STATISTICS-AIDED CODEBOOK DESIGNS FOR LIMITED FEEDBACK BEAMFORMING SYSTEMS OVER MILLIMETER WAVE CHANNELS

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ABSTRACT

In this paper, we consider statistics-aided beamformer codebook designs for millimeter wave (mmWave) channels with limited feedback. The dimension of the beamformer, which scales with the number of transmit antenna, is typically large in mmWave systems and a large dimension adds to the difficulty of codebook designs. Two low-cost design methods are proposed. The first one, a VQ based approach, reduces the dimensionality of the codebook design problem by using training AoDs (angles of departure) instead of training channels. The second one, requiring no codebook training, designs the beamformer codebook from the statistics of the channel alone based on spectral analysis of the channel. Simulations are given to demonstrate the usefulness of the proposed methods.

1. INTRODUCTION

Beamforming with multiple transmit and receive antennas are known to achieve a beamforming gain that improves with the number of antennas. Recent advances show that it is feasible to use a large number of antennas, particularly in millimeter wave (mmWave) communication systems where the wavelength is small [1]. The large number of antennas increases the difficulty of feeding back channel information in applications where the transmitter does not have the channel information, e.g., frequency division duplex systems. As the size of the channel increases with the number of antennas, the feedback of the channel information becomes a more daunting task. To reduce the amount of feedback information, antenna response vectors are used as the column vectors of the RF precoders in [2] and the feedback of the RF precoder reduces to that of only the angles of antenna response vectors. Product codebook design is suggested in [3] for uniform planar arrays (UPA). A projection-based differential feedback scheme is proposed in [4] for massive MIMO. Feedback overhead is reduced in [5] through the grouping of the antennas and the feedback of the grouping patterns. Limited feedback for MIMO-OFDM beamforming systems is considered in [6]. A joint design of RF precoders and subcarrier baseband precoders is considered in [7] for MIMO-OFDM with limited feedback.

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In this paper, we consider codebook designs of beamformers for mmWave systems with limited feedback. The sparse nature of mmWave channels motivates two codebook designs that exploit the statistics of the channel. The first one uses antenna response vectors as beamforming vectors. Instead of using optimal beamformer as training data as in earlier VQ design for general MIMO channels, we use training AoDs (angles of departure) to reduce the dimensionality of the design problem. The second one, requiring no training, designs the beamformer codebook from the statistics of the channel alone. Using the commonly adopted Saleh-Valenzuela clustered model for mmWave channels, we decompose the channel into subchannels, each corresponding to a particular cluster. Codewords are then chosen from the eigenvectors of the subchannel covariance matrices to take advantage of the channel statistics. Simulations show that the incorporation of statistics in codebook designs considerably improves the performance.

Notations: The variance of a random variable x is denoted as σ_x^2. The 2-norm of a vector $\mathbf{f}$ is denoted as $||\mathbf{f}||$. The notation $\mathbf{A}^\dagger$ denotes the transpose and conjugate of a matrix $\mathbf{A}$. The expectation of a random variable x is denoted by $E[x]$.

2. SYSTEM MODEL

Consider a point-to-point beamforming system in which the transmitter is equipped with N_t antenna and the receiver with N_r antenna. The channel is typically modeled by an $N_r \times N_t$ matrix $\mathbf{H}$ and the noise vector $\mathbf{n}$ is additive white Gaussian with zero mean and variance N_0.

We consider the clustered channel model based on the extended Saleh-Valenzuela Model [8]. The channel matrix is assumed to be a sum of contributions from $N_{c\ell}$ scattering clusters, each of which contributes N_{ray} propagation paths.

In particular,

$$\mathbf{H} = \gamma \sum_{i=1}^{N_{c\ell}} \sum_{\ell=1}^{N_{ray}} \alpha_{i,\ell} \mathbf{a}_r(\phi^r_{i,\ell}, \theta^r_{i,\ell}) \mathbf{a}_t^\dagger(\phi^t_{i,\ell}, \theta^t_{i,\ell}), \quad (1)$$

where γ is a normalization factor given by $\gamma = \sqrt{\frac{N_t N_r}{N_{c\ell} N_{ray}}}$. The scalar $\alpha_{i\ell}$ is the complex gain of the ℓth ray in the ith cluster, assumed to be zero-mean Gaussian with variance $\sigma^2_{\alpha,i}$. $\phi^t_{\ell,i}$ ($\theta^t_{\ell,i}$) denotes the azimuth (elevation) angle of departure (AoD)[1] and $\phi^r_{\ell,i}$ ($\phi^r_{\ell,i}$) the azimuth (elevation) angle of arrival (AoA). The AoDs $\phi^t_{\ell,i}$ ($\theta^t_{\ell,i}$) are independent, of mean $\overline{\phi}^t_\ell$ ($\overline{\theta}^t_\ell$) and standard deviations (also called angular spread) $\sigma_{\phi^t_\ell}$ ($\sigma_{\theta^t_\ell}$). The complex gain $\alpha_{\ell,i}$, AoD and AoA are assumed to be independent. The means of the ith cluster angles $\bar{\phi}^t_i$ and $\bar{\theta}^t_i$ are uniformly distributed over $[0, 2\pi)$. The array response vector for an UPA arranged on the yz-plane with size $N_z \times N_y$ (N_z in the z-direction and N_y in the y-direction) is given by

$$[\mathbf{a}(\phi, \theta)]_{m+nN_z} = \frac{1}{\sqrt{N_y N_z}} e^{j\xi(m\cos(\theta) + n\sin(\theta)\sin(\phi))}, \quad (2)$$

for $0 \le m < N_z$ and $0 \le n < N_y$, where $\xi = 2\pi d$ and d is the antenna spacing normalized by the wavelength. When $N_y = 1$, the antenna array becomes a uniform linear array (ULA) along the z-axis. We assume the transmit antenna array is $N_{t,z} \times N_{t,y}$.

The transmit symbol s has zero mean and variance P_t, where P_t is the transmission power. The output of the receiver is given by $y = \mathbf{g}^\dagger \mathbf{H} \mathbf{f} s + \mathbf{g}^\dagger \mathbf{n}$, where $\mathbf{f}$ is the $N_t \times 1$ beamformer at the transmitter and $\mathbf{g}$ the $N_r \times 1$ combiner at the receiver. Both $\mathbf{f}$ and $\mathbf{g}$ are normalized so that $||\mathbf{f}|| = ||\mathbf{g}|| = 1$. When maximal ratio combiner is used, i.e., $\mathbf{g} = \mathbf{H}\mathbf{f}/||\mathbf{H}\mathbf{f}||$, the SNR at the receiver output is $\frac{P_t}{N_0}||\mathbf{H}\mathbf{f}||^2$, where the term $||\mathbf{H}\mathbf{f}||^2$ corresponds to the beamforming gain.

3. VQ CODEBOOK DESIGN WITH LOW TRAINING COST

VQ codebook design method has been developed for general MIMO channels in [9]. Training channels are generated and SVD (singular value decomposition) are computed to find the optimal precoder for each training channels. Then the precoders obtained from the training channels are used in LBG algorithm, which iterates between the nearest neighbour and centroid conditions to generate the codebook. In such a design, SVD needs to be computed for each training channel whose dimensions scale with the number of antennas. In the following, we consider a design that is also VQ based but with much lower design cost and there is no need of SVD computations. As in [2] and [6] we use the antenna response vectors as beamforming vectors, but the beamformers are designed by exploiting the statistics of the channels.

Observe that the antenna response vector in (2) can be completely determined by the pair (α, β), where $\alpha = \cos(\theta)$ and $\beta = \sin(\theta)\sin(\phi)$. We will use such pairs as codewords. (We can also use (ϕ, θ) as codewords, which is not adopted due to reason to be explained later.) Let N_c be the number of codewords in the codebook and the codebook $\mathcal{C}_b$ has N_c pairs of (α, β). Each pair leads to a beamforming vector $\mathbf{v}$ given by

$$[\mathbf{v}(\alpha, \beta)]_{m+nN_z} = \frac{1}{\sqrt{N_y N_z}} e^{j\xi(m\alpha + n\beta)}, \quad (3)$$

for $0 \le m < N_z$ and $0 \le n < N_y$. Given the statistics of the channels, we generate samples of AoD and use the samples to form training vectors $\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_M$, where M is the number of training vectors. The ith training vector $\mathbf{p}_i$ is of the form $\begin{bmatrix}\cos(x_i) & \sin(x_i))\sin(y_i)\end{bmatrix}^T$, where x_i and y_i are the sample AoDs in azimuth and in elevation.

Given a set of training vectors, we apply the LBG algorithm [10] using the nearest neighbour and the centroid conditions to obtain a codebook of N_c codewords. First we initialize the codewords $\mathbf{w}_1, \mathbf{w}_2, \cdots, \mathbf{w}_{N_c}$.

1. Nearest neighbour condition: For a given training set $\mathcal{S} = \{\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_M\}$, we partition the set into N_c cells $\{\mathcal{R}_i\}_{i=1}^{N_c}$, where $\mathcal{R}_i$ is the ith Voronoi region given by

$$\mathcal{R}_i = \{\mathbf{p} \in \mathcal{S} : \|\mathbf{p} - \mathbf{w}_i\| \le \|\mathbf{p} - \mathbf{w}_j\|, \ \forall j \ne i\}.$$

2. Centroid condition: For a given partition $\{\mathcal{R}_i\}_{i=1}^{N_c}$, we compute the centroid of each Voronoi region,

$$\mathbf{w}_i = \frac{1}{|\mathcal{R}_i|} \sum_{j=1, \mathbf{p_j} \in \mathcal{R}_i}^{M} \mathbf{p}_j,$$

where $|\mathcal{R}_i|$ denotes the number of vectors in $\mathcal{R}_i$.

The LBG algorithm iterates between the above two conditions until the average error $\frac{1}{M}\sum_{i=0}^{N_c}\sum_{j=1,\mathbf{p}_j\in\mathcal{R}_i}^{M}||\mathbf{p}_j - \mathbf{w}_i||^2$ converges. The dimension of the vectors in the codebook is only 2. The training data can be easily generated and the LBG algorithm can be carried out with low complexity. Compared to the VQ method that works with general MIMO channels [9], there is no need of generating training channels that may be of large dimensions and no need of computing their SVDs. Having obtained the codebook, the actual beamforming codewords are antenna response vectors corresponding to the codewords in $\mathcal{C}_b$. In the above algorithm, we have chosen codewords in $\mathcal{C}_b$ to be of the form $\begin{bmatrix}\cos(\theta) & \sin(\theta)\sin(\phi)\end{bmatrix}^T$ instead of $\begin{bmatrix}\phi & \theta\end{bmatrix}^T$. That is because what really matters in determining the beamformer is

[1] The elevation angle of a ray is the angle between the ray and the z-axis whereas the azimuth angle is the angle between the x-axis and the orthogonal projection of the ray on the xy-plane.

the pair $(\cos(\theta), \sin(\theta)\sin(\phi))$. Applying LBG on (ϕ, θ) may result in two codewords that yield nearly the same antenna response vectors though the two codewords are reasonably separated. Notice that the above algorithm can also be used for the ULA case. For ULA, the dimension of the the training data is further reduced to one, i.e., $p_i = \cos(x_i)$, and the codewords are of dimension one.

4. CODEBOOK DESIGN WITHOUT TRAINING

In this section, we propose a codebook design method that requires no training and the codebook can be generated from the statistics of the channel alone.

In [11], the channel is decomposed into subchannels, each one corresponding to a cluster of rays. It is shown therein that the eigenvectors of the subchannel covariance matrices yield a good representation of the optimal statistical beamformer. This motivates us to use the eigenvectors of the subchannel covariance matrices as codewords.

Let us express the channel in (1) in a matrix form as

$$\mathbf{H} = \sum_{\ell=1}^{N_{c\ell}} \mathbf{A}_{r,\ell} \mathbf{D}_\ell \mathbf{A}_{t,\ell}^\dagger \tag{4}$$

where $\mathbf{A}_{t,\ell}$ is the $N_t \times N_{ray}$ matrix whose column vectors are the transmit antenna response vectors corresponding to the ℓth cluster, $\mathbf{a}_t(\phi_{\ell,1}^t, \theta_{\ell,1}^t), \ldots, \mathbf{a}_t(\phi_{\ell,N_{ray}}^t, \theta_{\ell,N_{ray}}^t)$, and $\mathbf{A}_{r,\ell}$ the $N_r \times N_{ray}$ matrix whose column vectors are the receive antenna response vectors. The matrix $\mathbf{D}_\ell$, of size $N_{ray} \times N_{ray}$, is a diagonal matrix with $[\mathbf{D}_\ell]_{ii} = \alpha_{\ell,i}$. Using the properties that the AoDs and the path gains $\alpha_{\ell,i}$ are independent, we can verify that $E[\mathbf{H}^\dagger \mathbf{H}] = \sum_{\ell=1}^{N_{cl}} \mathbf{C}_\ell$, where

$$\mathbf{C}_\ell = E[\sigma_{\alpha_\ell}^2 \mathbf{A}_{t,\ell} \mathbf{A}_{t,\ell}^\dagger]. \tag{5}$$

Let the eigen decomposition of $\mathbf{C}_\ell$ be

$$\mathbf{C}_\ell = \mathbf{V}_\ell \mathbf{\Lambda}_\ell \mathbf{V}_\ell^\dagger \tag{6}$$

where the diagonal elements of $\mathbf{\Lambda}_\ell$ are in non increasing order, $\lambda_{\ell,1} \geq \lambda_{\ell,2} \geq \ldots \geq \lambda_{\ell,N_t}$. The i-th eigenvector $\mathbf{v}_{\ell,i}$ of $\mathbf{C}_\ell$ is associated with eigenvalue $\lambda_{\ell,i}$. Collecting the eigenvectors from all the clusters together, we have a set $\mathcal{P}$ of $N_{cl}N_t$ potential codewords. To choose codewords from these vectors, let us consider the average beamforming gain $E[||\mathbf{Hf}||^2]$. Notice that the average gain $E[\mathbf{f}^\dagger \mathbf{H}^\dagger \mathbf{H} \mathbf{f}] \geq \mathbf{f}^\dagger \mathbf{C}_\ell \mathbf{f}$, which is equal to $\lambda_{\ell,i}$ when $\mathbf{f} = \mathbf{v}_{\ell,i}$. Therefore, choosing $\mathbf{v}_{\ell,i}$ as the beamformer yields a beamforming gain of at least $\lambda_{\ell,i}$. Using the average beamforming gain as a criterion, we choose the N_c eigenvectors from $\mathcal{P}$ that are associated with the N_c largest values of $\lambda_{\ell,i}$.

The beamforming codewords obtained from the eigenvectors do not have the unit modulus property [2]. Such beamformers can be implemented using the two-phase shifters-per-coefficient (THIC) structure [12]. Or the phase part of the vectors can be extracted [2] so that the beamformers can be implemented using a single phase shifter for each coefficient.

5. SIMULATIONS

In this section, we evaluate the performance of the two proposed codebook design methods, VQ with low training cost (termed simplified VQ) and eigenvector selection method that requires no training. The beamformer is chosen from a codebook that is known to both the transmitter and the receiver. Maximal ratio combining is used at the receiver. Consider the Saleh-Valenzuela channel model in (1) with two clusters $N_{cl} = 2$ and $N_{ray} = 4$ rays per cluster. We assume the AoDs and AoAs are of truncated Laplacian distribution [13] and the angle spreads in elevation and azimuth are equal to $7.5°$ at both the transmitter and receiver. The antenna spacing is half wave length.

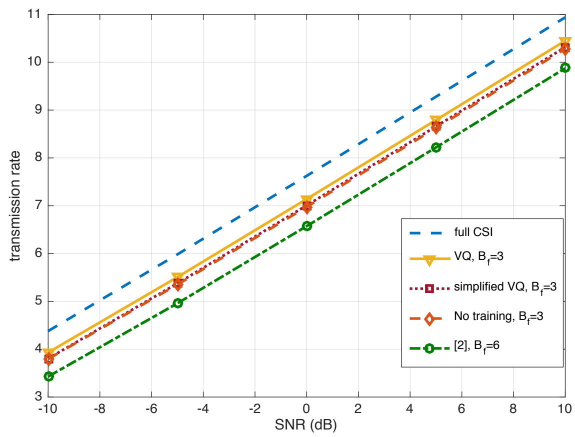

Figure 1: Transmission rate v.s. SNR (P_t/N_0).

Fig. 1 shows the average transmission rate for the two proposed designs of beamformer codebooks, 'simplified VQ' in Sec. 3 and 'no training' in Sec. IV, when $(N_t, N_r) = (16, 16)$. We have also shown the average transmission rate when the codebook is designed using VQ method developed for general MIMO channels [9]. It is labeled as 'VQ'. The 'full CSI' curve corresponds to the performance of the optimal beamformer when full channel state information is available to the transmitter. We can see that with 3 feedback bits, i.e, 8 codewords, the ''simplified VQ and the 'no training' methods behave similarly. They are around 0.7 bits away from the optimal beamformer and 0.2 bits worse than the VQ. For comparison, we have also shown the transmission rate of the limited feedback system in [2], which also employs antenna response vectors as beamforming codewords, with six feedback bits. We see that, with the aid of statistics, we can design codebooks that are better tailored to the channel statistics and fewer feedback bits are needed for comparable performances.

Fig. 2 show the transmission rate as a function of the number of transmitter antenna N_t when N_r is fixed at 16 for all the cases considered in Fig. 1. The difference between 'VQ' and 'no training' stays around 0.2 bits for different values of N_t. We also see that, as N_t increases, the 'full CSI' curves improves more than the systems with limited feedback. For the limited feedback systems in Fig. 2, the number of codewords is fixed. The average feedback bits per beamforming coefficient decreases with N_t, which restricts the achievable beamforming gain.

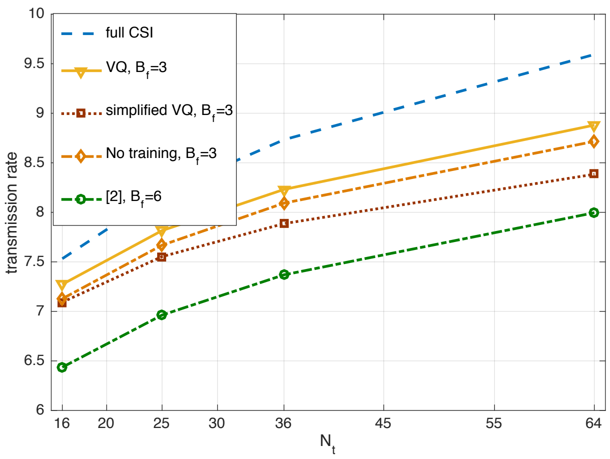

Figure 2: Transmission rate v.s. N_t (the number of transmit antennas).

6. CONCLUSION

In this paper, we consider codebook designs of beamforming mmWave systems with limited feedback. We proposed two methods that incorporate the channel statistics in the design. The first one is a VQ based approach that uses AoDs as training vectors and thus significantly reduces the dimension of training data. For the second one, the codewords are eigenvectors of appropriately defined subchannel covariance matrices and the design does not require codebook training. Simulation results show that the exploitation of channel statistics considerably improves the performance due to the sparse nature of the mmWave channels.

7. REFERENCES

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