}^2$, and the minimum transmission power $\widetilde{P}_t^*$. Suppose now we quantize $\tilde{b}_k^*$ to $\hat{b}_k$, the required transmission power using the quantized bit loading $\widehat{P}_t \approx \Gamma\sum_{k=0}^{M-1} 2^{\hat{b}_k}\hat{\sigma}_{e_k}^2$ can be rewritten as

$$\widehat{P}_t \approx \Gamma\sum_{k=0}^{M-1} 2^{\tilde{b}_k^*}\hat{\sigma}_{e_k}^2 2^{(\hat{b}_k-\tilde{b}_k^*)} = \frac{\widetilde{P}_t^*}{M}\sum_{k=0}^{M-1} 2^{(\hat{b}_k-\tilde{b}_k^*)},$$

where we have used $2^{\tilde{b}_k^*} = \widetilde{P}_t^*/(M\Gamma\hat{\sigma}_{e_k}^2)$. Hence, the transmission power is increased by a factor of $\widehat{P}_t/\widetilde{P}_t^* = \frac{1}{M}\sum_{k=0}^{M-1} 2^{(\hat{b}_k-\tilde{b}_k^*)}$. We define the power penalty due to the quantization of bit loading as

$$D_b = 10\log_{10} E\left[\frac{1}{M}\sum_{k=0}^{M-1} 2^{(\hat{b}_k-\tilde{b}_k^*)}\right].$$

When the precoder is not quantized, the optimal bit loading $\{b_k^*\}$ is in nonincreasing order [9]. When B_f is large and the quantization error of the precoder is small, we can assume that the optimal bit loading $\{\tilde{b}_k^*\}$ is also in nonincreasing order. In this case, $\tilde{b}_k^*$ are bounded as given in the following lemma.

Lemma 2. *When a bit loading $\{b_k\}$ satisfies $\sum_{k=0}^{M-1} b_k = R_b$ and $b_0 \geq b_1 \geq \ldots \geq b_{M-1} \geq 0$, b_k is bounded by $b_{k,min} \leq b_k \leq b_{k,max}$, where*

$$\begin{aligned} b_{k,max} &= R_b/(k+1),\ k = 0,\ \ldots,\ M-1, \\ b_{0,min} &= R_b/M,\ b_{k,min} = 0,\ k = 1,\ \ldots,\ M-1. \end{aligned} \tag{12}$$

Proof. When $\{b_k\}$ is in nonincreasing order, $b_{0,max} = R_b$ and $b_{k,min} = 0$ for $1 \leq k \leq M-1$. The minimum value of b_0 occurs when R_b is uniformly distributed among $b_0, \ldots, b_{M-1}$, so $b_{0,min} = \frac{R_b}{M}$. Similarly, b_k is at its maximum value when $b_{k+1} = \ldots = b_{M-1} = 0$. Thus, $b_{k,max} = \frac{R_b}{k+1}$. ☐

Suppose $B_{b,k}$ bits are used for scalar quantization of $\tilde{b}_k^*$. With a moderate number of quantization bits $B_{b,k}$, it is reasonable to assume that the quantization error $\delta_k = \hat{b}_k - \tilde{b}_k^*$ has a uniform distribution over $(-\Delta_k/2, \Delta_k/2]$, where $\Delta_k = (b_{k,max} - b_{k,min})2^{-B_{b,k}}$ is the quantization step size [16]. In this case, D_b can be given in a closed form.

Lemma 3. *When the quantization error $\delta_k = \hat{b}_k - \tilde{b}_k^*$ is uniformly distributed over $(-\Delta_k/2, \Delta_k/2]$, D_b is given by*

$$D_b = 10\log_{10}\left(\frac{1}{M\ln 2}\sum_{k=0}^{M-1}\frac{1}{\Delta_k}\left[2^{\Delta_k/2} - 2^{-\Delta_k/2}\right]\right). \tag{13}$$

$$D_b + D_f \approx 10\log_{10}\left(\frac{1}{M\ln 2}\sum_{k=0}^{M-1}\frac{2^{B_{b,k}}}{\ell_k}\left[2^{\left(\frac{\ell_k}{2}2^{-B_{b,k}}\right)} - 2^{\left(\frac{-\ell_k}{2}2^{-B_{b,k}}\right)}\right]\right)+$$

$$10\log_{10}\left(2^{(B-B_b)/M}(M_t-1)\left[\sum_{j=2}^{M_t-1}\left(\frac{1}{j-M_t}2^{\frac{-(M_t-j)}{M_t-1}(B-B_b)/M}\right) - \ln\left(1-2^{\frac{-(B-B_b)/M}{M_t-1}}\right)\right]\right), \quad (14)$$

where $\ell_k = b_{k,max} - b_{k,min}$.

Proof. It can be verified that $E[2^{\delta_k}] = \frac{1}{\Delta_k \ln 2}[2^{\Delta_k/2} - 2^{-\Delta_k/2}]$. The expression of D_b in (13) follows.

Rate allocation. Starting from the optimal precoder $\mathbf{F} = \mathbf{V}_M$ and the optimal bit allocation, the performance is degraded by D_f(dB) when the precoder is quantized. When we further quantize the bit loading, there is an additional degradation of D_b(dB). Therefore we can minimize the power penalty by allocating the rate such that the combined penalty $D_f + D_b$ is minimized. From (10), we see that the quantization of each $\mathbf{v}_k$ contribute to D_f in the same manner, so we choose $B_{v_k} = B_f/M$ for $0 \le k \le M-1$. Substituting $B_{v_k} = (B - B_b)/M$ into (11), the objective function $D_b + D_f$ becomes (14) that is shown at the top of this page. For $0 \le B_b \le B$, we evaluate $D_f + D_b$ for all possible integer $\{B_{b,k}\}$ that satisfy $\sum_{k=0}^{M-1} B_{b,k} = B_b$ and choose the one that has the smallest combined power penalty.

4. SIMULATIONS

In the following examples, the elements of channel matrix $\mathbf{H}$ are independent complex Gaussian random variables with zero mean and unit variance, $M_r = M_t = 4$ and $R_b = 16$. We use the bit loading codebook designed in [8] and the randomly generated precoder codebook in [3]. For codeword selection, the bit error rate (BER) criterion is employed. We have used 10^5 channel realizations in the simulations. The power is equally divided among all symbols carrying nonzero bits.

Example 1. Feedback rate allocation. We demonstrate the importance of proper feedback rate allocation between precoder and bit loading in this example. For $B = 5$, the optimal rate allocation is $(B_f, B_b) = (3, 2)$ using the method in Sec. 3. In Fig. 2, we show the BER for all possible (B_f, B_b) such that $B_f + B_b = B$. We see that the rate allocation $(B_f, B_b) = (3, 2)$ gives the best performance. For example, at BER$= 10^{-5}$, $(3, 2)$ is better than $(B_f, B_b) = (1, 4)$ by around 2.5 dB. This demonstrates that the performance is sensitive to rate allocation; by moving two bits from B_f to B_b the performance can differ by 2.5 dB. In the case $(B_f, B_b) = (5, 0)$, all feedback bits are used for precoder feedback and the bit loading is a fixed vector. Two cases of fixed bit loading are shown, a nonuniform one $\begin{bmatrix}6 & 5 & 5 & 0\end{bmatrix}$ and a uniform one $\begin{bmatrix}4 & 4 & 4 & 4\end{bmatrix}$. The fixed nonuniform bit loading is obtained by using the generalized Lloyd algorithm in [8] with only one codeword; the performance is considerably better than that of uniform bit loading. Therefore the design of bit loading is particularly important when $B_b = 0$.

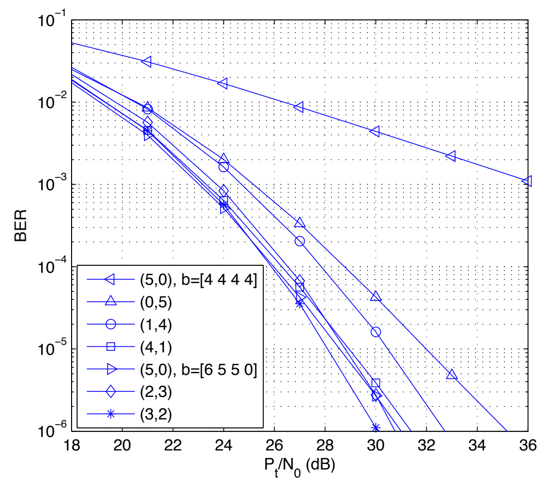

Fig. 2. Example 1. Bit error rate performance for all different feedback rate allocations when $B = 5$.

Example 2. BER comparison. In this example we show the BER of the proposed method and other limited feedback systems with a linear receiver for $B = 8$. The feedback rate allocation computed using (14) is $(B_f, B_b) = (5, 3)$. The two precoder systems [4][11] feed back the index of the precoder in the codebook. In [11], the precoder is quantized using sequential vector quantization (SVQ). For both [4] and [11], bits are uniformly loaded on all M substreams. In the multimode (MM) precoding system [5], the constellation on all substreams are the same, but the number of substreams transmitted can vary with the channel. The modified multimode precoding (modified MM) in [10] improves the performance of MM in [5] by introducing additional feedback of nonuniform bit loading. In [8], the feedback information is bit loading only; the precoder is allocated zero feedback bit. The results are shown in Fig. 3. The systems that allow the number of substreams to vary enjoy a better performance than

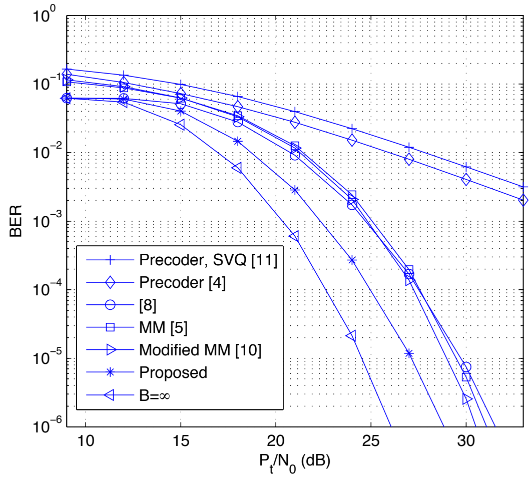

Fig. 3. Example 2. Comparisons of BER for systems with linear receivers for $B = 8$.

the two precoder systems that use uniform bit loading. At BER=10^{-4}, the gap between the proposed system and other systems is around 2.3 dB. By judicious allocation of feedback rate between precoder and bit loading, the proposed system can achieve a better performance. As a benchmark, the performance of the case $B = \infty$ is also shown, in which the precoder $\mathbf{F} = \mathbf{V}_M$, and the optimal integer bit loading is used. With 8 bits of feedback, the performance of the proposed system is around 2.4 dB away from that uses infinitely many feedback bits at BER=10^{-4}.

5. CONCLUSION

In this paper, we have developed a systematic approach to designing feedback rate allocation between precoder and bit loading. We have analyzed the power penalty due to quantization of precoding and bit loading. The analysis leads to an algorithm for finding the optimal feedback rate allocation that minimizes the power penalty.

6. REFERENCES

[1] D. J. Love, R. W. Heath, V. K. N. Lau, D. Gesbert, B. D. Rao, and M. Andrews, "An overview of limited feedback in wirelese communication systems," *IEEE J. Sel. Areas in Commun.*, vol. 26, no. 8, Oct. 2008.

[2] D. J. Love and R. W. Heath, "Limited feedback unitary precoding for spatial multiplexing systems," *IEEE Trans. Inf. Theory*, vol. 51, pp. 2967-2976, Aug. 2005.

[3] W. Santipach and M. L. Honig, "Capacity of a multiple-antenna fading channel with a quantized precoding matrix," *IEEE Trans. Inf. Theory*, vol. 55, no. 3, pp. 1218-1234, Mar. 2009.

[4] S. Zhou and B. Li, "BER criterion and codebook construction for finite-rate precoded spatial multiplexing with linear receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 5, pp. 1653-1665, May 2006.

[5] D. J. Love and R. W. Heath, "Multimode precoding for MIMO wireless systems," *IEEE Trans. Signal Process.*, vol. 53, no. 10, pp. 3674-3687, Oct. 2005.

[6] H. Zhuang, L. Dai, S. Zhou and Y. Yao, "Low complexity per-antenna rate and power control approach for closed-loop V-BLAST," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1783-1787, Nov. 2003.

[7] Y. Jiang and M. K. Varanasi, "Spatial multiplexing architectures with jointly designed ratetailoring and ordered BLAST decoding part-II: a practical method for rate and power allocation," *IEEE Trans. Wireless Commun.*, vol. 7, no. 8, pp. 3626-3271, Aug. 2008.

[8] Y.-P. Lin, H.-C. Chen and P. Jeng, "Bit allocation and statistical precoding for correlated MIMO channels with limited feedback," *IEEE Trans. on Vehicular Technology*, vol. 61, no. 2, pp. 597-606, Feb. 2012.

[9] C. C. Weng, C. Y. Chen, and P. P. Vaidyanathan, "MIMO transceivers with decision feedback and bit loading: theory and optimization," *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1334-1346, March 2010.

[10] M. Shin, S. Kim, J. Kang, and C. Lee, "An efficient multimode quantized precoding technique for MIMO wireless systems," *IEEE Trans. Vehicular Technology*, vol. 58, no. 2, pp. 733-743, Feb. 2009.

[11] J. C. Roh and B. D. Rao, "Efficient feedback methods for MIMO channels based on parameterization," *IEEE Trans. Wireless Commun.*, vol. 6, no. 1, pp. 282-292, Jan. 2007.

[12] J. Liu, D. Xu, R. Zhoa, L. Yang, and X. You, "Adaptive transmission for finite-rate feedback MIMO systems," *Proc. IEEE Wireless Communications and Networking Conference*, 2008.

[13] J. C. Roh and B. D. Rao, "Transmit beamforming in multiple-antenna systems with finite rate feedback: A VQ-based approach," *IEEE Trans. inf. Theory*, vol. 52, no. 3, pp. 1101-1112, Mar. 2006.

[14] S. Bergman, D.P. Palomar and B. Ottersten, "Joint Bit Allocation and Precoding for MIMO Systems With Decision Feedback Detection," *IEEE Trans. Signal Process.*, vol.57, no.11, pp.4509-4521, Nov. 2009

[15] P. P. Vaidyanathan, S. M. Phoong, and Y.-P. Lin, *Signal Processing and Optimization for Transceiver Systems*. Cambridge University Press, 2010.

[16] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression* . Norwell, MA: Kluwer Academic, 1992.

[17] R. Horn and C. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1985.

[18] A. Gupta and D. Nagar, *Matrix Variate Distributions*. London, U.K.:Chapman & Hall/CRC, 2000.