This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2008 proceedings.

COMPENSATION OF IQ IMBALANCE AND DC OFFSET FOR OFDM TRANSMISSION OVER FREQUENCY SELECTIVE CHANNELS

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ABSTRACT

In this paper a time-domain method for joint estimation of in-phase and quadrature-phase (IQ) imbalance, direct current (DC) offset and channel response for orthogonal frequency division multiplexing (OFDM) system is proposed. Using the fact that in an OFDM system the block size is usually much larger than the cyclic prefix length, we show that without knowing the channel impulse response, we are able to accurately estimate the IQ mismatch and DC offset, from which the channel response can be obtained. Only one OFDM block is needed for training. Simulation results demonstrate that the performance of the proposed method is very close to the ideal case when IQ mismatch, DC offset and channel response are perfectly known at the receiver.

1. INTRODUCTION

Recently the direct-conversion architecture has drawn a lot of attention as it is an attractive candidate for implementing many communication systems. Though it has a low cost, a direct-conversion receiver suffers from many nonidealities [1]. Two of the nonideal effects are the IQ imbalance and DC offset. The IQ imbalance is caused by the mismatch in the local oscillator and the DC offset is caused by the oscillator leakage. If these effects are not properly compensated at the receiver, the system can suffer from severe intersymbol interference (ISI) and the system performance degrades significantly [2].

In the literature, many methods have been proposed for the compensation of IQ imbalance and DC offset [2-7]. By carefully designing the OFDM training blocks, several methods for estimating the IQ imbalance were described in [2]. Based on the assumption that the channel frequency response is smooth, a frequency-domain method for jointly estimating the IQ imbalance and channel response was proposed in [3]. By using only one OFDM block for training, the authors are able to accurately estimate the IQ imbalance and it was demonstrated [3] that good BER performance is achieved. A pilot-based scheme for the compensation of both carrier offset and IQ imbalance is introduced in [4]. A blind compensation method for IQ imbalance based on a traditional adaptive interference canceler is proposed in [5]. The authors in [6] introduce an adaptive compensation of DC offset. A method for jointly estimating the IQ imbalance, DC offset and the channel response was introduced in [7]. By optimally designing the training block, both the IQ parameter and DC offset can be estimated accurately. The authors in [8] and [9] study the joint compensation of the transmitter and receiver IQ imbalance.

In this paper, we propose a new time-domain method for jointly estimating the IQ imbalance, DC offset and channel response for OFDM system. The method needs only one OFDM block for training and the training sequence can be arbitrarily chosen. Simulation results show that the proposed method can accurately estimate all the parameters and the BER performance is very close to the ideal case when all the parameters are known perfectly at the receiver. The paper is outlined as follows. Sec. 2 describes the effect of IQ mismatch and DC offset on the received signal and the OFDM system model. In Sec. 3, the proposed time-domain method is presented. Simulation results are given in Sec. 4 and conclusions are drawn in Sec. 5.

Notation: Boldfaced upper case and lower case letters represent vectors and matrices respectively. The matrix $\mathbf{A}^\dagger$ denotes transpose-conjugate of $\mathbf{A}$ and the vector $\mathbf{v}^*$ denotes the complex conjugate of $\mathbf{v}$.

2. SYSTEM MODEL

2.1. Effect of IQ Imbalance and DC Offset

Consider the transmission of a baseband sequence $x(n)$ through a channel with equivalent baseband channel response $h(n)$. In this paper, we assume that the channel is FIR with channel order L. In the absence of IQ imbalance and DC offset, the received baseband signal is given by

$$r(n) = \sum_{l=0}^{L} h(l)x(n-l) + q(n), \tag{1}$$

This work was supported in parts by National Science Council, Taiwan, ROC, under NSC 95-2752-E-002-006-PAE and NSC 96-2221-E-002-156

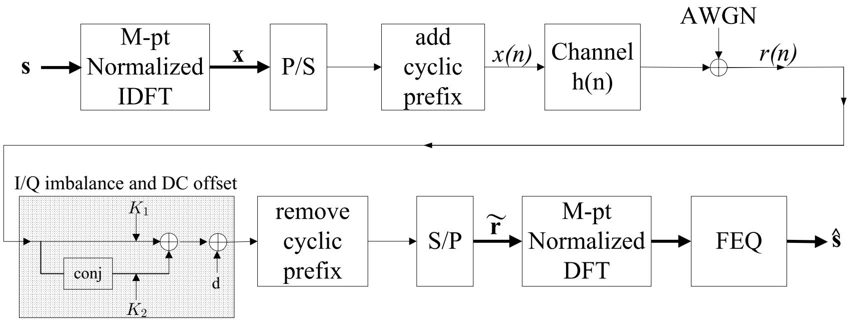

Figure 1: An OFDM system with receiver IQ imbalance and DC offset.

where $q(n)$ is the channel noise. Suppose now that there is IQ imbalance due to the mismatched local oscillator and let the oscillator signal be

$$Osc(t) \quad = \quad cos(2\pi f_c t) - j\epsilon sin(2\pi f_c t + \phi), \qquad (2)$$

where ϵ models the amplitude mismatch and ϕ models the phase mismatch. Define

$$K_1 = \frac{1 + \epsilon e^{-j\phi}}{2} \text{ and } K_2 = \frac{1 - \epsilon e^{j\phi}}{2}. \qquad (3)$$

Then it is known [5] [7] that in the presence of IQ imbalance and DC offset, the received baseband signal becomes

$$\widetilde{r}(n) = K_1 r(n) + K_2 r^*(n) + d, \qquad (4)$$

where d is the DC offset level and $r(n)$ is the desired baseband signal given in (1). When there is no mismatch, i.e. $\epsilon = 1$ and $\phi = 0^o$, then $K_1 = 1$ and $K_2 = 0$ and $\widetilde{r}(n)$ reduces to $r(n)$.

For convenience of discussions, we define the IQ parameter

$$\alpha = \frac{K_2}{{K_1}^*}. \qquad (5)$$

It can be verified that if α and d are known, we can recover a scaled version of $r(n)$ from $\widetilde{r}(n)$ by using the following expression:

$$K_1 r(n) = \frac{(\widetilde{r}(n) - d) - \alpha\,(\widetilde{r}^*(n) - d^*)}{1 - |\alpha|^2}. \qquad (6)$$

2.2. OFDM systems

In this paper, we study the compensation of IQ imbalance and DC offset for OFDM systems. Fig. 1 shows the block diagram of an OFDM system. The input $\mathbf{s}$ is an $M \times 1$ vector consisting of modulation symbols such as PAM, QAM, PSK, *etc.* After taking the M-point normalized IDFT of $\mathbf{s}$, the output vector $\mathbf{x}$ is first converted to a sequence and then a cyclic prefix (CP) of length L is added. The cyclic prefixed sequence $x(n)$ is then transmitted over the channel. In this paper, we assume that the CP length is equal to the channel order L. When there is no IQ mismatch and no DC offset, i.e. $K_1 = 1$, $K_2 = 0$ and $d = 0$, the $M \times 1$ received vector $\widetilde{\mathbf{r}}$ after CP removal (Fig. 1) reduces to $\mathbf{r}$ and it is given by

$$\mathbf{r} = \mathbf{H}_{circ}\mathbf{x} + \mathbf{q}, \qquad (7)$$

where $\mathbf{H}_{circ}$ is an $M \times M$ circulant matrix with the first column $[h(0)\ h(1)\ \cdots\ h(L)\ 0\ \cdots\ 0]^T$. In this case, it is well-known that zero-forcing equalization can be obtained by using simple one-tap frequency domain equalizers (FEQ).

However, in the presence of IQ imbalance and DC offset, the received vector $\widetilde{\mathbf{r}}$ is no longer equal to the vector $\mathbf{r}$ in (7). It is now given by

$$\widetilde{\mathbf{r}} \quad = \quad K_1\mathbf{r} + K_2\mathbf{r}^* + d\mathbf{1}, \qquad (8)$$

where the $M \times 1$ vector $\mathbf{1} = [\ 1 \quad 1 \quad \cdots \quad 1\ ]^T$ and $\mathbf{r}$ is as given in (7). If the IQ imbalance and DC offset are not properly compensated at the receiver, the orthogonality of subcarriers will be destroyed and this can result in severe inter-subcarrier interference [1][2]. Therefore it is important to estimate the IQ mismatch and DC offset and compensate their effect at the receiver. Our goal is to introduce a new method in Sec. 3 for jointly estimating the channel response, IQ imbalance and DC offset when only one known OFDM block $\mathbf{s}$ (or equivalently $\mathbf{x}$ in Fig. 1) is sent for channel estimation.

Suppose now that the IQ imbalance parameter α and DC offset d are known at the receiver. We can compensate these effects and obtain a scaled version of the desired baseband vector

$$\mathbf{r}_0 \stackrel{\triangle}{=} K_1\mathbf{r} = \frac{(\widetilde{\mathbf{r}} - d\mathbf{1}) - \alpha(\widetilde{\mathbf{r}}^* - d^*\mathbf{1})}{1 - |\alpha|^2}. \qquad (9)$$

One can apply DFT followed by FEQ for symbol recovery. As we will see later, the scale factor K_1 will be compensated by the FEQ because the estimated channel response will also be scaled by the same factor.

3. JOINT ESTIMATION OF IQ IMBALANCE, DC OFFSET AND CHANNEL RESPONSE

In practice, the block size M in OFDM systems is usually much larger than the CP length L so that the spectral efficiency is high. In this section we will assume that $M > L+1$. Below we will show how to exploit this fact to jointly estimate the IQ imbalance, DC offset and channel response. First let us assume that there is no IQ imbalance and DC offset. From earlier discussions, we know that the $M \times 1$ received vector after CP removal is given by $\mathbf{r}$ in (7), which can be rewritten as

$$\mathbf{r} \;=\; \mathbf{X}\mathbf{h} + \mathbf{q}, \tag{10}$$

where $\mathbf{X}$ is an $M \times M$ circulant matrix whose first column is $\mathbf{x}$ and the $M \times 1$ vector $\mathbf{h}$ is given by

$$\mathbf{h} \;=\; \left[\begin{array}{ccccccc} h(0) & h(1) & \cdots & h(L) & 0 & \cdots & 0 \end{array}\right]^T \tag{11}$$

Suppose that one OFDM block $\mathbf{x}$ is sent for channel estimation. Then from (10) we can get

$$\widehat{\mathbf{h}} = \mathbf{X}^{-1}\mathbf{r}. \tag{12}$$

The first $(L+1)$ entries of $\widehat{\mathbf{h}}$ will be the estimated channel response. When there is no noise, i.e., $\mathbf{q} = \mathbf{0}$, the bottom $(M-L-1)$ entries of $\widehat{\mathbf{h}}$ will be zero. Therefore for a moderate SNR, these $(M-L-1)$ entries will be small.

Now suppose that there are IQ mismatch and DC offset. In this case, we know that the vector $\mathbf{r}_0$ (which is equal to $K_1\mathbf{r}$) is related to the received vector $\widetilde{\mathbf{r}}$ as (9). Substituting (9) into (12), we will get

$$\widehat{\mathbf{h}}_0 = \mathbf{X}^{-1}\mathbf{r}_0 = \mathbf{X}^{-1}\frac{(\widetilde{\mathbf{r}} - d\mathbf{1}) - \alpha(\widetilde{\mathbf{r}}^* - d^*\mathbf{1})}{1-|\alpha|^2}. \tag{13}$$

Notice that α and d in the above expression are the parameters we would like to estimate. If α and d were known, the first $(L+1)$ entries of $\widehat{\mathbf{h}}_0$ will give an estimate of the channel response and the last $(M-L-1)$ entries of $\widehat{\mathbf{h}}_0$ will be small. Any error in the estimation of α and d will make the last $(M-L-1)$ entries of $\widehat{\mathbf{h}}_0$ larger. Thus one can estimate α and d by minimizing the energy of the last $(M-L-1)$ entries of $\widehat{\mathbf{h}}_0$. Below we will show that the estimates of α and d can be given in closed forms.

To simplify the problem, we make two assumptions : (a) the IQ parameter α satisfies $|\alpha|^2 \ll 1$, and (b) the second order term αd^* is negligible. With these assumptions, (13) can be expressed as

$$\widehat{\mathbf{h}}_0 \approx \mathbf{X}^{-1}\widetilde{\mathbf{r}} - \alpha\mathbf{X}^{-1}\widetilde{\mathbf{r}}^* - d\mathbf{X}^{-1}\mathbf{1}. \tag{14}$$

Define the matrix[1]

$$\mathbf{F} = \left[\begin{array}{cc} \mathbf{0}_{(M-L-1)\times(L+1)} & \mathbf{I}_{(M-L-1)\times(M-L-1)} \end{array}\right]. \tag{15}$$

Then pre-multiplying $\widehat{\mathbf{h}}_0$ by $\mathbf{F}$ will drop the first $(L+1)$ entries and retain only the last $(M-L-1)$ entries of $\widehat{\mathbf{h}}_0$. By doing so, we will have

$$\mathbf{F}\widehat{\mathbf{h}}_0 = \mathbf{b} - \mathbf{A}\left[\begin{array}{c} \alpha \\ d \end{array}\right], \tag{16}$$

where

$$\mathbf{b} = \mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}}, \quad \text{and} \quad \mathbf{A} = \left[\begin{array}{cc} \mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}}^* & \mathbf{F}\mathbf{X}^{-1}\mathbf{1} \end{array}\right]. \tag{17}$$

The problem of finding α and d to minimize $\|\mathbf{F}\widehat{\mathbf{h}}_0\|^2$ can be solved by using the least-squares method. The solution is given in closed form:

$$\left[\begin{array}{c} \widehat{\alpha} \\ \widehat{d} \end{array}\right] = (\mathbf{A}^\dagger\mathbf{A})^{-1}\mathbf{A}^\dagger\mathbf{b} \tag{18}$$

To get an estimate of the channel response, one can substitute $\widehat{\alpha}$ and $\widehat{d}$ from the above equation into (13) and the first $(L+1)$ entries of $\widehat{\mathbf{h}}_0$ will give a scaled version of the estimated channel response. Note that after compensation of IQ mismatch and DC offset, the received vector is given by $\mathbf{r}_0 = K_1\widetilde{\mathbf{r}}$ in (9). The scaled factor K_1 in $K_1\widetilde{\mathbf{r}}$ will be canceled by the FEQ as the estimated channel is also scaled by the same factor. As the proposed method estimates the parameters before the DFT operation, it is called a time-domain method. Some comments on the time-domain method are in order.

1. Only one OFDM input block $\mathbf{s}$ is needed for training.
2. There is no constraint on the training vector $\mathbf{s}$. It can contain any symbols, such as QPSK, PSK or QAM.
3. *Complexity:* The main computation of the algorithm is the calculation[2] of the vector $\mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}}$ (the vector $\mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}}^*$ can be obtained in the process of computing $\mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}}$). As $\mathbf{X}$ is fixed and known at the receiver, $\mathbf{F}\mathbf{X}^{-1}$ and $\mathbf{F}\mathbf{X}^{-1}\mathbf{1}$ can be pre-computed. Moreover as $\mathbf{A}$ has only two columns, $(\mathbf{A}^\dagger\mathbf{A})^{-1}$ is 2×2 and $(\mathbf{A}^\dagger\mathbf{A})^{-1}\mathbf{A}^\dagger\mathbf{b}$ can be obtained easily.

[1] As $M > L+1$, $\mathbf{F}$ is not a zero matrix.

[2] Let $\widetilde{\mathbf{r}}_1$ and $\widetilde{\mathbf{r}}_2$ be respectively the real and imaginary parts of $\widetilde{\mathbf{r}}$. Then the complexity of calculating $\mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}}_1$ and $\mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}}_2$ is equivalent to that of multiplying a complex vector by a complex matrix. One can easily obtain both $\mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}}$ and $\mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}}^*$ from the vectors $\mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}}_1$ and $\mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}}_2$ by using two additions.

4. *The two approximations:* In the formulation of (14), we make two assumptions: (a) $|\alpha|^2 \ll 1$, and (b) $\alpha d^* \approx 0$. The first assumption has little effect on the estimation accuracy because it simply scales the cost function by a factor. On the other hand, for a moderately large mismatch, omitting the term αd^* will affect the accuracy, especially when a very small estimation error is needed. Below a simple two-step approach is proposed to improve accuracy.

A two-step estimation method. When the second order term of αd^* is included, the estimated vector $\widehat{\mathbf{h}}_0$ can no longer be expressed as a linear function of α and d. To avoid a nonlinear problem, we propose a two-step procedure. In the first step, we obtain an initial estimate of IQ mismatch and DC offset using (18). Denote these estimates as $\widehat{\alpha}_0$ and $\widehat{d}_0$. Then in the second step, one can include the second-order term as follows

$$\widehat{\mathbf{h}}_0 = \mathbf{b}' - \mathbf{A}\begin{bmatrix} \alpha \\ d \end{bmatrix}, \tag{19}$$

where the vector $\mathbf{b}' = \mathbf{F}\mathbf{X}^{-1}\widetilde{\mathbf{r}} + \widehat{\alpha}_0\widehat{d}_0^*\mathbf{F}\mathbf{X}^{-1}\mathbf{1}$. An improved estimate of IQ parameter and DC offset is given by

$$\begin{bmatrix} \widehat{\alpha} \\ \widehat{d} \end{bmatrix} = (\mathbf{A}^\dagger\mathbf{A})^{-1}\mathbf{A}^\dagger\mathbf{b}', \tag{20}$$

Numerical simulation shows that the above expression gives a very accurate estimate of α and d. One can substitute $\widehat{\alpha}$ and $\widehat{d}$ into (13) to obtain the estimated channel response.

4. SIMULATION

We use Monte-Carlo experiments to verify the performance of the proposed method. The block size is $M = 64$ and the CP length is $L = 3$. The modulation symbols are QPSK for both training and data transmission. The channel order is $L = 3$ and the channel taps are i.i.d. complex Gaussian random variables with variance equal to 1/4 each. A total of 1000 random channels are generated. The channel noise is AWGN with power N_0. The SNR is defined as E_s/N_0 where E_s is the power of the QPSK symbols. The mean-squared errors (MSEs) for the estimation of the IQ parameter, the DC offset, and the channel response are respectively defined as:

$$\begin{aligned} MSE(\alpha) &= E\{|\widehat{\alpha}-\alpha|^2\}. \\ MSE(d) &= E\{|\widehat{d}-d|^2\}. \\ MSE(h) &= \frac{1}{L+1}\sum_{l=0}^{L} E\{|\widehat{h}(l)-h(l)|^2\}. \end{aligned}$$

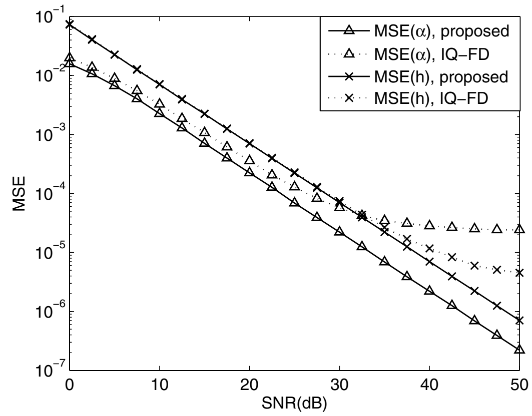

Figure 2: MSE performance (DC offset $d = 0$).

We compare the performance of our method with the IQ-FD method [3][3]. As DC offset is not considered in [3], we set $d = 0$ in the first experiment. The amplitude mismatch factor is $\epsilon = 1.1$, and the phase mismatch factor is $\phi = 10^o$. Because there is no DC offset, we do not need a two-step algorithm. Figures 2 and 3 show the MSE and BER results respectively. For the comparison, we also plot the BER of the case of no compensation where the IQ mismatch is not compensated and the ideal case where the IQ parameter and channel response are known perfectly at the receiver. From the figures, it is seen that both methods work well when SNR is moderate. When SNR is large, the IQ-FD method suffers from error flooring. There is no error flooring with the proposed method.

Suppose now that in addition to the IQ imbalance, there is also DC offset $d = 0.1+0.1j$. Fig. 4 shows the MSEs versus SNR. The estimation methods using (18) and (20) are referred to as the 'one-step' method and 'two-step' method respectively. From Fig. 4, we see that for the estimation of IQ parameter, the one-step and two-step methods have the same accuracy (the dotted and solid triangle curves overlap). For DC offset estimation, the figure shows that the MSE of the one-step method saturates at 2×10^{-4}. When SNR is larger than 20 dB, the two-step method gives a much better estimate. For channel estimation, the two-step method also enjoys a slightly better accuracy at high SNR (the dotted cross curve overlaps with the two triangle curves). Fig. 5 shows the BER. It is seen that the BER performance of the two-step method is very close to the ideal case where the IQ parameter, DC offset and channel response are known perfectly at the receiver. When SNR is smaller than 30 dB, the one-step method is as good as the ideal case and it deviates from the ideal case only when SNR > 30dB. This is due to the DC offset estimation error.

[3]The method proposed in [7] needs a training sequence that satisfying some orthogonality conditions. For $M = 64$, it is not easy to find such a sequence.

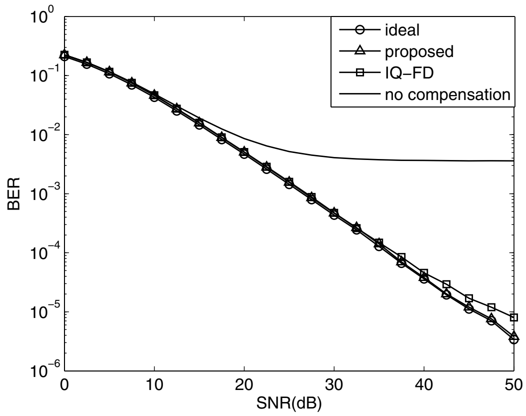

Figure 3: BER performance (DC offset $d = 0$).

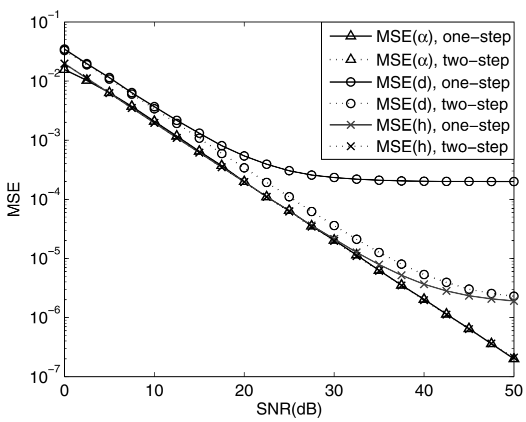

Figure 4: MSE performance.

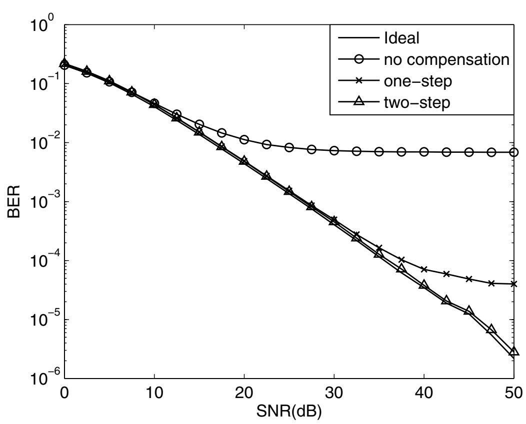

Figure 5: BER performance.

5. CONCLUSIONS

In this paper we propose a time-domain method for estimating the IQ mismatch factor, DC offset and channel response. Only one OFDM block is needed for estimating all the parameters. Moreover the training block can be arbitrarily chosen. Simulation results show that very accurate estimates and very good BER performance can be obtained by the proposed method.

6. REFERENCES

[1] B. Razavi, “Design Cosiderations for Direct-Conversion Recievers,” *IEEE Trans. Circuits and Systems*, Jun. 1997.

[2] A. Tarighat, E. Bahheri, A. H. Sayed, “Compensation schemes and performance analysis of IQ imbalances in OFDM receivers,” *IEEE Trans. Signal Process.*, Aug. 2005.

[3] J. Tubbax, B. Come, L. Van der Perre, S. Donnay, M. Engels, H. De Man, M. Moonen, “Compensation of IQ imbalance and phase noise in OFDM systems,” *IEEE Trans. Wireless Commun.*, May 2005.

[4] G. Xing, M. Shen, H. Liu, “Frequency offset and IQ imbalance compensation for direct-conversion receivers,” *IEEE Trans. Wireless Commun.*, Mar. 2005.

[5] M. Valkama, M. Renfors, V. Koivunen, “Advanced methods for IQ imbalnace compensation in communication receivers,” *IEEE Trans. Signal Process.*, Oct. 2001.

[6] J. K. Cavers, M. W. Liao, “Adaptive compesation for imbalnce and offset losses in direct conversion trancievers,” *IEEE Trans. Veh. Technol.*, Nov. 1993.

[7] I.-H. Sohn, E.-R. Jeong, Y.-H. Lee,“Data-aided approach to IQ mismatch and DC offset compensation in communication receivers,”*IEEE Communications Letters* Dec. 2002.

[8] A. Tarighat and A. H. Sayed,“Joint compensation of transmitter and receiver impairments in OFDM systems,” *IEEE Trans. Wireless Commun.,* pp. 240-247, Jan. 2007.

[9] D. Tandur and M. Moonen, “Joint adaptive compensation of transmitter and receiver I/Q imbalance under carrier frequency offset in OFDM based systems,” *IEEE Trans. Signal Process.,* accepted for publication 2007.